Housing Price and Fundamentals in A Transition Economy: The Case of Beijing Market

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Abstract

This paper develops a dynamic rational expectations general equilibrium framework that links house value to fundamental economic variables such as income growth, demographics, migration and land supply. Our framework is general and can handle non-stationary dynamics as well as structural changes in fundamentals. Applying the framework to Beijing, we find that the rational equilibrium housing price is substantially lower than the data under reasonable parameterizations of the model, unless the income level is seriously understated. We can rationalize the current housing price in Beijing in an extension of our model with endogenous migration of rich households from outside Beijing. Our model is able to capture the long-run historical trends of house price and rent in San Francisco.

JEL Classification: E20, E30, R10, R13, R21, R31

Keywords: housing price, fundamentals, transition economy, dynamic equilibrium, life-cycle overlapping generations model

1 Introduction

This paper develops a dynamic rational expectations general equilibrium framework to formalize the link between housing price and macroeconomic fundamentals such as income growth, demographics, migration and land supply. We provide a structural estimate of the rational fundamental value of house price and rent. Our model is general and flexible enough to handle non-stationary dynamics as well as structural changes in the fundamentals often encountered in a transtion economy.[1] The historical relation between prices and fundamentals is unlikely to repeat itself during the transition periods. Thus, a reduced-form estimation of the price-fundamental relation that implicitly relies on the stationarity assumption would lead to misleading results. Our equilibrium approach circumvents this problem by deriving such relation endogeneously and dynamically along the transition path.

We apply the model to the Beijing housing market and examine to what extent its prices can be rationalized by the evolution of economic fundamentals.[2] This is a challenging task due to the rapid changes of the economy and the strong non-stationarity of both housing price and economic fundamentals. During his visit to China in April 2014, Nobel Prize winner Robert Shiller told the media that "China is in such a rapid growth period. It is very hard to price assets when growth is at the high level. The future matters more. In a stable economy that is not going anywhere, you have a pretty good idea of what they are worth."

Our model considers a period of economic transition followed by a balanced growth path (BGP) of the economy. We assume that income, land supply and population grow at constant rates after the economic transition is completed. We derive closed-form solutions for house price and rent in BGP and show that house price, rent and housing supply all grow at constant rates during this phase of the economy. Then we use the equilibruim quantities in BGP as the terminal conditions and use backward inductions to solve for the trajectories of equilibrium house prices and rents during the transition phase.[3]

In our analysis, we focus on the effects of long-term trends (low-frequency movements)

[1] Throughout this paper, we refer an economy such as China that is undergoing structural transformations as a transition economy.

[2] Jim Chanos, the founder and president of the hedge fund Kynikos, has repeatedly asserted that China is in the midst of the biggest real estate bubble in human history.

[3] The transition period lasts about 100 years in the calibration of our model to the Chinese data. We develop an efficient numerical method to compute the trajectories of equilibrium house prices and rents during this period.

in macroeconomic fundamentals and ignore cyclical movements in variables such as productivity, mortgage rate and unemployment. Thus our work differs from and complements a number of recent studies that link house prices to the cyclical movements of fundamentals.[4] Due to the absence of cyclicality, housing price risks do not exist in the model. This may either overstate or understate the return of housing investment, because housing returns can vary positively with risk under some circumstances, but negatively in others (Han (2013)). On the other hand our model admits two types of idiosyncratic risks: household's income shocks and medical expense shocks. These shocks lead to precautionary saving in the form of housing investment, which is found to be important (Chen (2010), Gan (2010) and Iacoviello and Pavan (2013)).

We choose Beijing as a leading example of developing markets. It has witnessed rapid yet declining growth of income, as well as large influx of young immigrants. These factors are often used to justify, without quantitative analysis, the unusually high level and growth rate of Beijing housing price. We also study the San Francisco market as a further validation of our model.

Our analyses produce interesting findings. First, the equilibrium rational house price in Beijing that can be justified by the fundamentals is around 15,000 RMB per square meter in 2014, which is much lower than the market price of about 30,000 RMB in the data. Alternative assumptions on land supply, income growth and population structure do not help much in narrowing the gap between the model-implied house price and the observed price in the data. Second, the current high price-rent ratio and price-income ratio in Beijing are consistent with an extended version of the model where rich households in other cities optimally choose to migrate to Beijing. Alternatively, if the average income of house buyers in Beijing is severely understated so that their actual average income is 2.5 times that reported by National Bureau of Statistics (or about the same level as in Hong Kong), then model-implied rational house price in 2014 is again consistent with the data. Third, price-income ratio declines over time as income growth slows down and the economy converges to BGP. Thus housing will become more affordable, despite the limited land supply and influx of young workers that keep up the housing demand. Fourth, our model explains well the long-run trends of house price and rent in developed markets such as San Francisco.

[4] See Chu (2014), Diaz and Jerez (2013), Garriga, Tang, and Wang (2014), Head, Lloyd-Ellis, and Sun (2014), Liu, Wang, and Zha (2013), and Sommer, Sullivan, and Verbrugge (2013).

Our results highlight that high price-income and price-rent ratios in themselves are not necessarily indicative of a price bubble in a transition economy. High ratios may be consistent with the evolution of economic fundamentals and converge to those found in developed markets as the economy matures. It is also misleading to compare price-income and price-rent ratios over time or across countries because these ratios are evidently not stationary during the economic transitions.

This paper is organized as follows. Section 2 reviews the related literature and highlights our contributions. Section 3 presents the model and discusses properties of the economy in the BGP. Section 4 specifies the dynamics of exogenous fundamental variables and calibrates the model using the Beijing housing market data. Section 5 reports quantitative results for the model equilibrium outcomes such as price and rent under the baseline scenario, a variety of robustness checks as well as an extended version of the basic model. Section 6 applies our model to the San Francisco market. Section 7 concludes the paper.

2 Literature Review

Empirical studies have shown that house prices can exhibit prolonged period of deviation from the fundamentals, but in the long run house prices tend to return to the fundamental values progressively and their growth rates can be explained by the fundamentals to a large extent.[5] While the empirical analysis has gained tremendous insights into the correlation between house price and fundamentals, the structural model in this paper is able to articulate their dynamic links. An important advantage of our framework is that it is general enough to deal with emerging markets where fundamentals can be non-stationary and rapidly changing, and the price-fundamentals relation is unstable. In addition, our model is amenable to calibration and quantitative analysis. In particular, as a general equilibrium model, it is a useful tool for policy analysis.

This paper is related to the emerging literature that study the structural link between house price and fundamentals. Using a dynamic general equilibrium model, Sommer, Sullivan, and Verbrugge (2013) and Chu (2014) find that a large fraction of the observed house

[5]See, e.g., Capozza, Hendershott, Mack, and Mayer (2002), Ambrose, Eichholtz, and Lindenthal (2013), Glaeser and Gyourko (2005), Himmelberg, Mayer, and Sinai (2005), Hott and Monnin (2009), Poterba, Weil, and Shiller (1991) and Shiller (2003)

price increase in the decades since 1995 can be explained by the changes in credit constraints, low interest rates and growing income. Introducing search and match friction into the housing market, Diaz and Jerez (2013) reproduce the cyclical time series properties of house prices and the comovement of prices with number of sales and time on the market. Head, Lloyd-Ellis, and Sun (2014) use a dynamic search model to understand the short-run dynamics of average house prices, home sales, construction and population growth. Head and Lloyd-Ellis (2015) study the valuation of owned versus rental housing under a user-cost model in which house prices equate the costs or renting and owning for the marginal buyer. They focus on the relation between price-rent ratio and real interest rate under various assumptions regarding expected future interest rates. The innovation in this paper lies in its focus on persistent fundamentals and long-run trends of house price and rent, in a non-stationary environment with rapid changing income, population size and age structure.

Our modelling strategy resembles Kiyotaki, Michaelides, and Nikolov (2011) in that we have a representative firm that issues equity to finance land purchase and new capital, produces houses taking land and capital as inputs. This enable us to endogenize housing supply, which is critical in a transition economy. Rather than studying the perturbation of the economy around the balanced growth path as in Kiyotaki, Michaelides, and Nikolov (2011), we focus on house price and rent dynamics during the economic transition phase.

Our paper is also related to Mankiw and Weil (1989) and voluminous ensuing empirical studies on the link between demography and house price. An important difference is that Mankiw and Weil (1989) ignore housing supply. Our model takes two dimensions of population structure into account: population size and age distribution. The latter is important because housing demand has a clear life-cycle profile, as shown in Yang (2009), but the effect of population age distribution has been largely ignored in the previous studies. Our calibration exercise shows that population structure indeed has a large impact on house price and rental rate.

Different from our approach, several papers study the housing fundamental value using Lucas type asset-pricing model (e.g., Piazzesi, Schneider, and Tuzel (2007) and Han (2013)). These studies emphasize the risk-return trade-off of housing investment which we abstract away from. Another approach is based on the present value model (e.g., Campbell and Shiller (1988), Campbell, Davis, Gallin, and Martin (2006), Brunnermeier and Julliard (2008), Ambrose, Eichholtz, and Lindenthal (2013)) where interest rates, dividends or rents are used as drivers of house prices. In contrast, the economic determinants of housing price in our model are more fundamental variables such as income, demographics, land supply, urbanization, and medical expense. In particular, we derive rents as part of the equilibrium. Morec recently, Giglio, Maggiori, and Stroebel (2014) test the existence of housing bubble (in UK and Singapore) by taking advantage of the co-exsitence of two forms of residential property ownership: leaseholds and freeholds, and test directly for the failures of the transversality condition.

In a contemporaneous related work, Garriga, Tang, and Wang (2014) (GTW) examines a dynamic general equilibrium rational expectations model with endogenous rural-urban migration decision. They show that the quick rise of house price between 1998-2007 in China can be largely explained by the process of relocating workers to cities combined with the typical stages of economic development. GTW takes urbanization rate and ratio of residential land to urban land from the US as the terminal conditions that are necessary for solving the dynamic rational expectations model, while we take the price-income and price-rent ratios for the economy in balanced growth path as the terminal conditions. GTW focuses on the growth rate of price during 1998-2007. By contrast, our paper studies both the level and growth of house price since 2005.

Our model differs from GTW in several important aspects and the two papers are complementary. First, we use a life-cycle overlapping-generations model rather than an infinite horizon model as in GTW. This allows us to study the impact of the population age distribution on housing price, which can not be studied in the framework of GTW. Second, in GTW housing is modelled as pure consumption, thus the investment incentive is absent. We model housing as both a consumption good and an investment, thus the expected return to housing investment is a key driver of housing demand. Third, GTW model studies only the extensive margin of the rising housing demand – migrants moving from rural areas to cities. In contrast, housing demand in our model is determined by both extensive margin and intensive margin (home owners adjusting housing demand in response to changes in fundamentals). Fourth, asset accumulation and inter-temporal substitution are absent in GTW, but they play important roles in our model.

Our paper contributes to a growing literature that studies housing market in China. Wu, Gyourko, and Deng (2012) provide an empirical assessment of the housing market in Beijing and seven other cities, showing that high expected house price appreciation is needed to justify the low rental return, which is an equilibrium outcome in our structural model. Chen and Wen (2014) use a model with rational bubble to understand the high growth rate of house price in China. Fang, Gu, Xiong, and Zhou (2015) offer a comprehensive overview of Chinese housing market. In particular, their paper constructs house price indices for major cities in China based on sequential sales of new homes within the same housing developments, and shows that Beijing house price level has increased 660% from 2003 to 2013.

Our model abstracts away from the mortgage market. One feature of the housing market in China is high downpayment, which is traditionally 30% for the first (primary) residence and 50% for the second unit.[6] Mortgage refinancing does not exist in China. The absence of a mortgage market in our model understates housing investment demand of young households because they can not borrow. However, it also overstates housing investment demand of middle-age and old households because precautionary saving tends to increase with more stringent credit constraint (e.g., see Carroll and Kimball (2001) and Lee and Sawada (2010)). The net effect of credit constraints on house price in our model is ambiguous and likely small. In fact, Glaeser, Gottlieb, and Gyourko (2010) find no convincing evidence that cheap credit can explain the bulk of the changes in house prices in U.S. Although credit constraints play an important role explaining the cyclical movement of housing prices, our framework is not designed to address the cyclical aspects of housing markets.[7] Finally, our model is already complex to solve without the mortgage market.

3 Model

In the economy there exist overlapping generations of households and a long-lived representative firm. The households receive exogenous income which is directly consumable, and purchase housing service from the firm. The firm uses land and capital to build houses. We assume land supply is exogenously determined by the government who sells some new land

[6]More stringent mortgage rules have been implemented but failed to restrain the fast pace of house price growth in China. For example, Chinese government in 2011 raised the down payment on second mortgages to 60 percent, and required the interest rate to be at least 10 percent higher than the central banks benchmark.

[7]Credit constraints can magnify the effects of demand shocks on the housing market resulting in large short-term price volatility. Collateralized borrowing may be a propagation mechanism for the cyclical movement of the economy. See, e.g., Stein (1995), Ortalo-Magne and Rady (2006), Liu, Wang, and Zha (2013), Chu (2014).

to the market in each period. But land price is endogenously determined so that land market clears. To finance the land purchase and capital investment, the firm issues shares. Following Kiyotaki, Michaelides, and Nikolov (2011), we assume households are share holders of the firm. Each household optimally decides whether to own or rent houses.

There exists no aggregate uncertainty in this economy. Households face a common deterministic growth rate of income, although they experience idiosyncratic income shocks and medical expense shocks. There is also no productivity shock. Consequently house price is non-stochastic.[8]

3.1 Firm

The representative firm maximizes the value of its shareholder. It combines land and capital to produce houses. The production function and dynamic optimization problem are laid out below.

3.1.1 Production Function

Let K and L denote capital and land input. Following Kiyotaki, Michaelides, and Nikolov (2011), we assume the firm's production function is

$$H_t = Z L_t^{\theta} K_t^{1-\theta}, \tag{1}$$

where Z is a scaling parameter, and $\theta \in (0, 1)$ measures the relative importance of land in housing construction.[9] We abstract from labor input in housing construction for simplicity and transparency. Land price is much more important than labor cost in determinging the house price for big cities, as shown in Davis and Heathcote (2007) for the US market, and in Deng, Gyourko, and Wu (2012) for the Chinese market.

3.1.2 Timing and Flow of Funds

Before the start of period t, the firm already owns H_{t-1} units of housing produced using K_{t-1} unit of capital and L_{t-1} unit of land. At the beginning of period t, the firm issues

[8] This should lead to higher housing demand and higher house price relative to the case of risky housing investment.

[9] Our specification assumes that the firm can continuously adjust the housing production. In reality, downward adjustment of housing stock would be difficult, at least in the short run. However, we focus in this paper on a growing economy with no productivity shocks, hence downward adjustment never happens.

new shares to raise new capital and purchase land, and then construct new housing. At the end of period t, the firm has H_t units of housing and H_t shares outstanding. We use p_t to denote house price in period t, and use r_t to denote the rental rate.[10] The amount of rental payment collected in period t is $r_t H_t$, which is assumed to be paid out as dividends to share holders. The firm's flow of funds in period t is

$$p_t(H_t - H_{t-1}) = K_t - (1-\delta)K_{t-1} + q_t(L_t - L_{t-1}), \tag{2}$$

where δ is the depreciation rate of capital, and q_t is land price in period t.[11] The left side of the equation is the proceeds from issuing new shares, which is used to purchase additional capital and land, as shown in the right side.

3.1.3 Optimization Problem

In the beginning of period t, the firm decides on the purchase of new capital and land to maximize the value of existing share holders after the issuance of new shares, $p_t H_{t-1}$. From equation (2), we have

$$\begin{aligned} p_t H_{t-1} &= p_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) \\ &= r_t H_t - [K_t - [1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) + (p_t - r_t)H_t \\ &= r_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{p_t - r_t}{p_{t+1}} p_{t+1} H_t \\ &= r_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{1}{R_{t+1}} p_{t+1} H_t. \end{aligned} \tag{3}$$

In the last equation, we define $R_{t+1} = p_{t+1}/(p_t - r_t)$ which is the return to housing investment and can also be thought of as the firm's cost of financing.

Evidently, in the beginning of period t, before the issuance of new shares, the firm's value and optimal decision depend on the capital and land stock carried over from the previous period, denoted K_{t-1} and L_{t-1} respectively. In the dynamic programming problem, (K_{t-1}, L_{t-1}) is the state vector of the firm. Using $V(K_{t-1}, L_{t-1})$ to denote the value of the

[10]If the firm adjusts the number of equity shares outstanding in period t to be equal to the number of housing units, then p_t is also the per share price of the housing firm.

[11]The depreciation of housing stock is captured by the depreciation of capital (δ). We do not directly model the depreciation of housing stock H_t, because we assume H_t is also the number of shares.

firm given the state vector, the firm's optimization is

$$\begin{aligned}
\max_{K_t, L_t} \quad & r_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{1}{R_{t+1}} V(K_t, L_t) \\
s.t. \quad & \qquad\quad H_t = Z K_t^{1-\theta} L_t^{\theta}.
\end{aligned} \tag{4}$$

First order conditions with respect to K_t and L_t are

$$Z(1-\theta) r_t \left(\frac{K_t}{L_t}\right)^{-\theta} = 1 - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial K_t} = 1 - \frac{1-\delta}{R_{t+1}} \tag{5}$$

$$Z\theta r_t \left(\frac{K_t}{L_t}\right)^{1-\theta} = q_t - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial L_t} = q_t - \frac{q_{t+1}}{R_{t+1}} \tag{6}$$

In the last equality of (5) and (6), we substitute the envelope conditions: $\frac{\partial V(K_t, L_t)}{\partial K_t} = 1 - \delta$ and $\frac{\partial V(K_t, L_t)}{\partial L_t} = q_{t+1}$.

From equation (5), capital input relative to land, $\frac{K_t}{L_t}$, decreases with R_{t+1}, because higher R_{t+1} means higher cost of capital. On the other hand, equation (6) shows that $\frac{K_t}{L_t}$ increases with the cost of land. The firm can acquire land at the price of q_t in period t, and then sell it for q_{t+1} next period. Discounting land price in period $t+1$ to period t using R_{t+1}, the cost of one unit of land is thus $q_t - q_{t+1}/R_{t+1}$. Intuitively, higher land price induces substitution of capital for land, leading to higher $\frac{K_t}{L_t}$.

3.1.4 Housing Supply

In this subsection, we first derive the equilibrium housing supply as a function of the exogenous land supply denoted by L_t^*, which is also the land used in housing construction in period t. Then we determine the market clearing land price by adjusting the land price so that land demand by the firm equals exogenous land supply.

The optimal level of capital given land supply L_t^*, housing rental rate r_t and firm's financing cost R_{t+1} can be obtained from the first order condition (5):

$$K_t^* = \left[\frac{Z(1-\theta) r_t}{1 - (1-\delta)/R_{t+1}}\right]^{1/\theta} L_t^* \tag{7}$$

Plugging this expression into the housing production equation (1), we derive the following housing supply function:

$$H_t = Z^{1/\theta} \left[\frac{(1-\theta) r_t}{1 - (1-\delta)/R_{t+1}}\right]^{(1-\theta)/\theta} L_t^* \tag{8}$$

Thus in our model, housing supply depends critically on land supply. When more land is supplied by the government, the firm optimally chooses more capital investment (equation (7)), and hence housing supply is increased.

From equation (8), housing supply also increases with rental rate r_t, because rental income is the only source of firm revenue. On the other hand, higher financing cost (R_{t+1}) reduces the continuation value of the firm, hence decreases housing supply. Note that financing cost increases with r_t, which generates an indirect negative effect of rental rate on housing supply.

To further understand how housing price affects supply, let $G_{p_t} = p_{t+1}/p_t$ be the growth factor of housing price, then $R_{t+1} = p_{t+1}/(p_t - r_t) = G_{p_t} p_t/(p_t - r_t)$. Assuming that G_{p_t} is independent of p_t, we have the following partial derivatives:

$$\partial R_{t+1}/\partial p_t = -\frac{G_{p_t} r_t}{(p_t - r_t)^2} < 0 \tag{9}$$

$$\partial R_{t+1}/\partial G_{p_t} = \frac{p_t}{p_t - r_t} > 0 \tag{10}$$

These relations imply that, everything else being equal, a higher p_t implies a lower financing cost, which leads to more housing supply. On the other hand, a larger G_{p_t} implies a higher financing cost, hence less housing supply. Thus, when there is excessive investment demand, p_t is raised and p_{t+1}/p_t falls, so that supply rises; when housing consumption demand exceeds housing supply, rental rate r_t increases so that supply rises. This helps us understand how the housing market clears.

3.1.5 Market Clearing Land Price

Using the land market equilibrium condition that the firm uses L_t^* unit of land, equation (6) becomes

$$\theta r_t H_t = \left(q_t - \frac{q_{t+1}}{R_{t+1}}\right) L_t^*. \tag{11}$$

Recall that θ is the land share in Cobb-Douglas housing production function, and $r_t H_t$ is the firm's revenue in period t. The left side of the equation (11) is the share of revenue attributed to land as one of the production factors, while the right side is the cost of land.

Equation (11) can be rewritten into the following relation which can be used to solve land price recursively:

$$q_t = \frac{\theta r_t H_t}{L_t^*} + \frac{q_{t+1}}{R_{t+1}} \tag{12}$$

Intuitively, current land price q_t equals the land share of the firm's revenue (per unit of land), plus the discounted future land price. We will show that the economy has a balanced growth path in which land price grows at a constant factor of G_q, and the firms financing cost is a constant, denoted R_{BGP}. Therefore in the BGP, land price can be expressed as a function of G_q, R_{BGP}, and the rental rate r_t.

To solve for land price in the BGP, we plug housing supply (8) into (11), and obtain the following dynamic relation for land prices:

$$q_t - \frac{q_{t+1}}{R_{t+1}} = \theta Z^{1/\theta} r_t^{1/\theta} \left[\frac{1-\theta}{1-(1-\delta)/R_{t+1}} \right]^{(1-\theta)/\theta} \tag{13}$$

In the BGP, $q_{t+1} = q_t G_q$ and $R_{t+1} = R_{BGP}$, thus

$$q_t = M \frac{r_t^{1/\theta}}{1 - G_q/R_{BGP}}, \tag{14}$$

where $M = \theta Z^{1/\theta} \left[\frac{1-\theta}{1-(1-\delta)/R_{BGP}} \right]^{(1-\theta)/\theta}$ is a function of firm's financing cost R_{BGP} in stationary equilibrium which is a constant.

In the ***Proposition*** we will show that rental rate r_t grows at a constant factor G_r in the BGP. Moreover, $G_q = G_r^{1/\theta}$. Plugging this into equation (14), we have:

$$\begin{aligned} q_t &= M \frac{r_t^{1/\theta}}{1 - G_r^{1/\theta}/R_{BGP}} \\ &= M r_t^{1/\theta} \left[1 + \frac{G_r^{1/\theta}}{R_{BGP}} + \left(\frac{G_r^{1/\theta}}{R_{BGP}} \right)^2 + \left(\frac{G_r^{1/\theta}}{R_{BGP}} \right)^3 \ldots \right] \\ &= M \left[r_t^{1/\theta} + \frac{(r_t G_r)^{1/\theta}}{R_{BGP}} + \frac{(r_t G_r^2)^{1/\theta}}{R_{BGP}^2} + \frac{(r_t G_r^3)^{1/\theta}}{R_{BGP}^3} \ldots \right] \\ &= M \left[r_t^{1/\theta} + \frac{r_{t+1}^{1/\theta}}{R_{BGP}} + \frac{r_{t+2}^{1/\theta}}{R_{BGP}^2} + \frac{r_{t+3}^{1/\theta}}{R_{BGP}^3} \ldots \right] \end{aligned} \tag{15}$$

That is, in the BGP, land price q_t is the sum of discounted rental rates from period t on raised to the power of $1/\theta$.

Denote by $r_{_{BGP}}$ and $q_{_{BGP}}$ the rental rate and land price at the time the economy reaches the BGP, then

$$q_{_{BGP}} = M \frac{r_{BGP}^{1/\theta}}{1 - G_r^{1/\theta}/R_{BGP}} \tag{16}$$

In the quantitative analysis that follows, we will derive $q_{_{BGP}}$ from $r_{_{BGP}}$ and R_{BGP} which in turn are obtained from a set of regularity conditions, then we will back out the land price paths during the economic transition based on equation (13) and the paths of R_{t+1} and r_t. A major task of our study is to compute equilibrium paths of p_t and r_t.

3.2 Households

The economy is populated by a growing mass of households. A household works since age J_0 and retires at J_1, then live up to a maximum age of J. At each age, a household is faced with an age-specific death probability. In the numerical analysis, $J_0 = 21$, $J_1 = 60$ and $J = 96$.

3.2.1 Heterogeneity and Uncertainty

The households are ex ante homogeneous, but they are heterogeneous ex post because they receive idiosyncratic income shocks and medical expense shocks. In addition, the households face mortality risks which increases with age as in the data. As mentioned earlier, we assume there is no aggregate uncertainty in the economy.

We include out-of-pocket medical expense shocks in the model for two reasons. First, recent studies show that stochastic medical expense is an important determinant of wealth accumulation/decumulation for retirees (see De Nardi, French, and Jones (2010) and the references therein). Second, medical insurance is usually under-provided in the emerging markets, and our framework can serve as a tool to examine the potential impact of better provision of medical insurance.

The model admits two major incentives of wealth accumulation: precautionary savings and retirement savings. The vehicle of saving is equity shares of the firm.

3.2.2 Income and Medical Expense

Income growth is one of the key driving forces of housing demand. Household income consists of two components, one deterministic and the other stochastic. Let $y(i, a, t)$ be the income of the i^{th} household at age $a \leq J_1$ and year t, then

$$y(i, a, t) = \tilde{y}(i, a, t) \times \overline{y}(a, t), \forall a \leq J_1, \tag{17}$$

where $\tilde{y}(i, a, t)$ and $\overline{y}(a, t)$ are the stochastic and the deterministic components respectively. The deterministic income, $\overline{y}(a, t)$, includes an age effect (a) capturing the hump-shaped life-cycle profile of income and a time effect (t) for the growth of the aggregate income. We assume an AR(1) process for the logarithm of stochastic component of income:

$$\ln \tilde{y}(i,a,t) = \rho_y \ln \tilde{y}(i,a-1,t-1) + \epsilon(i,a,t), \forall a \leq J_1, \tag{18}$$

where $\epsilon_{i,a,t}$ is the idiosyncratic shock to the i^{th} household in year t, and ρ_y determines the persistence of the shock. Regardless of time and the household's age, $\epsilon_{i,a,t}$ is drawn from a normal distribution with mean zero and standard deviation of σ_y. For a household just entering the labor market, the age is J_0 and we assume $\tilde{y}(i,J_0,t) = \epsilon(i,J_0,t)$.

After retirement, households are no longer subject to income shocks. At the time of retirement, income of a household is assumed to be κ fraction of its income right before retirement. Further, post-retirement income is assumed to grow at the same rate as the aggregate income.

Retirees are faced with stochastic out-of-pocket medical expenses, denoted by $m(i,a,t)$, which is assumed to have an idiosyncratic stochastic component $\tilde{m}(i,a,t)$ and a deterministic component $\overline{m}(a,t)$ that is common for all individuals of the same age a at time t:

$$m(i,a,t) = \tilde{m}(i,a,t) \times \overline{m}(a,t), \forall a > J_1, \tag{19}$$

We assume that $\ln \tilde{m}(i,a,t)$ follows an AR(1) process:

$$\ln \tilde{m}(i,a,t) = \rho_m \ln \tilde{m}(i,a-1,t-1) + \eta(i,a,t), \forall a > J_1, \tag{20}$$

where $\eta(i,a,t)$ is drawn from a normal distribution with mean zero and standard deviation σ_m. Since out-of-pocket medical expense is small relative to income before retirement, we assume it is zero for simplicity.

3.2.3 Timing

A household of age a enters period t with $s_{a-1,t-1}$ shares of equity. In the beginning of the period, upon the revelation of non-financial income $y_{a,t}$ and out-of-pocket medical expense $m_{a,t}$, the household decides on the quantity of housing consumption $h_{a,t}$, nonhousing consumption $c_{a,t}$ and shares of equity $s_{a,t}$. We assume that equity is traded only in the beginning of the period. Immediately after trading, the household receives dividend $s_{a,t}r_t$. At the end of period t, with probability ν_a the household dies, leaving $s_{a,t}$ as a bequest.

3.2.4 Household's Optimization Problem

We omit the household index i and the time index t in the household's dynamic optimization problem below. Using $V(s_{a-1}, y_a, m_a)$ to denote the value function of a household with s_{a-1} shares of equity, income y_a and medical expense m_a in the beginning of a period, we have

$$V(s_{a-1}, y_a, m_a) = \max\nolimits_{c_a, h_a} u(c_a, \psi h_a) + \beta \mathcal{E}\left[(1-\nu_{a+1})V(s_a, y_{a+1}, m_{a+1}) + \nu_{a+1} V_b(s_a)\right], \tag{21}$$

$$s.t. \qquad r_t h_a + c_a = y_a - m_a + p_t(s_{a-1} - s_a) + r_t s_a, \tag{22}$$

$$\text{and} \;\; s_a > 0.$$

where $V_b(s_a)$ is the bequest function, and the expectation operator $\mathcal{E}$ is taken with respect to the income and medical expense distributions of the next period. We assume the Cobb-Douglas form direct utility $u(c_a, \psi h_a)$:

$$u(c, \psi h) = \frac{\left[c^{1-\omega}(\psi h)^{\omega}\right]^{1-\gamma}}{1-\gamma},$$

where γ is the inverse of inter-temporal elasticity of substitution (EIS). This utility form implies a unit elasticity of substitution between housing and nonhousing consumption for which Morris and Ortalo-Magne (2011) find strong data support. Further, it implies that in balance growth economy of our model, consumption, investment, house price and rent grow at constant rates.

Following Kiyotaki, Michaelides, and Nikolov (2011), we assume that the same housing stock provides less utility if it is occupied by a renter rather than by an owner, as captured by the parameter ψ. Specifically, we have

$$\psi \begin{cases} < 1, & \text{for renter;} \\ = 1, & \text{for owner.} \end{cases}$$

Owners are defined as households whose housing equity value exceeds d fraction of the housing consumption, i.e., $s \geq dh$, with d defined as the down payment rate. Renters are those with $s < dh$.

We assume that the value of bequeathing s_a shares of equity at age a is

$$V_b(s_a) = \max_{c,h} \quad \mathcal{B}u(c, h),$$

s.t.

$$c + r_t h = p_t s_a.$$

where $\mathcal{B}$ determines the strength of bequest motive. In other words, the deceased evaluates the utility as if the beneficiaries consume the bequeathed wealth in just one period, optimally splitting it between housing and non-housing consumption.

With Cobb-Douglas preference, we have the following analytical form of bequest value:

$$V_b(s_a) = \mathcal{B}\left[(1-\omega)^{1-\omega}(\psi\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_t}\right)^{\omega(1-\gamma)}\frac{(p_t s_a)^{1-\gamma}}{1-\gamma} \tag{23}$$

3.2.5 First Order Conditions

In each period, a household makes both intra-temporal and inter-temporal decisions. The inter-temporal decision involves the number of shares to hold and carry over into the next period. This is essentially a consumption-saving decision, governed by the following Euler equation:

$$\frac{\partial u(c_a, \psi h_{a,t})}{\partial c_a} = \beta R_{t+1}\mathcal{E}\left[(1-\nu_{a+1})\frac{\partial u(c_{a+1}, \psi h_{a+1})}{\partial c_{a+1}} + \nu_{a+1}V_b^{'}(s_a)\right], \tag{24}$$

where $R_{t+1} = p_{t+1}/(p_t - r_t)$ is the return on housing investment.[12]

Intra-temporally, the decision is to allocate between housing and non-housing consumption. It is straightforward to show the following:

$$\frac{c_a}{h_a} = \frac{1-\omega}{\omega} r_t \tag{25}$$

3.3 General Equilibrium

The general equilibrium is defined as sequences of prices $\{p_t, r_t, q_t\}$, and sequences of choice variables of the firm and the households that satisfies: (i) the firm's choice of L_t, K_t, H_t are consistent with the firm's optimization problem; (ii)the households choices of $c(i,a,t)$, $h(i,a,t)$, and $s(i,a,t)$ are consistent with the household's optimization problem; and (iii), the paths of prices and rents satisfy the following market clearing conditions.

Housing market clears for each t:

$$H_t = \sum_{a=J_0}^{J}\mu_{a,t}\int_i h(i,a,t)di \tag{26}$$

[12] Equation (24) holds only when $s_a > 0$ is not binding.

where $\mu_{a,t}$ is the fraction of individuals of age a in period t.

Equity market clears for each t:

$$H_t = \sum_{a=J_0}^{J} \mu_{a,t} \int_i s(i,a,t)di \tag{27}$$

Land market clears for each t:

$$L_t = L_t^* \tag{28}$$

3.4 Balanced Growth Path

Assume that from year T_{BGP} on, aggregate income, land supply and population grow at fixed factors G_Y, G_L and G_N respectively. In addition, age distribution of population no longer changes over time. Then we have the following proposition.

Proposition *A balanced balanced growth path (BGP) exists and is characterized by:*

1. *Aggregate capital grows at a factor of* $G_K = G_Y$*;*
2. *Aggregate housing supply/demand grows at a factor of* $G_H = G_Y^{1-\theta} G_L^{\theta}$*;*
3. *Housing demand per capita grows at* $G_s = (G_Y/G_N)^{1-\theta}(G_L/G_N)^{\theta}$ *and* $G_h = G_s$*;*
4. *Consumption per capita grows at* $G_c = G_Y/G_N$*;*
5. *Housing price grows at* $G_p = (G_Y/G_L)^{\theta}$*;*
6. *Housing rental rate grows at* $G_r = (G_Y/G_L)^{\theta}$*;*
7. *Land price grows at* $G_q = G_Y/G_L$*;*
8. *Floor-area ratio, defined as* H/L*, grows at* $G_{FAR} = (G_Y/G_L)^{1-\theta}$*.*
9. *Neither price-income ratio nor price-rent ratio is time-varying.*

Proof of this Proposition is provided in the Appendix. Equilibrium in BGP has a set of properties that are consistent with stylized facts. For example, housing price is driven by income growth rate and land supply. A more restrictive land supply leads to a higher price-rent ratio and a higher price growth rate. The importance of income is shown in Shiller

(2003), and the importance of land is emphasized in Glaeser and Saks (2005) and Saiz (2010). In addition, price-income ratio and price-rent ratio are both constants in BGP.[13]

The fifth point of the proposition states that a smaller G_L is associated with a higher growth rate of housing price. Further, with high growth rate of housing price, low rental return is need to maintain a reasonable overall return of housing investment. Low rental return is equivalent to high price-rent ratio, which leads to the sixth point of the proposition. These points are consistent with recent findings in Bracke (2015).

It has been observed that house price indices, after controlling for inflation, can exhibit extremely low growth rates in the long run (see, e.g., Chart 4 in Shiller (2007)). This phenomenon can be generated in our model when land supply and aggregate income grow at similar rates. On the other hand, when land supply grows at a lower rate than income, the model predicts a growing trend of housing price, consistent with the pattern of housing price experienced in the past decades by cities such as Hong Kong, San Francisco and New York.

Further insights are gained from comparing the fifth and seventh points in the ***Proposition***. Growth rate of land price is always higher than that of house price since $\theta < 1$. This is consistent the empirical observations of major cities in both the U.S. (Davis and Heathcote (2007)) and China (Deng, Gyourko, and Wu (2012)).

It should be noted that the economy does not operate in the BGP immediately after the stabilization of the exogenous variables. It needs to wait until the age distribution and the asset distribution of households become time-invariant. In the quantitative analysis below, we assume that after year 2044, all exogenous variables grow at constant rates. After another 70 years, i.e., after 2114, the age distribution and asset distribution will be time-invariant. In addition, households who enter the economy before 2044 will almost completely phase out by then. Therefore, we take T_{BGP} as year 2114.

The key variables grow at constant rates in the BGP, therefore they can be re-scaled so the the economy operates as if it is in a steady state. In the quantitative analysis, we start with finding the house price and rent in this "steady state", then we use them as the terminal conditions to solve for the paths of price and rent during the transition periods.

[13] Based on 355 years of data, Ambrose, Eichholtz, and Lindenthal (2013) show that house price and rents are cointegrated and price-rent ratio exhibits long-run stationarity.

4 Quantitative Analysis: Projection and Calibration

Having studied the analytical features of the housing market in the BGP, we now turn to the transition path where the non-stationarity of prices and fundamental variables become important. Like most of the incomplete market models in which idiosyncratic income and medical expense shocks are non-insurable, the model does not admit an analytical solution for the transition path. Therefore we resort to numerical method. In the context of Beijing market, this section explains in details the projection of exogenous processes, the estimation of initial conditions, and the calibration of model parameters.

4.1 Projections of Population, Income, and Land Supply

4.1.1 Evolution of Population Structure

Two dimensions of the population structure are relevant for our model: population size and age distribution of population. Population size directly affects housing demand. Age distribution matters because housing demand is age-specific. Both housing consumption demand and housing investment demand have hump-shaped age profiles. Therefore an aging population generates lower aggregate housing investment demand.

The population data are obtained from the 2010 National Census, and from sample surveys in other years between 2005-2013.[14] The upper-left panel of Figure 1 shows the age distribution of Beijing residents, defined as individuals who either have formal registration (Hu Kou) or have lived in Beijing for more than half a year. Compared with the overall urban population, Beijing population is much younger due to the influx of young in-migrants.

To project the population structure after 2013, we need to predict fertility rate, mortality rate and immigration rate of Beijing population. Using the data between 2005-2013, we calculate the age-specific fertility rate and mortality rate of Beijing population, shown in the upper-right (Old Rate) and lower-left panel of Figure 1. The low fertility rate in the data is due to China's one-child policy (which is currently being relaxed) and implies ever decreasing total population if we assume it remains the same in the future. Thus, we consider

[14] The 2005 sample is 1% of Beijing population, associated with the nationwide 1% *Population Sample Survey*. Sample size is about 0.1% between 2006-2009 which are associated with nationwide *Sample Survey on Population Change*. Since 2011, Beijing Bureau of Statistics has routinized annual sample survey to be 2% of the population in non-census years.

an alternative specification for the age-specific fertility rate (see the line labeled "New Rate" in the upper-right panel of Figure 1): for each age and in the first 10 years starting from 2014, it is the same as that estimated using the data between 2005-2013, but it rises linearly for the next 10 years so that the overall population growth rate reaches 0.4% in 2034 and then fertility rate is assumed to be time-invariant afterwards. Mortality rate is assumed to be constant over time since life expectancy in China is already close to those in industrialized countries.

As emphasized in Henderson (2010), rapid urbanization is one of the key issues in population dynamics for a developing country. Urbanization is reflected in the increasing city population in our model. We define immigration rate as the number of new immigrants to Beijing as a fraction of existing Beijing population. The rate has been declining since 2008, and averages around 2.88% between 2010-2013. In the baseline model, we assume that immigration rate decreases linearly from 2.88% in 2014 to zero after 30 years. In the data it is clear that immigrants to Beijing are mainly young workers, therefore we assume that only those aged between 20-30 migrate to Beijing in each year.

Based on the 2013 data and using the fertility rate, mortality rate and immigration rate discussed above, we extrapolate the population structure after 2013. The lower-right panel of Figure 1 plots the projected age structure of population in 2020, 2060 and 2100. Upon the completion of urbanization which is represented by a zero immigration rate after 2044, the peak age of population moves to 65 in 2060. In year 2100, the population structure stabilizes to a profile that decreases with age, due to the increasing age-profile of mortality rate.

4.1.2 Evolution of Land Supply

In China, local governments own land and auction land use right. The amount of land to be auctioned depends on a multitude of considerations, including policies from the central government, fiscal balance of the local governments and growth rate of local GDP.

We obtain data on supply of residential land between 2005 and 2013 from the National Bureau of Statistics (NBS), and project the land supply onward. For each major cities in China, NBS reports the amount of new residential land acquired by housing developers.[15] This is the flow of land. The stock of land in 2009 is available from the 2010 *China Statistical Year Book of Environment* complied by NBS. Table 11-3 of the year book is "Basic Statistics

[15] http://data.stats.gov.cn/workspace/index?m=csnd.

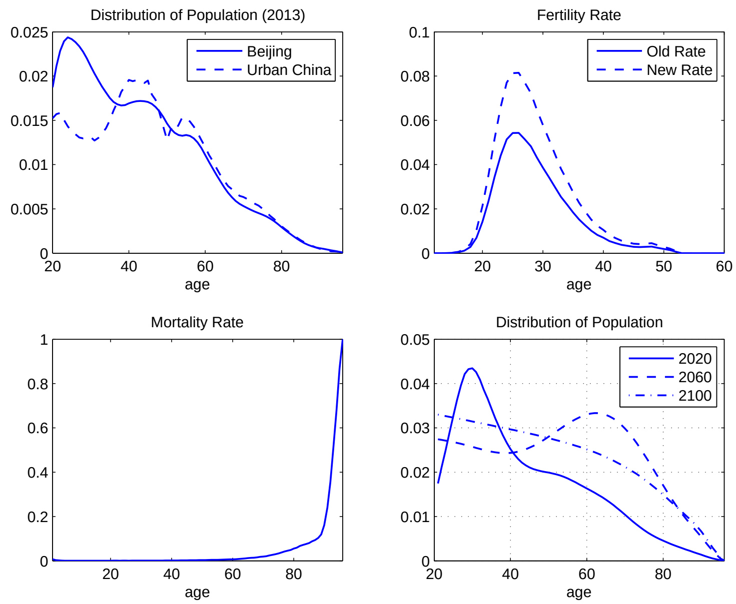

Figure 1: Fertility rate, mortality rate, and population structure

This figure shows the initial age distribution of population, fertility rate, mortality rate and projected population structure. The fertility rate is presented as one half of women's fertility rate in the data, interpreted as fertility rate per couple. The "Old Rate" is the average fertility rate between 2005-2013 in the data. The "New Rate" is the projected fertility rate after 2034.

on Urban Area and Land Used for Construction by Region" which reports that the area of residential land is 383.3 hectares in Beijing at the end of 2009. Based on the stock of land in 2009 and the annual flows, we obtain the total stock of residential land in Beijing. Finally, we divide total land stock by the population of Beijing residents to obtain land supply per capita.

Table 1 reports total residential land, newly-acquired residential land, and residential land per capita. Although the simulation of the economy starts from 2005, we report land supply since 2001. From the last row of the table, it is clear that the growth rate of land falls far behind that of population after 2005, leading to declining land supply per capita, which is partly responsible for the soaring house price since 2005.

We assume that land supply grows at a constant rate of 0.05% from 2014 onwards in the baseline model. The evolution of land per capita is given by the solid line in the left panel of Figure 2. Land supply per capita falls gradually due to the inflow of population during the

Table 1: Land and population of urban Beijing

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	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Population (million)	13.9	14.2	14.6	14.9	15.4	16.0	16.8	17.7	18.6	19.6	20.2	20.7	21.4
Growth rate (%)	–	2.75	2.33	2.49	3.03	4.10	4.68	5.67	5.03	5.48	2.89	2.51	3.61
New land (ha)	1472	2093	1391	1572	774	295	392	823	625	859	507	306	906
Total land stock (ha)	29518	30990	33082	34474	36046	36820	37115	37507	38330	38955	39814	40321	40627
Growth rate (%)	–	4.99	6.75	4.21	4.56	2.15	0.80	1.05	2.20	1.63	2.20	1.27	0.76
Land per capita (m^2)	21.31	21.77	22.72	23.09	23.44	23.00	22.14	21.18	20.61	19.86	19.72	19.49	18.95
Growth rate (%)	–	2.18	4.32	1.67	1.48	-1.87	-3.71	-4.37	-2.69	-3.65	-0.67	-1.21	-2.75

urbanization periods. As the growth of population plateaus, land supply per capita becomes time-invariant. The two broken lines show the evolution of land per capita when the growth of aggregate land supply is either 1% or 0%, which will be used in the sensitivity analysis.

4.1.3 Evolution of Aggregate Income

The average disposable income of Beijing residents, as reported by the NBS, is 36.47, 39.30 and 41.96 thousand Renminbi (RMB) in year 2012, 2013 and 2014 respectively, each in terms of 2012 RMB. These are roughly one-third of the disposable income of Hong Kong residents in the corresponding years. We assume that average income grows at a constant rate of 7% in 2015, then the growth rate declines linearly from 7% to 3% over the next 30 years. Given the assumption that population grows at a constant rate of 0.4% in the BGP, the growth rate of average income is $(1 + 3\%)/(1 + 0.4\%) - 1 = 2.59\%$ per year. In the sensitivity analysis, we consider two alternative cases where income growth plateaus to 3% after either 20 or 40 years. Figure 2 plots the projected evolution of the per capita income under the baseline and the two alternative scenarios.

4.2 Other Exogenous Inputs

4.2.1 Initial Assets

The initial distribution of households' asset by age is an important input in the model. The best survey data about household assets in China is the China Household Finance Survey (CHFS http://www.chfsdata.org/). However, thus far there is only one wave of data available publicly – the 2012 wave. From the survey, we estimate the ratio of financial wealth to income and the age profiles of financial asset and housing equity for urban households in

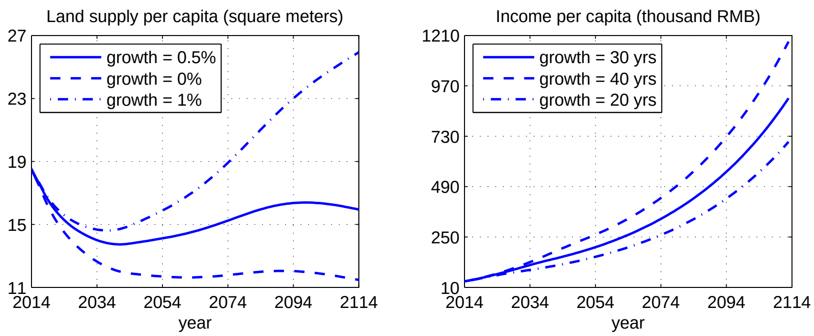

Figure 2: Projected land supply and income

This figure shows the projected land supply and income under different assumptions. In the left panel, "growth" refers to the growth rate of land per capita. In the right panel, "growth" refers to number of years it takes before the growth rate of aggregate income plateaus. In the baseline mode we use "growth = 0.5%" and "growth = 30 yrs"

China. The estimated ratio of financial wealth to income is 3.68.

Data from Beijing Bureau of Statistics show that the disposable income per capita is 19.13 thousand in 2005 (in terms of 2012 RMB). Therefore the estimated average financial wealth is 70.37 thousand RMB for the Beijing residents in 2005. In addition, the average housing size for Beijing residents is 19.5 square meters in 2005. We distribute these assets across different ages, assuming the same age profiles of financial wealth and housing equity as in the 2012 wave of CHFS.[16] These are the initial assets of households who enter the economy in period $t = 0$ (i.e., in year 2005). For those who enter at $t > 0$, their initial assets are bequests from those who die in period $t - 1$. The total amount of bequest wealth is endogenously determined the model and distributed evenly among new households just entering the economy.

[16]In the model, financial wealth is also treated as housing equity, therefore the initial housing stock in the model would be higher than in the data. To address that, in simulation we distribute the 70.37 thousand RMB financial wealth to households who enter the economy in 2005 over a period of 10 years, between 2005-2014.

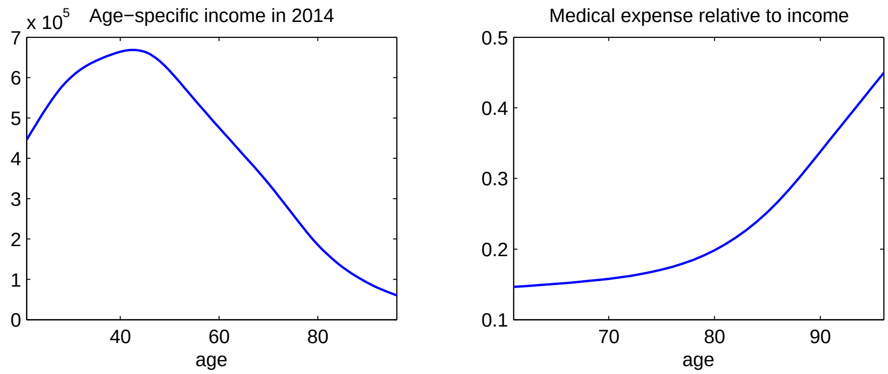

Figure 3: Age profile of income and medical expense

4.2.2 Income and Medical Expense

The age profile of income, defined as the term $\overline{y}(a,t)$ in equation (17), is estimated from China Health and Nutrition Survey (http://www.cpc.unc.edu/projects/china/). We use all the available waves of survey prior to 2011 (i.e. 1989, 1991, 1993, 1997, 2000, 2004, 2006, 2009), and regress the logarithm of income on age and year dummies. The left panel of Figure 3 plots the smoothed age-profile of income, re-scaled to match the average income in 2014. In addition to the age profile of income, our numerical analysis takes into account the growth trend of income over time.

The age-specific medical expense as a proportion of income is estimated from the 2011 wave of China Health and Retirement Longitudinal Study (http://charls.ccer.edu.cn/en). As shown in the right panel of Figure 3, this ratio is 0.15 at age 61, and it reaches 0.45 at age 96.

The AR(1) processes of stochastic income and out-of-pocket medical expense are estimated from China Health and Nutrition Survey and China Health and Retirement Longitudinal Study respectively. The persistence parameters and variances of shocks are reported in Table 2.

Table 2: Stochastic Processes of Income and Medical Expense

	var. of shocks	persistence
Income	0.064	0.864
Medical expense	0.25	0.922

4.3 Model Parameters

Our model has three parameters related to housing production (Z, θ, δ) and five preference parameters $(\gamma, \psi, \mathcal{B}, \beta, \omega)$ respectively. These parameters are pined down by calibrating our model to match some key features of the Beijing market when it reaches the BGP.[17] We assume that Beijing housing market in the BGP will resemble the current state of Hong Kong in terms of price-income ratio, price-rent ratio and growth rate of real house price. Hong Kong is a reasonable reference city for Beijing because both adopt a land lease policy which has been shown to have a significant impact on housing price dynamics (Anglin, Dale-Johnson, Gao, and Zhu (2014)). These two cities have similar cultural background which should lead to similar preferences over housing, risk tolerance and other parameters.

The Hong Kong Rating and Valuation Department reports data for different housing classes in the three main regions of Hong Kong – Hong Kong island, Kowloon and New Territory. Averaging over different classes in the three regions, the annual housing rental rate in Hong Kong is about 3.39 thousand HK dollars per square meter in 2012. Using an exchange rate of 0.82 RMB per HK dollar in 2012, the annual rental rate is about 2.78 thousand RMB per square meter. The average house price is 121 thousand HK dollars in 2012 which is about 99 thousand RMB.

Price rent ratio is calculated as *(average house price) / (average rental rate)* $= 99/2.78 \approx 35.6$. We validate this ratio by calculating the ratio for each class of housing in each of the three regions in 2012, then taking the average of these ratios. The resulting number is 34.8. Price income ratio is calculated as *(price per square meter)×(average number of square meters per capita)/(average income per capita)*. Based on the 2012 data, housing per capita is about 12 square meters and average disposable income per capita is around 100 thousand RMB in Hong Kong. Therefore price income ratio is $99 \times 12/100 = 11.88$.

The growth rate of real house price in Hong Kong is calculated from house price index and CPI, the former is available from the Hong Kong Rating and Valuation Department, and the latter is available from the Census and Statistics Department. During the period of 1981-2012, the geometric mean of the growth rate of house price in Hong Kong, adjusted for inflation, is 2.14%. Thus, we take the growth factor of house price for Beijing in the BGP

[17] Our goal is to estimate current house prices and rents under rational expectations of realistic future evolutions of fundamentals and compare to the market data. Thus, we calibrate our model based on moments at BGP and then back out the implications for current values of house price and rent.

to be $G_p = 1.0214$.

Real housing return is calculated from price-rent ratio and the real appreciation of house price. Given the price-rent ratio of 35.6 and real appreciate rate of house price of 2.14%, return on housing investment is $R = 4.95\%$, which we take as the return on housing investment for Beijing in the BGP.

4.3.1 Parameters Related to Housing Production

Three parameters are related to housing production: land share in production (θ), capital depreciation rate (δ) and scaling parameter in housing production (Z).

Land share in housing production depends on a variety of factors, among which land availability and land price are the leading ones. We pin down θ by comparing the growth rate of house price to the growth rate of land price. As is evident from the ***Proposition***, in the BGP, the growth factors of house price (G_p) and land price (G_q) satisfy $G_p = G_q^\theta$, therefore

$$\theta = \frac{logG_p}{logG_q} \tag{29}$$

We calculate the growth factors based on Hong Kong data between 1987 and 2012. During this period, θ implied by the growth rates of house price and land price is 0.839, 0.497 and 0.819 for Hong Kong island, Kowloon and New Territory respectively. We take the average value $\theta = 0.72$.

As noted earlier, depreciation of housing is reflected in the capital depreciation parameter δ. Leigh (1980) estimates the annual depreciation rate of housing in the United States to be between 0.0036 and 0.0136. Let δ^h denote this housing depreciation rate, capital depreciation rate is $(1-\delta^h)^{1/(1-\theta)}$ based on our housing production function. Therefore capital depreciation should be between 0.01-0.03, and we choose $\delta = 0.02$.

To pin down the scaling parameter Z, we use the housing supply equation (8). Since price-income ratio (pH/Y) and price-rent ratio (p/r) are constants in the BGP, housing supply H in the BGP satisfies

$$H = \frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{r} \tag{30}$$

where $ratio^{py}$ and $ratio^{py}$ are price-income ratio and price-rent ratio respectively, Y is the average income per capita and r is the housing rental rate. Substitute equation (30) into equation (8) yields

$$Z = \left(\frac{1}{r}\right)\left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{L}\right)^{\theta}\left[\frac{1-(1-\delta)/R}{1-\theta}\right]^{1-\theta} \tag{31}$$

In Hong Kong, housing per capita is about 12 square meters and the average floor-area ratio is about 4.5, thus land use per capita is $L = 12/4.5 \approx 2.67$ square meters per capita. Substitute into equation (31) the per capita income Y=100 thousand RMB, rental rate of $r = 2.78$ thousand RMB, price-income ratio $ratio^{py} = 11.88$ and price-rent ratio $ratio^{pr} = 35.6$ as well as return on housing investment $R = 4.95\%$, we obtain $Z = 1.47$.

An additional input parameter in the model is down payment rate (d). For the Beijing market, down payment is generally 30% for the first home and about 50-60% for the second home. We take the average and set $d = 0.4$. This is consistent with the average down payment rate of the middle income households in first-tier cities in China, as calculated in Fang, Gu, Xiong, and Zhou (2015).

4.3.2 House price, Rent and Land Price in the BGP

We have used information from the Hong Kong market such as per-capita income, rental rate, land use, price-income and price-rent ratios to identify the scaling parameter Z. Now we use the projected per capita income and land in Beijing to obtain house price and rental rate when Beijing market converges to the BGP.

From the housing supply equation (8), it is straightforward to obtain

$$r_t = \left(\frac{1}{Z}\right)\left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y_t}{L_t}\right)^{\theta}\left[\frac{1-(1-\delta)/R_{t+1}}{1-\theta}\right]^{1-\theta} \tag{32}$$

To calculate the rental rate when the economy enters the BGP, we need to predict Y_{BGP} and L_{BGP}, the income and land supply in Beijing at $t = T_{BGP}$.

Based on the projected evolution of income, land supply and urban population structure, by the time Beijing market reaches the BGP, $Y_{BGP} = 832$ thousand in 2012 RMB, and $L_{BGP} = 12.83$ square meters per capita. Substitute these numbers and other related values into equation (32), we obtain $r_{BGP} = 4.12$ thousand RMB per square meter. This is the market clearing annual rental rate for Beijing $t = T_{BGP}$ (i.e., year 2114).

House price at the BGP is calculated as r_{BGP} divided by the price-rent ratio. That is $P_{BGP} = 4.12 \times 35.6 \approx 146.7$ thousand in 2012 RMB per square meters. Land price at the

BGP is 816.0 per square meter, calculated using equation (16). Thus land price is 5.6 times that of house price at the time the economy converges to the BGP.

At the BGP, housing size is $H_{BGP} = 67.36$ square meters per capita based on equation (30). The implied floor-area ratio is $\text{FAR}_{BGP} = H_{BGP}/L_{BGP} = 67.36/12.83 \approx 5.25$. By contrast, housing size is about 30 square meters per capita and FAR is below 1.8 in 2014.

4.3.3 Parameters Related to Consumer Preference

To pin down the five parameters related to consumer preference, $(\gamma, \psi, \mathcal{B}, \beta, \omega)$, we take a moment-matching approach. Specifically, we pick the set of parameters so that the following six moments generated from the model match as closely as possible those from the data: (i) average price-income ratio; (ii) average price-rent ratio; (iii) home ownership rate; (iv) the average age of first-time home buyers; (v) clearing of the rental market in the BGP (equation (26)); (vi) clearing of the housing equity market in the BGP (equation (27)).

To generate model moments, we simulate 1,000 paths of income and medical expenses for each generation of households, compute the optimal decisions of households in the BGP for each path, then calculate the related moments by taking the average values across the 1,000 simulated households.

For each of the moments used in model calibration, Table 3 shows its target value and the fitted value from our calibrated model. The average age of first-time home buyers contains important information about preference for home ownership as well as parameters related to wealth accumulation, such as β and γ. We could not find data on the average age of first-time home buyers in Hong Kong. But the age at first marriage is about 30 according to the Census and Statistics Department of Hong Kong. The age of first-time home purchase is usually older than the age at first marriage, therefore we use 33 as the age of first-time home purchase. As a further reference, in the US, age at first marriage is 28 on average and age of first-time home purchase is 34 according to the 2009 American Housing Survey.

Home ownership rate target is 0.75 for Beijing market in the BGP, higher than the rate of 52% in Hong Kong. According to data from 2012 wave of China Household Finance Survey, average home ownership rate in the first-tier cities in China is over 80%. Based on the history of economies that experienced successful economic transition, we do not expect home ownership rate in Beijing to decline significantly in the future.

Table 4 reports the set of preference parameters with which the model fits the data

Table 3: Moments

Moments	Target	Model
price/income	11.88	11.94
price/rent	35.6	35.6
home ownership rate	0.75	0.77
age of first time buyers	33	34
surplus (consumption MKT)	0	0
surplus (investment MKT)	0	0.02

moments best. Our model is able to generate high price-income ratio as well as other moments for Beijing housing market in the BGP, if households are very patient (high β), have a high EIS (low γ) and have a very strong bequest motive (high $\mathcal{B}$).

Table 4: Parameters

Production parameters		
land share in production	θ	0.72
capital depreciation rate	δ	0.02
scaling parameter in production	Z	1.47
Preference parameters		
inverse of EIS	γ	1.22
renter's utility discount	ψ	0.92
discount factor	β	0.99
housing share in utility	ω	0.35
strength of bequest motive	$\mathcal{B}$	25.0

5 Quantitative Analysis: Results

This section reports the main quantitative results. We first report results from the baseline model, and then conduct several sensitivity analyses.

5.1 Equilibrium Path of Price, Rent and Other Outcome Variables

Figure 4 reports the trajectories of house price, rent and land price (all in terms of 2012 RMB) that are consistent with the rational decisions of the developer and consumers. Based on the terminal values (in year 2114) for Beijing housing market in the BGP that are determined in subsection 4.3.2, we solve for house price and rent trajectories that clear the housing equity market, housing rental market and land market for each year prior to 2114. Land prices are then calculated from house prices and rents using equation (12).

The calibrated baseline model implies a rational house price in 2014 of 14.92 thousand per square meter (in 2012 RMB), which is significantly lower than the actual observed house price, despite the fact that price-rent and price-income ratios of Beijing in the BGP under the model match with the high values of the Hong Kong market. An important factor underlying the relatively low rational fundamental value of Beijing housing market is the more abundant land supply. Given that housing per capita is about 12 square meters and the average floor-area ratio is about 4.5 in Hong Kong, land use per capita is about $12/4.5 \approx 2.67$ square meters per capita. By contrast, residential land in Beijing is 18.95 square meters per capita. In other words, current land use per capita in Beijing is over seven times larger than that in Hong Kong.

The equilibrium annual rental rate is about 460 RMB per square meter in 2014, also much lower than in the data. For land price, the market clearing price at 2014 is 31.5 thousand per square meter.

In the upper-right panel of figure 4, the thick lines reports the growth rate of real house price. It has a period of relatively sluggish growth between 2040-2080, due to an aging population and a period of slow growth of population size after the completion of urbanization. The growth rate of house price converges to 2.14% which is the growth rate in the BGP. The growth rates of population, aggregate income, aggregate land supply are also plotted. The correlation between these fundamentals and house price is clearly discernable.

Figure 5 reports the trajectories of a number of interesting outcome variables. The upper panels plot two affordability measures: price-income ratio and the ratio of annual income over house price (i.e., how many square meters of housing can be purchased by one year of income). Our model implies that price-income ratio will decline rapidly between 2005-2035, then gradually converge to the level in Hong Kong which is about 11.88. The intuitive reason is that the currently high price-income ratio is supported by the high growth rate of

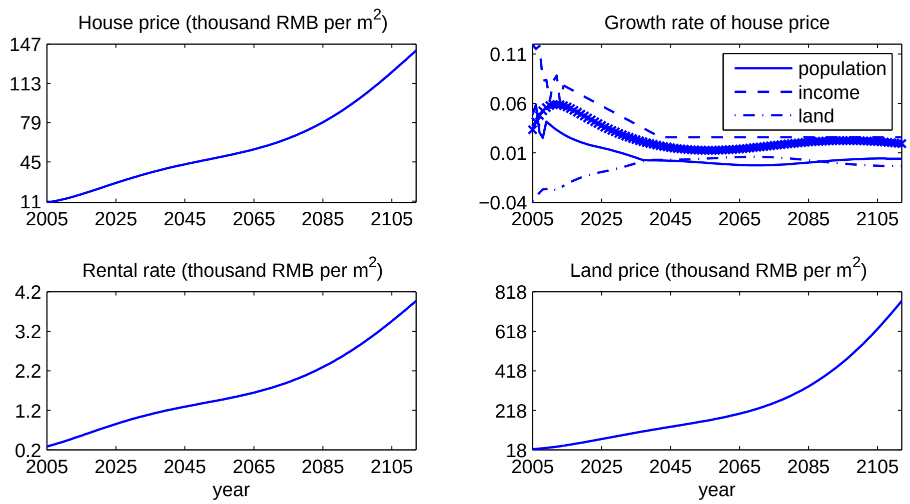

Figure 4: Equilibrium house price, rent and land price

This figure plots the trajectories of house price, rent and land price (in 2012 RMB) under the baseline model. The thick line in the upper-right panel is the growth rate of house price, plotted along with the projected growth rates of population, income and land supply.

income expected for the future. When income is growing quickly, the actual ability to pay of households is much higher than what is captured by the current income, therefore a high price-income ratio does not necessarily indicate the over-pricing of houses. As we project the income growth rate to decline from 7% to 3% within 30 years, the price-income ratio also declines. Due to the perfect foresight of households, price-income ratio decreases over time right from the beginning (year 2005), although growth rate of income starts to fall only after 2014. The ratio of annual income over house price displays an increasing time trend. In other words, housing becomes more affordable over time. In the ***proposition***, we have shown that aggregate income grows more quickly than house price in the BGP. This is also true during the transition periods.

The middle panels of Figure 5 plot the price-rent ratio and return to housing investment. The equilibrium price-rent ratio declines quickly between 2005-2015, then rises gradually until it reaches the level in the BGP. The period of declining price-rent ratio coincides with a period of low rental return and high house price growth, consistent with the finding of Wu, Gyourko, and Deng (2012). Return to housing investment combines rental return and capital gain. It declines after 2014, which is consistent with our projections about exogenous

Figure 5: Housing affordability and quantity

This figure plots the evolution of several measures related to housing affordability, husing quantity as well as return to housing investment during the economic transition periods for Beijing market.

variables: income growth and immigration rate both decline over time. In the end, housing return converges to the level of 4.95% in the BGP.[18]

The bottom left panel of Figure 5 shows that under our model, the ratio of land price over house price rises steadily. This indicates the increasing importance of land relative to structure in house price, which is consistent with the patterns found in the major U.S. cities (Davis and Heathcote (2007)). Our model also implies that Beijing will witness increasingly higher density and higher floor-area ratio, as shown in the bottom right of Figure 5.

[18]This 4.95% includes both rental return and price appreciation. Gyourko, Mayer, and Sinai (2013a) report that the average annual real house price growth between 1950 and 2000 for several "superstar cities" in U.S to be 2.5% or higher. They argue the high price growth of superstar cities is due to an inelastic supply of land in some unique locations combined with an increasing number of high income households nationally.

5.2 Instability of Empirical Relations

In an emerging market undergoing transitions, the empirical relation between house price (or rent) and economic fundamentals may not be stable. To elaborate on this, we revisit point 5-6 in the ***Proposition***. In the BGP we have the following:

$$log(G_p) = \theta log(G_Y) - \theta log(G_L) \tag{33}$$

$$log(G_r) = \theta log(G_Y) - \theta log(G_L) \tag{34}$$

Therefore, in the BGP the relation between house price (rent) and economic fundamentals can be well captured by linear regressions, as is done in Shiller (2003) using the US data. For the Beijing market, using simulated data in the BGP, the regression of G_p (or G_r) on G_Y and G_L returns a regression coefficient of 0.72, the calibration value for θ.

Now we run the same regression for different subperiods during the economic transition. Table 5 reports the results. The difference in coefficients from different periods is evident. Theoretically, for emerging markets undergoing transitions, the relation between house price (or rent) and the fundamentals such as income, land supply and age distribution of population can be nonlinear and time varying. Linear regressions would fail to capture these complex dynamic dependences. An attempt to estimate the fair housing value based on its historical relation with the fundamentals would produce misleading results for an emerging market. Therefore it is necessary to rely on structural models such as the one developed in this paper.

Table 5: Linear Regression Results

Dependent variable:	$log(G_p)$		$log(G_r)$	
	income	land	income	land
2005-2040	0.898	-1.212	0.754	0.017
2041-2080	0.605	-0.126	0.662	-0.696
2081-2114	0.867	-0.504	0.854	-0.881

This table reports the regression coefficients of equation (33) and (34) for several subsamples of equilibrium house prices, rents and exogenous variables.

5.3 Sensitivity Analysis

The baseline model implies a rational fair value of house price in 2014 that is about one half of that observed in Beijing market. In this section, we check the robustness of model-implied house prices and rents by changing some of the parametric assumptions about the exogenous processes. Four categories of sensitivity analysis are conducted, related to projection of land supply, projection of income growth, initial wealth, and evolution of age distribution respectively.

Table 6: Price and rent under alternative assumptions (in thousands of 2012 RMB)

		Price		Rent	
		2014	2114	2014	2114
(0)	Baseline	14.92	138.45	0.459	3.893
(1)	Growth of land supply = 1%	11.12	73.80	0.240	2.195
(2)	Growth of land supply = 0.0%	16.61	187.63	0.479	5.277
(3)	Income stabilizes in 20 years	15.35	114.84	0.455	3.231
(4)	Income stabilizes in 40 years	14.22	167.04	0.455	4.698
(5)	Income = 2/3 × Hong Kong	22.53	228.12	0.728	6.400
(6)	Income = Hong Kong	29.87	297.38	0.963	8.617
(7)	Income = 2 × Hong Kong	46.17	502.44	1.578	14.091
(8)	Initial Fin'l wealth = 0.2 Million	18.52	144.18	0.484	3.916
(9)	FAR≤3	15.56	267.59	0.465	7.136

This table reports the equilibrium price and rent for Beijign housing market in 2014 and 2114 under alternative model specifications.

5.3.1 Alternative Projection of Land Supply

In the baseline model, we have assumed that aggregate land supply grows at a rate of 0.5% since 2014. To gauge the sensitivity of the quantitative results to land supply, we examine two alternatives: the growth rate of land supply being either 0% or 1% after 2014. The corresponding land supply per capita is shown in the right panel of Figure 2 (dashed and dotted lines respectively). The market clearing paths of prices and rents are re-computed, and the results are summarized in rows labeled (1) and (2) in Table 6.

With more abundant land supply, house price and rent are uniformly lower compared with the baseline case. In addition, the growth rates of price and rent are lower too. For example, house price increases by about ten-fold from 2014 to 2114 under the baseline model, but it grows less than seven times when the growth of aggregate land supply is 1% per year.

When land supply is less abundant, both house price and rent are higher. By the time the economy converges to the BGP, both house price and rent in the case of zero land supply growth are about 36% higher than the baseline case of 0.5% land supply growth. House price in 2014 under no land supply growth is 16.61 thousand RMB per square meter, which is still significantly lower than the level of 30 thousand RMB reported by the National Bureau of Statistics. Thus we conclude that more stringent land supply does not help much to narrow the gap between house price in the real data and that implied by the model.

5.3.2 Alternative Projection of Income Growth

The baseline scenario assumes that over the next 30 years, the growth rate of aggregate income declines from 7% to 3%. Table 6 rows (3)-(4) show how shorter (20 years) or longer (40 years) periods of high income growth affects the equilibrium house prices and rents.

Two effects arise: substitution effect and income effect. Compare the case of shorter period of high income growth (i.e., aggregate income declines from 7% to 3% over the next 20 years) with the baseline model. First, because of the slower income growth, households have a stronger incentive to save for the future, which leads to a higher housing investment demand but weaker housing consumption demand. This substitution effect is immediate. Second, as illustrated by the right panel of Figure 2, the level of aggregate income is uniformly lower, generating less housing demand – for both consumption and investment purpose. The impact on rental rate is always negative because both effects work in the same direction. But the two effects have opposing implications for house price, so the net impact on house price depends on the relative strength of the two effects.

We first consider house price and rent in 2114 when the economy converges to the BGP. Shorter period of high income growth implies significantly lower house price and rent in the BGP, due to the strong income effect (which lasts throughout the transition periods because the difference in income level between the two cases continues to widen as shown in the right panel of Figure 2) and the phasing out of the substitution effect (there is no difference in income growth rate between the two scenarios after 30 years). However, for the current (i.e., year 2014) house price and rent, it turns out the opposite is true: the income effect is weak but the substitution effect is strong. If high income growth stops sooner, the implied current house price is higher. Conversely, if the high income growth persists longer, the implied current house price is lower. In both cases the rental rate stays almost the same. It is only marginally affected by the income and substitution effects.

5.3.3 Sudden Change in Income Growth

In the baseline model, the income growth rate is assumed to change gradually. This subsection aims to gain some insights about how a sudden drop of income growth rate impacts the housing market, which is a concern of policy makers and practitioners.

Assume the trajectory of income growth in the baseline model is a common expectation among all the households before 2015, and households make decisions about consumption and housing investment based on such expectation. In the beginning of 2016, households observe a sudden drop in the growth rate of income, then they adjust expectation of future income growth and re-optimize their housing demand accordingly. Here we deviate from perfect foresight assumption, because the sudden drop of income growth in 2016 is not expected before it occurs.[19]

Figure 6 shows the effects of a sudden drop in income growth in 2016 from about 7% to either $GY_{2016} = 6\%$ or $GY_{2016} = 4\%$ which then declines to 3% linearly between 2016-2044. Rental rate drops immediately, then grows steadily again. The response of house price is mild initially, but the cumulative effect is large: by the time the economy converges to the BGP, the equilibrium house price is lower than the baseline price by 17.4% and 31.9% for the case of $GY_{2016} = 6\%$ or $GY_{2016} = 4\%$ respectively.

Note that the response of rent here is quite different from its response to gradual changes of income growth. The reason is that a sudden decline of income growth brings strong and immediate income effect. For house price, the response still depends on the relative strength of income and substitution effects. Therefore, the model predicts a higher growth rate of house price in 2016 (compared to the baseline model) when $GY_{2016} = 6\%$, but a lower growth rate when $GY_{2016} = 4\%$ because the income effect dominates in the later case.

[19]Our model does not incorporate the possibility of default by home owners. In reality, a sudden and unexpected decline of income growth income may cause some low income households to default, generating larger price declines, as discussed in Fang, Gu, Xiong, and Zhou (2015).

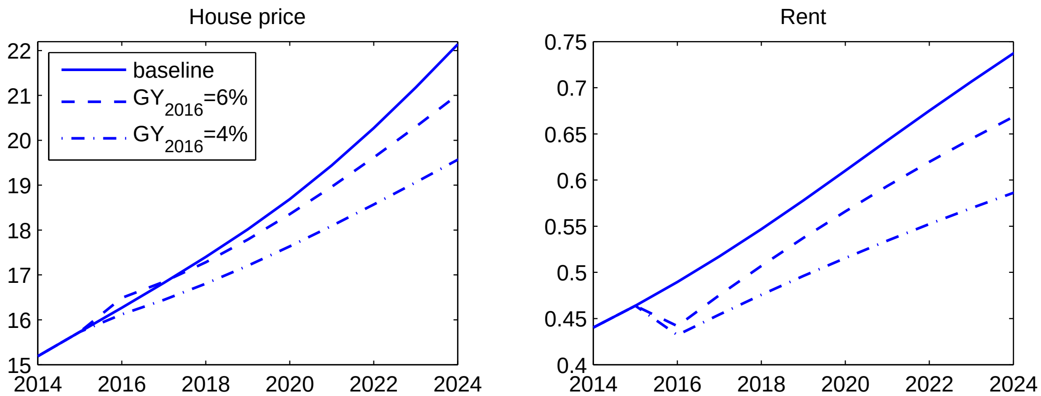

Figure 6: House price and rent in response to a sudden drop in income growth

This figure shows the paths of price and rent when the income growth rate drops suddenly in 2016.

5.3.4 Higher Income Level

This subsection considers the possibility that the current level of income reported by the NBS may be understated. The reported income is based on annual surveys that sample 5000 households in Beijing. The sample includes two major components: residents with Beijing Hukou and residents without Beijing Hukou but living in Beijing for over 6 months every year.

The reported income can be understated for several reasons. First of all, a large number of rich households in the rest of China purchase house in Beijing for investment purposes, for easier access to facilities, services and conveniences. Some of these home buyers live in Beijing for only short periods of time hence are not included in the survey. Secondly, corrupted officials would report extremely mediocre income although their actual income is considerably higher than the average. Recent years see increasing news reports about officials owning dozens of condominiums in Beijing.

Rows (5)-(7) of Table 6 report the results corresponding to three different income levels for year 2014: two-thirds, or one time or twice of the average disposable income in Hong Kong in 2014. For each case, income growth rate is assumed to remain the same as in the baseline model.

Given higher income levels, both house price and rent increase unequivocally. When Beijing has the same average income as Hong Kong, the implied house price in 2014 is 29.87

thousand per square meter (in terms of 2012 RMB), close to the actual price level in the data. The implied rental rate is 963 RMB per square meter per year, higher than 700 RMB observed in the data. In other words, the current high price in Beijing can be rationalized by assuming that average income of Beijing house buyers in 2014 is similar to that in Hong Kong, but then according to our model, the current rental rate is too low at this level of income.

5.3.5 Alternative Initial Wealth

We consider the case of higher initial financial wealth, raising it from 70.37 thousand to 200 thousand RMB. Results are reported in row (8) of Table 6. In this case, the implied house price is 18.52 thousand RMB per square meter in 2014, a rise of 24% relative to the baseline. The rise of rental rate, however, is only 5.5%. Intuitively, rational households will spread the additional initial wealth over time for extra consumption, hence the rental rate (the price of housing consumption) rises only marginally. As the households consume the additional wealth gradually, the extra demand for housing investment diminishes over time, thus the gap between the new house price path and the baseline price path shrinks over time. By the time the economy converges to the BGP, the effect of higher initial wealth is almost negligible.

In unreported exercise, we find that in order to match the price of 30 thousand RMB per square meter as in the 2014 data, the initial wealth of Beijing residents (as of year 2005) needs to be 500 thousand RMB, much higher than the estimate of 70.37 thousand RMB based on the data (see Section 4.2.1). This illustrates from another angle that house price in Beijing is too high relative to the fundamentals based on our model.

5.3.6 Upper Bound on Floor-area Ratio

We have shown that, in the absence of building restriction, floor-area ratio (FAR) in Beijing should be between 5 and 6 when the economy converges to the BGP. Afterwards it will grow at the constant annual rate of $(G_Y/G_L)^{1-\theta} - 1$ which is about 0.82% according to our calibration. Here we ask what happens if the government imposes an upper-bound on FAR, denoted $\overline{FAR}$, so that urban density is restricted. This question has realistic relevance. In March 2008, Beijing City Planning Committee and Beijing City Land Resources Bureau jointly issue a regulation called "*Beijing City Construction Land Saving Standards*", stipulating that the FAR of residential area should not exceed 2.8.

In row (9) of Table 6, we assume $\overline{FAR} = 3.0$ and re-compute the equilibrium house prices and rents.[20] By potentially limiting house supply, this policy has a positive impact on price and rent. The impact is quite small in year 2014. By 2114, however, the policy raises price by 93% and rent by 83% relative to the baseline scenario.

5.4 Endogenous Migration

In this subsection, the exogenous projection of migration between 2014-2044 in the baseline model is replaced by endogenous migration. We allow rich households outside Beijing choose to migrate to and buy houses in Beijing, which generates the sorting of rich households into a superstar city, a process discussed in Gyourko, Mayer, and Sinai (2013b).

5.4.1 Migration Decision

In this extended model, the benefit of migration to Beijing is measured by a utility gain. In line with the theoretical discussion in Henderson and Becker (2000), we assume that, given the same consumption level, households enjoy less utility if they do not migrate to Beijing. Specifically, the utility function becomes $u(\phi c, \phi\psi h)$ where ϕ measures, in the form of consumption equivalence, the disadvantage of not having a home in Beijing in the following way:

$$\phi \begin{cases} = 1, & \text{if migrating to Beijing;} \\ < 1, & \text{otherwise.} \end{cases}$$

The migration decision of a household is the result of a standard discrete choice model. In each period of the life cycle, the household compares the value of migrating with the value of not migrating. When the former is larger than the latter, the household sells her existing houses and becomes a home owner in Beijing.[21] This generates extra demand in the Beijing market. The demand of an immigrant evolves over the remaining lifetime. Upon death of immigrants, the housing equity is distributed evenly to all the households in Beijing.

[20] Technically, whenever the implied FAR exceeds $\overline{FAR}$, we set housing supply to $\overline{FAR} \times L^*$, where L^* is the exogenous land supply by the government.

[21] As in the baseline model, the down payment is 40% of the house price. We assume that housing size in Beijing is at least 30 square meters. The downpayment requirement implies that only relatively rich households elsewhere can afford to migrate to Beijing.

5.4.2 Home Market of Migrants

We need to characterize the home market of potential immigrants to Beijing. Immigrants to Beijing and Shanghai are mainly from other cities in China, especially the so-called second-tier cities. We consider a representative second-tier city based on Wuhan, Chengdu, Dalian and Xi'an. As can been seen from the upper panel of Table 7, the average house price and annual rent in 2014 are around 9000 RMB and 300 RMB per square meter respectively for the representative second-tier city, less than half of Beijing. Average disposable income per capita is about 3/4 of Beijing. Thus, the implied price-rent ratio and price-income ratio are much lower than in Beijing.

For the second-tier city, we project the current price-income ratio and price-rent ratio to persist through the transition periods and in the BGP. Return on housing investment is assumed to be 4.95%, the same as Beijing in the BGP. Together with the price-rent ratio of 30, this implies that the growth rate of house price in the BGP is 1.62% for the second-tier city, lower than the 2.14% for Beijing, which is consistent with the fact that over the past decade, the growth rate of house price for second-tier city in China has been about 1/3 of that in Beijing.

Since the income growth rate in the second-tier city is about the same as that in Beijing, their lower growth rate of house price comes necessarily from higher growth rate of land supply, which is clear from point 5 of the ***Proposition***. We use G_L^0, G_p^0 and G_Y^0 to denote the growth factors of land, house price and income for the second-tier city in the BGP, and use G_L, G_p and G_Y to denote the same variables for Beijing. Since income growth rates are equal ($G_Y^0 = G_Y$), we have

$$G_L^0 = G_L \left(\frac{G_p}{G_p^0} \right)^{\frac{1}{\theta}} . \tag{35}$$

Based on equation (35), we find $G_L^0 = 0.077\%$, as opposed to $G_L = 0.05\%$ for Beijing. For simplicity, we assume the growth factor of house price and rent equal $(G_{Y_t}^0 / G_{L_t}^0)^{\theta}$ for the second-tier city during the transition, where $G_{Y_t}^0$ and $G_{L_t}^0$ are the growth factors of income and land supply during the transition. We also assume that out-migration does not negatively affect the house prices and rents of their home cities since the out-migrants amount to less than 0.3% of the population in the data. Under the above assumptions (summarized in the lower panel of Table 7), we compute the path of house prices and rents for the second-tier city, together with their endogenous migration decision.

Table 7: A comparison of out- and in-migrating cities

	Second-tier City	Beijing
Data		
House Price (per m^2)	9,000	30,000
Rent (per m^2 per year)	300	700
Income	30,000	40,000
Initial Housing Asset	30 m^2	30 m^2
Initial Financial Asset	3.68 × income	3.68 × income
Assumptions		
Growth of Price (BGP)	1.62%	2.14%
Growth of Rent (BGP)	1.62%	2.14%
Price/Rent (BGP)	30	35.6
Growth of Rent (transition)	exogenous	endogenous
Growth of Price (transition)	exogenous	endogenous
Growth of Land supply	0.077%	0.05%

5.4.3 Calibration

The extended model has one additional parameter ϕ that measures the consumption equivalence disadvantage of not migrating to Beijing. This parameter is pinned down by matching the share of immigrated workers in Beijing population between 2010-2013, which is 2.88% as discussed in Section 4.1.1. The migration rate also depends on the growth rate of price and rent in Beijing. For example, a higher growth rate of house price attracts more immigrants. In our analysis, for each guess of ϕ, we solve for the market clearing paths of price and rent, and then obtain the implied migration rate. This process repeats until the implied migration rate matches 2.88%. We find that $\phi = 0.723$, thus the disadvantage of not migrating to Beijing amounts to 27.7% of the average annual consumption of residents in the second-tier cities.[22]

[22]As a reference, the price of a Bejing Hukou is 300,000 RMB on the black market, according to a news report by *The Beijing News* on August 25, 2014.

Table 8: Prices and Rents from Endogenous Migration Model

	Price		Rent	
	2014	2114	2014	2114
Baseline	14.92	138.45	0.459	3.893
Income = Hong Kong	29.87	297.38	0.963	8.617
Endogenous Migration	31.65	219.29	0.814	6.155

5.4.4 Equilibrium House Prices and Rents

Table 8 reports the prices and rents from the endogenous migration model. For comparison, we include in the table the results from baseline model and the hypothetical case where the 2014 income in Beijing equals that of Hong Kong. As of 2014, house price from the extended model is 31.65 thousand RMB, which is fairly close to the observed market price. The rental rate is RMB 814 per square meter per year, slightly higher than the data. The endogenous migration case matches the rent rate in the data better than the case of "income = Hong Kong". It implies a lower rental rate because all the rich households who endogenously migrate to Beijing are home owners.

Therefore, the high price-income ratio and price-rent ratio in Beijing can be rationalized in the model with endogenous migration. There are a number of contributing factors. First, immigrants have much higher (on average 7-10 times higher) income than the average residents of Beijing. Second, these rich immigrants bring large amount of assets to Beijing, which boosts up housing demand immediately. These assets become bequest to the future generations, keeping housing demand at a high level.

5.5 Equilibrium House Prices Compared with the Historical Data

We now compare equilibrium house prices implied by various implementations of our model with the data between 2005-2014. Tsinghua Hang Lung Center for Real Estate provides nominal constant-quality house price indices of Beijing between 2006-2014. Adjusting the levels by urban CPI, the real growth rates between 2007-2014 are calculated. Real growth rates between 2005-2006 are taken from Fang, Gu, Xiong, and Zhou (2015). From these growth rates we calculate the price levels.

Figure 7 reports the results from the following four cases: (i) the baseline; (ii) initial

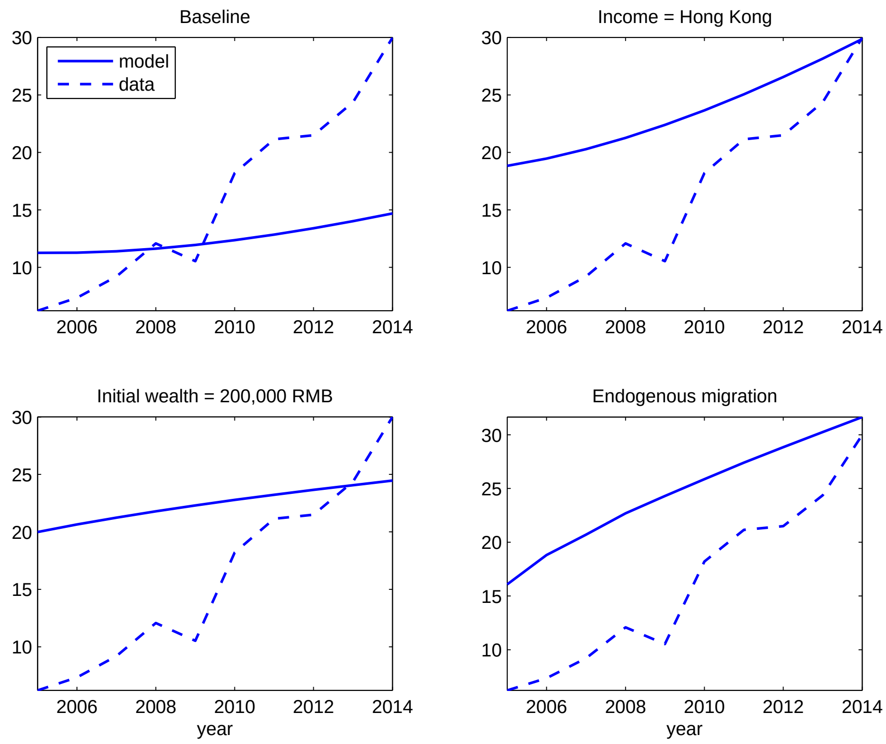

Figure 7: House Price 2005-2014

This figure compares equilibrium house prices from the model with those from the data between 2005-2014. The "income = Hong Kong" case assumes income in Beijing and Hong Kong are equal in 2014. The "initial wealth = 200,000 RMB" case assumes that average financial wealth in 2014 is 200 thousand RMB.

financial wealth equals 0.2 million; (iii) income level equals Hong Kong; and (iv) endogenous migration. In the first three cases (without endogenous migration), growth rate of house price is lower than in the data. Overall, the model with endogenous migration performs the best in terms of matching the data. Nevertheless, it still implies lower house growth rates than the data between 2005-2014. This indicates that market prices are not fully rational or some additional considerations are needed to explain the market prices based on the fundamentals. Interestingly, in all four cases, the model implied house price is significantly above the market price in 2005, suggesting underpricing of houses in the early stage of housing sector privatization.

6 San Francisco Housing Market

This section applies our framework to study the housing market of the San Francisco-San Mateo-Alameda metropolitan area between 1970-2013. We assume that the SF market operates in the BGP since 2013. Using the 2013 price-income and price-rent ratios as the terminal condition, we go backward in time and derive the model equilibrium house prices and rents since 1970 based on rational expectation of evolutions of income, population structure and land supply observed in the data.

Table 9: Moments and Parameters for SF Market

Preference Parameters		Matched Moments	Target	Model
γ	1.695	Price-income ratio	7	7.05
ψ	0.999	Price-rent ratio	25	25.00
β	0.977	Home ownership rate	0.37	0.487
ω	0.318	Age of first-time buyers	31	33.77
$\mathcal{B}$	5.017	surplus (consumption MKT)	0	0.011
		surplus (investment MKT)	0	0.00
Production Parameters		**Other statistics (year 2013)**		
θ	0.803	Income per capita	40,000	(dollar)
δ	0.020	Average house price	7,000	(dollar/m^2)
Z	0.797	Down payment ratio	20%	

This table reports the parameters used for solving paths of price and rent in the SF market, along with the moments used to pin down these parameters.

The data on population and disposable income are obtained from regional economic account of the Bureau of Economic Analysis (http://www.bea.gov/regional/). We are not able to find historical data on land stock for this area, and have to resort to estimation. Based on the relation that housing size=(price-income ratio)×(average income)/price, we estimate per capita housing size in SF to be 40 square meters (430.6 square feet) in 2013. Based on houses listed on the market, we find it a reasonable assumption that the ratio of lot size to house size is 0.7. Therefore, residential land per capita in 2013 is $40/0.7 = 57.1$ square meters (615 square feet). To obtain residential land supply prior to 2013, we employ the equation $G_L = G_Y/G_q \approx 1.01$ implied in the ***Proposition***. Using the time series data of land price and house price during 1982-2013, we back out G_L, the growth factor of land, then derive the path of land stock since 1970.[23]

The same strategy as in section 4.3 is used to pin down parameter values for the SF market. For the production function, θ is determined by the growth factors of house price and land price. During the period of 1982-2013, the average value of $log(G_p)/log(G_q)$ is 0.802, hence $\theta = 0.802$. Using equation (31), Z=0.815. The value of δ is again set to 2%. The model parameters and related moments are reported in Table 9.

The stochastic processes of income and medical expense specific to the SF area are not available. Instead we rely on the US national data. We use $\rho_y = 0.96$, $\sigma_\epsilon^2 = 0.044$, $\rho_m = 0.922$ and $\sigma_\eta^2 = 0.524$. Here the income process is estimated from 1989-2009 PSID data; while the medical expense process is from French and Jones (2004).

Distribution of wealth in 1970 requires household survey on wealth which is not available. From Survey of Consumer Finance (SCF) 1989-2007, we estimate the average wealth to income ratio of 1.643. The average disposable income in 1970 is 19.83 thousand in terms of 2013 dollars. Therefore the average initial wealth is $1.643 \times 19.83 = 32.58$ thousand dollars in 1970. Age-profile of wealth is also estimated from the SCF data.

Results for the SF market are reported in Figure 8. The main interest is to compare the price path implied by the model to that in the data represented by the dashed line. Here price path is the CPI-adjusted Case-Shiller house price index for SF area which dates back to 1982, and the rental rate is calculated based on price-rent ratio in 2013 and the shelter price index of San Francisco-Oakland-San Jose area provided by BLS. To account for inflation, we adjust the rental rate by the price index of non-shelter goods.

The figure shows that our model captures the growth trend of house price in SF from 1982 quite well. The model-implied rental path is consistent with the data except briefly for the periods of 1970-1974 and after 2005. In the data, these two periods exhibit strong growth of per capita income. Income growth is translated into growth of rental rate in the model, but it is not reflected in the rental rate from the data.

[23] Land price data are available from the website of Land and Property Values in the U.S., Lincoln Institute of Land Policy. http://www.lincolninst.edu/resources/. For details about the data, see Davis and Palumbo (2007).

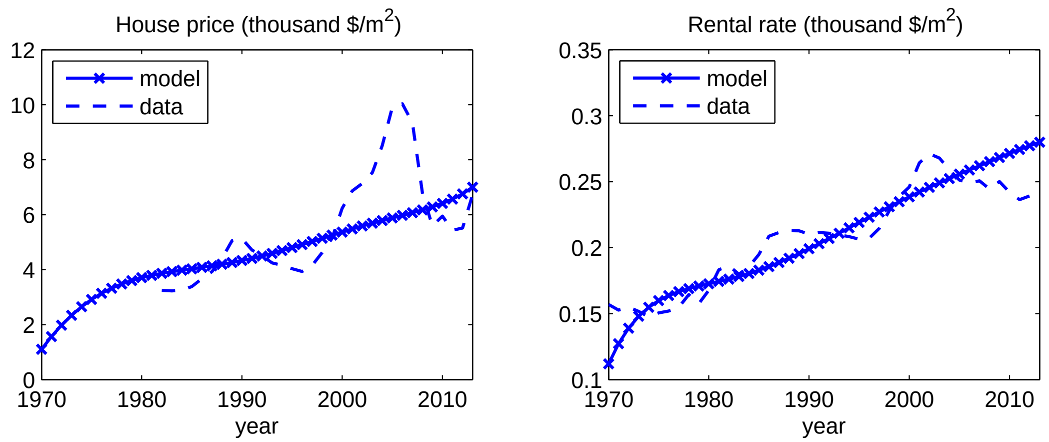

Figure 8: House price and rent in San Francisco

This figure shows the paths of house price and rent implied by the model for San Francisco. House price labeled "data" is Case-Shiller price index, and rent in the data is calculated from price index of shelter in the SF area.

7 Conclusion

This paper presents a dynamic rational expectations general equilibrium model that links the house price and rent to economic fundamentals including income growth, land supply, change of population structure and migration. Our model is general enough to deal with non-stationary fundamentals in emerging markets. We apply the model to the Beijing housing market, and examine to what extent house price and rent can be rationalized by rapidly changing economic fundamentals. We solve prices and rents in closed forms for the balanced growth path, and develop an efficient numerical method to compute the trajectories of equilibrium house price, rent and land price during the transition periods.

We find that the current house price in Beijing is too high compared to the model-implied rational price under most reasonable parameterizations of our model. However, our model explains well the historical growth trends of house price and rent in the San Francisco market. In addition, we find that the current high price-rent ratio and price-income ratio in Beijing are consistent with an extended version of the model where households in second-tier cities choose optimally whether to migrate to Beijing. Therefore, high price-income and price-rent ratio themselves may not be indicative of price bubble in a transition economy. We also find that, as the Chinese economy develops further, price-income ratio declines and housing affordability improves over time.

Based on simulated data from the model, we show that the empirical relations between house price (or rent) and economic fundamental variables during the economic transitions are dynamic and complex, not well captured by linear regressions, although they work well in the balanced growth path. Therefore while regression analysis is useful for studing price-fundamentals relation in developed markets, it can be quite misleading for emerging markets.

Since the model provides a structural link between price and fundamentals, it can be used to analyze the impacts of a number of policies, including the implementation of property tax, tightening of immigration restrictions, and provision of universal medical insurance. We leave these for future studies.

Appendices

A Proof of the *Proposition*

Given the exogenous growth factors of income and land supply $\{G_Y, G_L\}$, we will show that the growth factors of prices $\{G_p, G_r, G_q\}$ and the growth factors of the choice variables of the firm and households, as proposed in the ***Proposition***, satisfy the general equilibrium conditions. Specifically they are consistent with: (i) the optimization problem of the firm; (ii) the optimization problem of households; (iii) the market clearing conditions.

We use K_t^*, L_t^* and H_t^* to denote the firm's optimal decisions in period t, and $c_{a,t}^*$, $h_{a,t}^*$, $s_{a,t}^*$ to denote an individual's optimal decisions in period t. Further let p_t, r_t, and q_t be the market clearing prices. By definition, these choice variables and prices satisfy the general equilibrium conditions. We will show that, using the proposed growth factors, $K_{t+1} = K^* G_K$, $L_{t+1} = L_t^* G_L$, $H_{t+1} = H_t^* G_H$, $c_{a,t+1} = c_{a,t}^* G_c$, $h_{a,t+1} = h_{a,t}^* G_h$, $s_{a,t+1} = s_{a,t}^* G_s$, $p_{t+1} = p_t G_p$, $r_{t+1} = r_t Gr$ and $q_{t+1} = q_t G_q$ will also satisfy the above three conditions in period $t+1$. Note that for households, what we need to show is that the decisions of a later cohort (denoted by the subscript (a,t+1)) is the same as the earlier cohort (denoted by (a,t)) up to the scale factors as proposed in the ***Proposition***, which guarantees that average consumption and investment will grow at the proposed factors because the age distribution of households is time-invariant in the BGP.

Firm's Optimization Problem First we show the growth factors are consistent with the firm's flow of fund which is the following in period t.

$$p_t(H_t^* - H_{t-1}) = K_t^* - (1-\delta)K_{t-1} + q_t(L_t^* - L_{t-1}).$$

To show the above equation also holds in period $t+1$, we multiply both sides of the above equation with G_Y and apply the proposed growth factors. The left side of the equation is

$$\begin{aligned}
p_t(H_t^* - H_{t-1})G_Y &= p_t(H_t^* - H_{t-1}) \left(\frac{G_Y}{G_L}\right)^{\theta} G_Y^{1-\theta} G_L^{\theta} \\
&= p_t(H_t^* - H_{t-1}) G_p G_H \\
&= p_t G_p (H_t^* G_H - H_{t-1} G_H) \\
&= p_{t+1}(H_{t+1}^* - H_{t-1} G_H).
\end{aligned}$$

The right side is

$$\begin{aligned}\left[K_t^* - (1-\delta)K_{t-1} + q_t(L_t^* - L_{t-1})\right] G_Y &= K_t^* G_Y - (1-\delta)K_{t-1}G_Y + q_t(L_t^* - L_{t-1})G_Y \\ &= K_t^* G_K - (1-\delta)K_{t-1}G_K + q_t(L_t^* - L_{t-1})\frac{G_Y}{G_L}G_L \\ &= K_{t+1}^* - (1-\delta)K_{t-1}G_K + q_t(L_t^* - L_{t-1})G_q G_L \\ &= K_{t+1}^* - (1-\delta)K_{t-1}G_K + q_{t+1}(L_{t+1}^* - L_{t-1}G_L).\end{aligned}$$

Therefore, given state variables $\{K_t, L_t\} = \{K_{t-1}G_K, L_{t-1}G_L\}$ we have

$$p_{t+1}(H_{t+1}^* - H_t) = K_{t+1}^* - (1-\delta)K_t + q_t(L_{t+1}^* - L_t)$$

Next, we show the proposed growth factors are consistent with the firm's first-order condition with respect to K which is

$$A(1-\theta)r_t\left(\frac{K_t^*}{L_t^*}\right)^{-\theta} = 1 - \frac{1-\delta}{R_t} \tag{36}$$

First of all, notice that in the BGP, $G_p = G_r$, so

$$R_t = p_t/(p_{t-1} - r_{t-1}) = p_t G_p/(p_{t-1}G_p - r_{t-1}G_r) = p_{t+1}/(p_t - r_t) = R_{t+1}.$$

Thus, given (36), to prove the firm's first-order condition with respect to K in period $t+1$, it suffices to show $r_t(K_t^*/L_t^*)^{-\theta} = r_{t+1}(K_{t+1}^*/L_{t+1}^*)^{-\theta}$, which is straightforward using the proposed growth factors. Thus we omit the details and conclude that

$$A(1-\theta)r_{t+1}\left(\frac{K_{t+1}^*}{L_{t+1}^*}\right)^{-\theta} = 1 - \frac{1-\delta}{R_{t+1}}$$

Third, we show the proposed growth factors are consistent with the firm's first-order condition with respect to L which is

$$A\theta r_t\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta} = q_t - \frac{q_{t+1}}{R_t}$$

We need to show that

$$A\theta r_{t+1}\left(\frac{K_t^* G_K}{L_t^* G_L}\right)^{1-\theta} = q_{t+1} - \frac{q_{t+2}}{R_{t+1}}$$

Starting from the left-side, we have

$$
\begin{aligned}
A\theta r_{t+1}\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta} &= A\theta r_t G_r \left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_K}{G_L}\right)^{1-\theta} \\
&= A\theta r_t \left(\frac{G_L}{G_Y}\right)^{\theta}\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_Y}{G_L}\right)^{1-\theta} \\
&= A\theta r_t \left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_L}{G_Y}\right)^{\theta}\left(\frac{G_Y}{G_L}\right)^{1-\theta} \\
&= A\theta r_t \left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\frac{G_Y}{G_L} \\
&= \left(q_t - \frac{q_{t+1}}{R_t}\right) G_q \\
&= q_{t+1} - \frac{q_{t+2}}{R_{t+1}}
\end{aligned}
$$

where the last equality holds because $R_t = R_{t+1}$.

Household's Optimization Problem Using the same strategy as with the firm's flow of fund equation, it is straightforward to show that the growth factors of $c_{a,t}$, $s_{a,t}$, $h_{a,t}$ are consistent with the households budget constraint as in equation (22). We omit the algebraic details. For reader who are interested, keep in mind that the medical expense should growth at the same factor as income, because we have assumed that medical expense is a fixed proportion of income.

The intra-temporal optimal allocation in household's problem is governed by equation (25). It is also straightforward to show that the proposed growth factors are consistent with this equation.

The household's inter-temporal first-order condition may fail to hold due to the no short-sale constraint. To complete the proof that the proposed growth factors are consistent with household's optimization problem, we show that the functional equation (21) is re-scalable so that if $c_{a,t}^*$, $h_{a,t}^*$, $s_{a,t}^*$ solve the optimization in period t, $c_{a,t+1} = c_{a,t}^* G_c$, $h_{a,t+1} = h_{a,t}^* G_h$, $s_{a,t+1} = s_{a,t}^* G_s$ will solve the problem of households with the same age in period $t+1$.

First, using the growth factors proposed the ***Proposition***, we show both the bequest value and the utility function can be re-scaled by $\left(G_Y^{1-\theta\omega} G_L^{\theta\omega}/G_N\right)^{1-\gamma}$. From equation (23), in period $t+1$ we have

$$\begin{aligned}
V_b(s_{a,t+1}) &= \mathcal{B}\left[(1-\omega)^{1-\omega}(\psi\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_{t+1}}\right)^{\omega(1-\gamma)}\frac{(p_{t+1}s_{a,t+1})^{1-\gamma}}{1-\gamma}\\
&= \mathcal{B}\left[(1-\omega)^{1-\omega}(\psi\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_{t}}\right)^{\omega(1-\gamma)}\frac{(p_{t}s_{a,t})^{1-\gamma}}{1-\gamma}\left(\frac{G_pG_s}{G_r^{\omega}}\right)^{1-\gamma}\\
&= V_b(s_{a,t})\left(\frac{G_pG_s}{G_r^{\omega}}\right)^{1-\gamma}\\
&= V_b(s_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}, \qquad (37)
\end{aligned}$$

where we used the proposed growth factors for the last equality. Similarly we have

$$u(c_{a,t+1},h_{a,t+1}) = u(c_{a,t},h_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}. \qquad (38)$$

Finally, using equation (37) and (38), we can show via backward induction that the household's value function at any age can be re-scaled by $\left(G_Y^{1-\theta\omega}G_L^{\theta\omega}/G_N\right)^{1-\gamma}$. This property, combined with the fact that the household's budget constraint is consistent with the proposed growth factors, implies that the functional equation (21) is re-scalable.

Market Clearing We need to show that the proposed growth factors are consistent with the clearing of land market, housing rental market and equity market. For the land market, it is sufficient to show that land demand grows at the same factor as the exogenous land supply which is denoted G_L. The firm chooses land input in housing production according to land price. This optimization mechanism is represented by the first-order condition with respect to L_t, i.e., equation (6). Using G_L^d to denote the growth factor of land demand in the BGP, we have the following from equation (6).

$$G_r\left(\frac{G_K}{G_L^d}\right)^{1-\theta} = G_q,$$

which leads to

$$G_L^d = \left(\frac{G_r}{G_q}\right)^{1/(1-\theta)} G_K. \qquad (39)$$

Replacing G_r, G_q, and G_K with expressions in the ***Proposition***, we get $G_L^d = G_L$, hence the growth factors satisfy the land market clearing condition.

Now we show that the growth factors satisfy the housing market clearing condition. Aggregate housing supply grows at a factor of G_H, while individual housing demand grows at a factor of G_h. We need to prove that $G_H/G_N = G_h$. Given the housing production function, and given that $G_Y = G_K$, we have $G_H = G_Y^{1-\theta} G_L^{\theta}$, hence

$$\frac{G_H}{G_N} = \frac{G_Y^{1-\theta} G_L^{\theta}}{G_N} = \left(\frac{G_Y}{G_N}\right)^{1-\theta} \left(\frac{G_L}{G_N}\right)^{\theta}. \tag{40}$$

Therefore, these growth factors G_H and G_h as given in points 2-3 of the ***Proposition*** satisfy the housing market clearing condition. The same argument applies to home equity market clearing with G_h replaced by G_s.

FAR, Price-income Ratio and Price-rent ratio We have shown that growth factors in points 1-7 of the ***Proposition*** satisfy all the general equilibrium conditions. Using these growth factors, we have $FAR_{t+1}/FAR_t = \frac{H_{t+1}/L_{t+1}}{H_t/L_t} = \frac{G_H}{G_L} = \frac{G_Y^{1-\theta} G_L^{\theta}}{G_L} = (G_Y/G_L)^{1-\theta}$. Also it is straightforward to show that both price-income ratio and price-rent ratio are time-invariant. Hence points 8-9 of the ***Proposition*** are both true.

B Computation Strategy

The model is solved numerically. House price, rental rate, land price and housing quantity for the economy in BGP can be directly calculated from the terminal conditions discussed in details in Section 4.3.2. For the transition periods, the following procedure is used to solve for housing supply, demand and market clearing paths of house price, rent and land price.

1. Guess a path of house price and a path of rent.
2. Compute the corresponding supply and demand of housing consumption and housing equity at each point in time.
3. Derive the path of land price using equation (12).
4. Check the difference between supply and demand; iterate the steps above until markets clear.

One major technical challenge we face is to find the market clearing prices and rents for about one hundred years during the transition period. We assume that growth rates of income, population and land all become constant 30 years after 2014, i.e., 2044. Then the economy will eventually converge to the BGP when the last generation that enters into the economy before 2044 dies out. We assume that a household can live up to 76 years after entering into the economy, so there exists a total of 76 cohorts in the population at any given point in time, and the whole transition process takes 30+76=106 years. In addition, we solve for market clearing prices and rents between 2005-2014. In total, we need to solve for 116 house prices and 116 rental rates. This is extremely difficult to achieve using standard search algorithms because of the large dimensionality in the unknowns.

Instead of relying on mechnical updating schemes from the standard search algorithms, we update the price and rent paths in the direction to balance the supply and demand in the housing equity and rental markets under the model. The rental market is relatively easy to clear because supply and demand of rental housing are determined by current rental rate while future rental rates and house prices do not play any role. Thus our search algorithm simply increases the rental rates for any periods when demand exceeds supply, and vice versa.

To clear the housing equity market, it is important to take into account the current and future returns to equity investment which consist of dividend (rent) and capital gain (price appreciation). Suppose that demand exceeds supply between periods t and $t+j$, our search algorithm consists of the following three adjustments: (i) increase price in period t; (ii) decrease the growth rates of price between period t and $t+j$; (3) decrease the rents between period t and $t+j$. These adjustments not only reduce demand for housing equity, but also increase the supply because they reduce the financing cost of the firm.

We approximate the paths of house price and rent as functions of time, using six-order polynomials. Since there is no aggregate uncertainty in the model, both price and rent paths are smooth. Therefore polynomial approximations work very well. Each iteration to update price and rent paths boils down to updating the polynomial coefficients.

In practice, we start with a path of price and a path of rent, both increasing over time. We update next the path of house price based on the supply and demand of housing equity, and then update the path of rent based on the supply and demand of rental market. We use large-grid updating in the beginning, then gradually reduce the size of updating, until the paths of supply and demand converge.

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