

A Dynamic Model of Systemic Bank Runs*

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This draft: March 2019

Abstract

This paper develops a tractable dynamic model to study bank runs in a financial system, featuring the linkage between bank runs and asset market prices. The model speaks to the evolution of a systemic crisis. In our model economy, there are many banks and they share a common asset market. The interim liquidation value of a bank is endogenous and depends on its asset fundamentals, on the status of other banks and hence the aggregate fire sales in the system, and on the endogenous path-dependent market liquidity condition. We analytically characterize the dynamics of the interplay between market liquidity and bank runs. Our model shows runs-illiquidity traps with two steady-state equilibria and a crisis process consisting of an initial gradual decline followed by a sudden sharp crash in the transitional dynamics. Our theory explains empirical facts and gives new policy implications.

JEL classification: G01; G20; E50; D82

Keywords: Systemic crises, bank runs, market liquidity, strategic complementarities

*For helpful comments on an earlier version of the paper, I am grateful to Liang Dai, Itay Goldstein, Zhiguo He, Christian Hellwig, Yan Ji, Ron Kaniel, John Kuong, Agnese Leonello, Abhiroop Mukherjee, John Nash, Deniz Okat, Tianxi Wang, Wei Xiong, Ming Yang, and seminar participants at HKU, CUHK, Peking University, Zhejiang University, City University of Hong Kong, HKUST, Society for Economic Dynamics (SED) Annual Meeting 2018, International Conference on Economic Theory and Applications 2018, PHBS Workshop in Macroeconomics and Finance 2018, and HK-SZ Great Bay Area Summer Finance Conference 2018.

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1 Introduction

At the heart of the recent financial crisis of 2007-2009 was a series of bank runs that caused failures or impairments of many financial institutions (Bernanke (2010), Gorton (2010), Gertler et al. (2017)). Empirical research has revealed two key facts about the *evolution* of the crisis:¹

Fact 1: (Co-movement between bank runs and asset prices) Since summer 2007, runs on financial institutions had been on a steady rise and *at the same time*, “discount rates” in asset (credit) markets — fire-sale discounts of asset-backed securities, repo rates for securitized bonds, and interbank lending rates — exhibited an upward trend.² Gorton and Metrick (2011b) and Covitz et al. (2013) document that the probability of runs at the bank or program level was strongly correlated with the LIBOR-OIS spread (a primary measure of interbank lending rates).

Fact 2: (Two-stage crisis) The initial gradual deterioration was followed by a sudden sharp crash — a jump discontinuity in asset prices together with systemic bank runs in September 2008, in the absence of any apparent large exogenous shock to economic fundamentals. In an urgent and aggressive response, the Federal Reserve implemented a series of unconventional interventions to boost asset market liquidity (Bernanke (2009)).

As an illustration, Figure 1 shows the time series data for the LIBOR-OIS spread (an indicator for the discount rates) and runs on asset-backed commercial paper (ABCP) programs.³

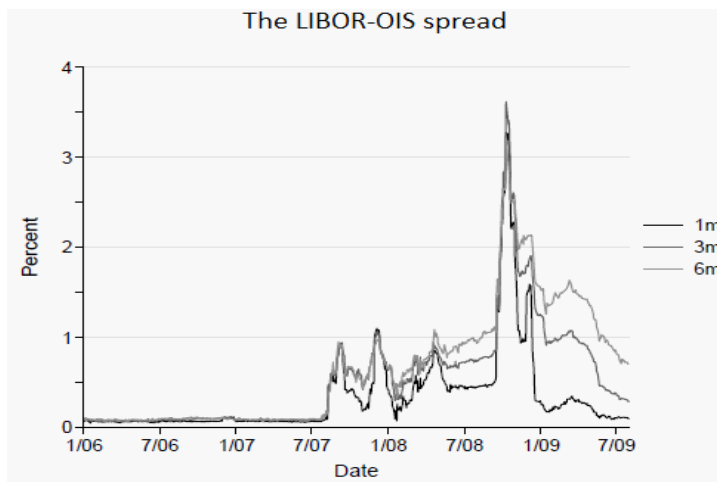


Figure 1(a): Interbank lending rates (source: British Bankers Association and Bloomberg)

¹Besides well-documented runs on commercial banks and investment banks, the modern-day bank runs occurred in the shadow banking system, such as the repo market (Copeland et al. (2014), Gorton and Metrick (2010, 2011a, 2011b), Krishnamurthy et al. (2014)), money market mutual funds (Duygan-Bump et al. (2013)), and the ABCP market (Covitz et al. (2013), Kacperczyk and Schnabl (2010), Acharya and Schnabl (2010)).

²All these rates are “discount rates” used in pricing assets. They all are essentially “exchange rates” of cash flows across time from the point of view of financial contracting. See also Vives (2014a).

³Covitz et al. (2013) report the result only in 2007. Based on the indirect evidence in Duygan-Bump et al. (2013), who study runs on ABCP money market mutual funds (while these funds in turn invest and run on ABCP programs), it is reasonable to conjecture that the trend in Figure 1(b) would continue in 2008 and there would be a spike in September 2008.

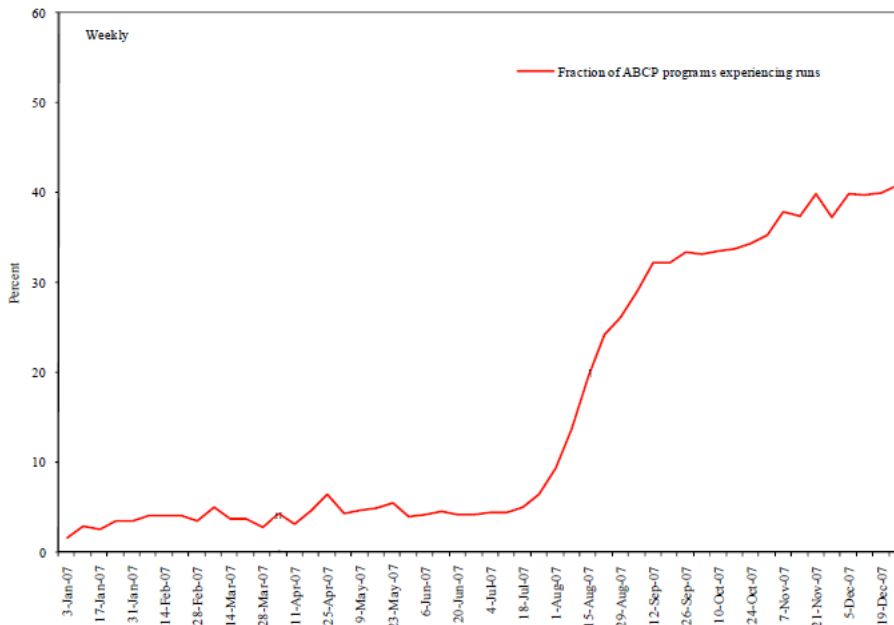


Figure 1(b): Runs on ABCP programs (source: Covitz et al. (2013))

Covitz et al. (2013) define that a program experiences a run in weeks when it does not issue paper but has at least 10% of paper maturing or when the program continues to not issue paper after experiencing a run in the previous week.

Why is there a co-movement between bank runs and asset prices? What gives rise to the two-stage crisis dynamics? Despite a large and growing literature studying banking and liquidity crises, very few theoretical papers have analyzed the joint phenomena and offered explanations for the joint dynamics. Indeed, the theory literature on bank runs pioneered by Bryant (1980) and Diamond and Dybvig (1983) typically does not feature a link to asset prices. In recent decades, however, the banking system has shifted heavily toward the “market-based” business model, including the rise of the originate-to-distribute model of securitization, special-purpose investment and financing structures to attract cheap short-term runnable funding, and an elongated intermediation chain tied with wholesale funding markets (see, e.g., Brunnermeier (2009) and Greenwood and Scharfstein (2013)). In this process, a large chunk of financial institutions (e.g., commercial and investment banks, broker dealers, and shadow banks) have gotten involved in and linked to asset markets.

In this paper, we develop a dynamic model that links bank runs and asset prices. Our model explains the empirical facts and gives new policy implications. Specifically, our model studies bank runs in a financial system, in which there are many banks and they share a common asset market. In the static setting with exogenous market liquidity (i.e., market depth), our model demonstrates two-way feedback between asset liquidation prices and creditor runs and shows the possibility of fundamental-based multiple equilibria at the system level. In the extension to the dynamic setting with endogenous market liquidity, our model analytically characterizes the dynamics of the interplay

between market liquidity and bank runs. The dynamic model generates runs-illiquidity traps with two steady-state equilibria and a crisis process consisting of first a gradual decline followed by a sudden sharp crash in the transitional dynamics.

We first present a three-date static model. There is a continuum of financial institutions (“banks”). At the initial date, each bank finances its investment (asset) by borrowing from a continuum of creditors under a demand-deposit-like contract. At the interim date, creditors of a bank receive noisy private signals about the fundamental value of the bank’s asset (i.e., the payoff at the final date) and decide whether to roll over or to withdraw. If too many creditors of a bank decide to withdraw, the bank will be unable to satisfy these early withdrawals and will therefore fail at the interim date. All failing banks liquidate their assets in a common asset market, where the liquidation value of a bank depends on its asset fundamentals, on aggregate fire sales in the system, as well as on exogenous market liquidity (depth). Under this setting, the equilibrium is characterized by the joint determination of three endogenous variables: the rollover decision of creditors, the interim liquidation value of a bank in the case of fire sales, and the number of banks failing in the system. Particularly, when creditors of a bank make rollover decisions, they need to form expectations of the liquidation value of their bank, as the value determines the extent to which the bank can withstand early withdrawals and thus their coordination risk.

We show that two-way feedback between liquidation prices and creditor runs (coordination risk) arises in our model. If creditors of a bank think the liquidation price of the bank’s asset to be low, they will optimally choose to run more often because a lower liquidation price increases coordination risk for them; if creditors run more often, more banks will fail in the system, increasing aggregate fire sales and thus reducing every bank’s liquidation price. In particular, we show that the feedback is strong enough, resulting in multiple equilibria. In fact, because banks share a common asset liquidation market, this gives rise to strategic complementarities among creditors of different banks, besides complementarities among creditors of the same bank. More concretely, when the creditors of a bank run, that bank is forced to conduct fire sales in the asset market; this decreases the liquidation prices of all other banks’ assets because of the limited risk-absorbing capacity of investors in the asset market. As a lower liquidation price increases coordination risk, the creditors of every other bank thus also have incentives to run. The increased degree of complementarity among creditors in the system results in a higher likelihood of equilibrium multiplicity. Indeed, multiple equilibria exist even if the precision of creditors’ private signals approaches infinity.

We then extend the static model to a dynamic OLG setting, where market liquidity (depth) is endogenous. In the OLG setting, the older generation of bankers (i.e., the bankowners in the last period), provided that they do not go bankrupt from creditor runs, become investors in the asset market in the current period. The asset market in this case can be interpreted as an interbank market. The number of investors in the asset market affects market liquidity. This means that

market liquidity in the current period depends on the number of banks that do not suffer creditor runs in the last period. The equilibrium is therefore dynamically linked across periods through endogenous market liquidity. We analytically characterize the dynamics of the interplay between market liquidity and bank runs.

We show that our OLG dynamic model can have two steady equilibria, meaning that the economy can exhibit runs-illiquidity traps. When the economy is trapped in the “bad” steady state, the asset market is illiquid and the banking sector is plagued by runs. Interestingly, transitions between the two steady states can be *asymmetric*. A change in the (self-fulfilling) beliefs of bank creditors can throw the economy onto the equilibrium path that departs from the “good” steady state toward the “bad” steady state. Once the economy reaches the “bad” steady state, however, there is no equilibrium path along which the self-fulfilling beliefs of bank creditors can get the economy out because in that state market liquidity is too low and beliefs play only a limited role. In studying the transitional dynamics under an aggregate shock to fundamentals, we show an equilibrium path featuring an initial gradual decline followed by a sudden crash. That is, at the beginning, there is a unique equilibrium path along which more runs and lower market liquidity form a vicious cycle with asset prices declining even more rapidly. After some tipping point, however, the second (lower) equilibrium path emerges. The economy can then abruptly switch to the new equilibrium path due to creditors’ self-fulfilling beliefs driven by sentiments or sunspots, in which case market liquidity and asset prices will drop discontinuously and at the same time, the number of runs on financial institutions will surge.

We derive policy implications of our model. First, we analyze means of intervention, by comparing the unconventional intervention of supporting market liquidity (asset prices) and the conventional intervention of providing funding liquidity. Our model suggests that the former can be more effective in dealing with a systemic liquidity crisis than the latter. Providing funding liquidity helps to mitigate the coordination problem among creditors *within* a bank, while supporting market liquidity targets directly the coordination problem among creditors *across* different banks in the system. Supporting market liquidity is crucial not only for breaking the runs-prices feedback along a stable equilibrium but also for eliminating self-fulfilling multiple equilibria. Second, we discuss the timing of intervention. Our model implies that the government’s ex post intervention should not be left too late. If intervention is timely enough, policy to boost bank creditors’ confidence with zero cost can improve efficiency. However, if intervention is too late, the actual intervention of the asset market may be necessary. Third, we consider a temporary shock versus a permanent shock that triggers a crisis. The model implies that even for a temporary shock, intervention may be necessary particularly because the equilibrium path of economic recovery is partially beliefs-driven.

Related literature. Our paper is related to the literature that uses global games methods to study creditor runs and provides the first dynamic model in this literature. The pioneering

works of Rochet and Vives (2004) and Goldstein and Pauzner (2005), who realistically assume that creditors receive some private information about a bank’s fundamentals, link a creditor run to bank fundamentals and thus show that a creditor run is both fundamental-based and panic-based. The interim liquidation value (function) of the bank in these models is exogenous. Vives (2014) and Bebchuk and Goldstein (2011) build general frameworks to incorporate the results in this literature. Liu (2016) and Eisenbach (2017) extend these earlier works to study a financial system in which there are many banks with interactions, so that the interim liquidation value of a bank is endogenous and depends on the situation of other banks in the system.⁴ Our present paper advances this literature by studying systemic bank runs in a dynamic framework, so that a bank’s interim liquidation value also depends on the endogenous market liquidity condition which is dynamically path-dependent.⁵ This helps to explain the evolution of a crisis and derive new policy implications. Our present paper also makes a methodological contribution by building a model framework (the static model in Section 2) to formalize the result that fundamental-based multiple equilibria can exist at the system level under a weaker condition than that in Liu (2016).⁶ This model framework also makes our dynamic model (Section 3) convenient and tractable. Figure 1 illustrates the position of our present paper in this literature.

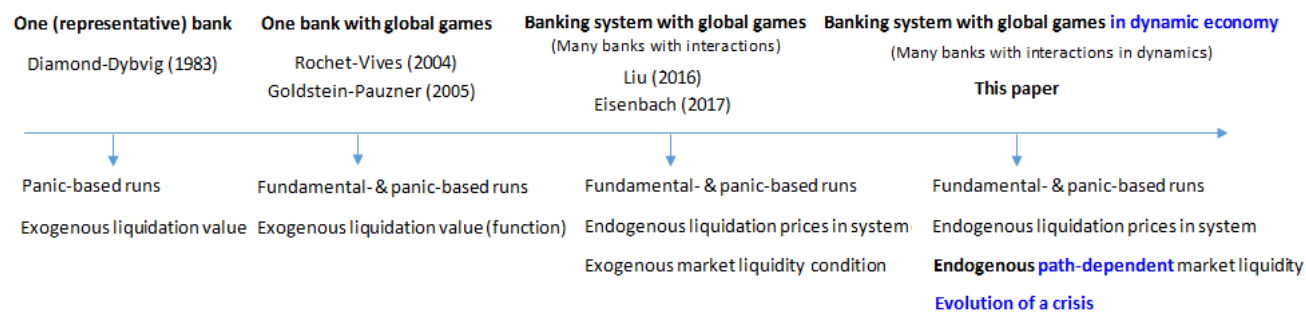


Figure 2: The position of the paper in the global-games bank runs literature

Our paper is connected to several contributions that study bank runs in a dynamic model. Martin, Skeie and von Thadden (2014a, 2014b), nesting Qi’s (1994) OLG version of Diamond-Dybvig, study runs on short-term funding markets. Their paper (2014a) deals with the case of perfect markets for distressed assets, where they show how market fragility due to sudden collective expectation changes can be addressed by individual banks and regulators. Their paper (2014b) focuses on how the different market microstructures (e.g., bilateral and tri-party repo) influence expectation-driven

⁴Eisenbach (2017) uses a reduced-form approach by assuming an exogenous downward-sloping aggregate demand curve of assets (while his paper has other focuses). Liu (2016) explicitly models and endogenizes the interbank market.

⁵A bank’s liquidation value in our model also depends on its own fundamentals, different from the case where all banks have a common liquidation value in Eisenbach (2017).

⁶Concretely, in Liu (2016) the existence of multiple equilibria is possible under the condition that the noise of creditors’ private signals is not small, while our present paper shows it is possible even when the noise of creditors’ private signals approaches zero.

runs. Our model studies systemic bank runs and runs are both fundamental-based and expectation (panic)-based, which gives novel implications for the dynamics of a crisis. He and Xiong (2012) study dynamic debt runs in a model where a firm has a time-varying fundamental and a staggered debt structure. They consider one firm, not various firms (banks). Every creditor, instead of receiving noisy private information as in a standard global games model, observes the same public information. Their contribution is on formalizing the intertemporal coordination problem among creditors of a firm and characterizing the comparative statics. Our paper abstracts away from the intertemporal coordination problem and instead focuses on the intertemporal linkage of runs and market liquidity/asset prices in a system.⁷ Gertler, Kiyotaki and Prestipino (2017) incorporate banks and banking panics within a conventional macroeconomic framework. They assume a representative bank for the entire banking sector while modelling the interaction among different banks in the system (i.e., the coordination problem among creditors of the same bank and among creditors of different banks) is a key emphasis of our model.

Our model demonstrates the (dynamic) interaction between market liquidity and creditor runs (coordination risk), which is related to, but different from, the feedback loop between market liquidity and margin requirements in Brunnermeier and Pedersen (2009). Bernardo and Welch (2004) model market runs and show in a rational framework that investors can rush to sell in a stock market. In our model, banks conduct asset sales passively based on their creditors' requests — there is no coordination problem among banks and hence no market runs. Doh (2018) studies the interaction between primary markets and secondary markets. His paper focuses on analyzing how segmentation (entry barrier friction) of secondary asset markets affects financial contagion and welfare. The key friction in the primary debt markets in Doh's model is the conflict of interest between equityholders and creditors. Unlike in his paper, the key friction in our model is the coordination problem among creditors under imperfect information (in the primary market).

The paper is organized as follows. Section 2 presents the static model. Section 3 studies the dynamic model. Section 4 discusses policy implications. Section 5 includes robustness analyses. Section 6 concludes.

2 Static Model

In this section, we present the static model, to highlight some key mechanisms and also to provide a tractable model framework that can be readily embedded into and extended to the dynamic setting in the next section.

⁷Angeletos, Hellwig and Pavan (2007) study dynamic global games of regime change, focusing on the situation where agents take actions in many periods and learn about one fixed underlying fundamental value over time.

2.1 Model Setting

Figure 3 illustrates the model framework, where the notations will be explained in due course. There are three dates in the model: $t = 0, 1$ and 2 . We discuss banks, the asset market, and creditor runs, in order.

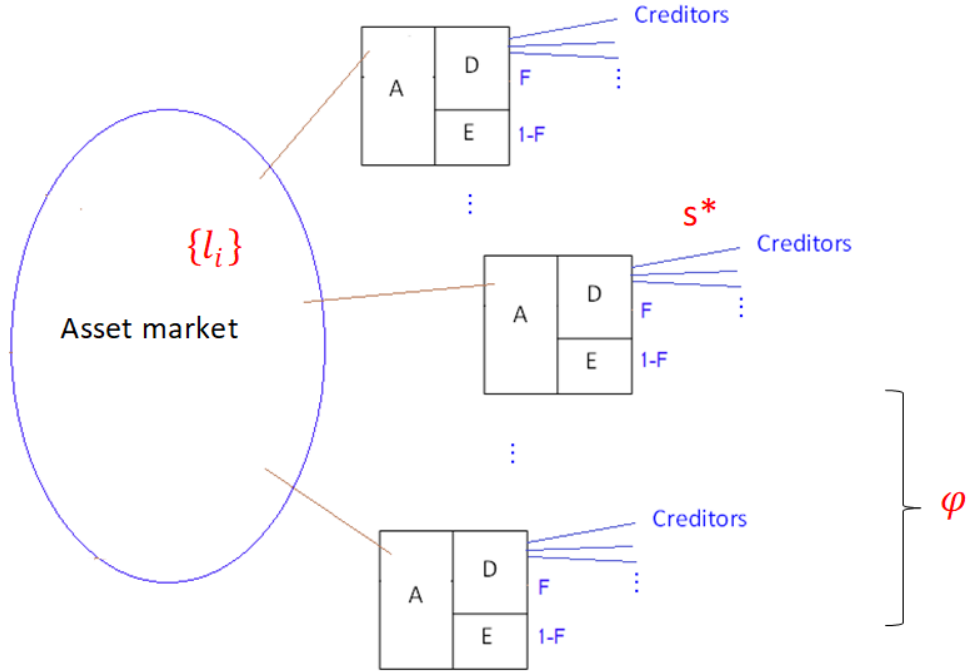


Figure 3: Illustration of the model framework with the equilibrium

2.1.1 Banks

There is a continuum of banks with unit mass, indexed by $i \in [0, 1]$. At $t = 0$, each bank invests in one unit of its own assets at the cost of 1. The cost is financed from two sources: an amount F comes from a continuum of its creditors (depositors) with F mass, with each creditor contributing 1, and an amount $1 - F$ comes from its equityholder (bankowner).⁸ At $t = 1$, a creditor of a bank has the right to decide whether or not to roll over his lending to the bank. If he decides not to roll over, his claim is the par value 1 at $t = 1$; if, instead, he decides to roll over, the (promised) notional claim to him is R at $t = 2$, where $R > 1$ is the gross interest rate.⁹ Creditors are risk-neutral.

The payoff of bank i 's assets at $t = 2$ is

$$v_i = \theta_i + e_i,$$

⁸We assume that each bank has its own creditor base (for example, these banks are regional banks).

⁹Without loss of generality, we normalize the interim notional claim to 1. What matters to the model is the interest rate between $t = 1$ and $t = 2$, i.e., the R .

which follows a normal distribution as in Grossman and Stiglitz (1980). That is, the uncertainty of the payoff is resolved gradually. Specifically, the term θ_i , interpreted as asset quality, has its realization at $t = 1$, which is independently drawn from an identical distribution $\theta_i \sim N(\mu_\theta, \sigma_\theta^2 = \tau_\theta^{-1})$ across banks. μ_θ corresponds to the aggregate state of the economy and is common knowledge. The term e_i is a random variable with distribution $e_i \sim N(0, \sigma_e^2 = \tau_e^{-1})$ and resolves its uncertainty at $t = 2$. For simplicity and without loss of generality, we assume that $e_i \equiv e$ is perfectly correlated across banks.¹⁰ Figure 4 illustrates the timeline of the asset payoff's uncertainty realization.

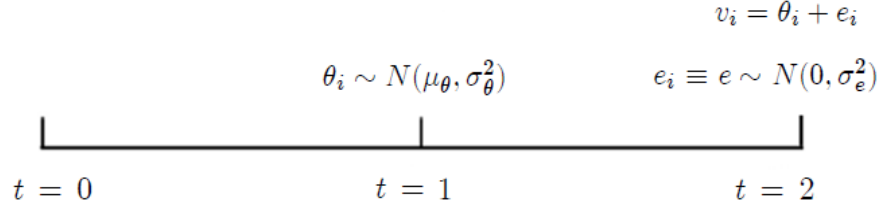


Figure 4: Timeline of the asset payoff's uncertainty realization

Although the asset quality of a bank is realized at $t = 1$, its creditors are not informed. Nevertheless, at $t = 1$ a creditor of a bank receives imperfect information (a signal) about the asset quality of the bank. Specifically, the signal for creditor h of bank i (about asset quality θ_i) at $t = 1$ is $s_i^h = \theta_i + \sigma_s \epsilon^h$, where $\sigma_s > 0$ is constant, and the individual-specific noise $\epsilon^h \sim N(0, 1)$. ϵ^h is independent across different creditors h of a bank, and each is independent of θ_i . Denote $\tau_s \equiv 1/\sigma_s^2$.

2.1.2 Asset Market

If a bank suffers a creditor run (to be elaborated), its assets must be liquidated or under fire sales at $t = 1$ in a competitive asset market, which consists of a continuum of competitive investors with a total mass of n . Investor $j \in [0, n]$ has utility function

$$U(W^j) = -\exp(-\gamma W^j),$$

where W^j is the end-of-period wealth at $t = 2$, and γ is the risk-aversion (CARA) coefficient. Without loss of generality, the gross risk-free interest rate between $t = 1$ and 2 is normalized to 1.

Investors have private information (signals) about banks' asset qualities. Specifically, the signal for investor j about asset quality θ_i at $t = 1$ is $x_i^j = \theta_i + \sigma_x \epsilon_i^j$, where $\sigma_x \geq 0$ is constant, and the individual-specific noise $\epsilon_i^j \sim N(0, 1)$. ϵ_i^j is independent across i 's and j 's, and each ϵ_i^j for a given i is independent of θ_i .

Suppose that the system has, in total, a mass of φ ($\in [0, 1]$) of banks suffering creditor runs.

¹⁰As long as e_i is correlated across banks to some degree (i.e., not perfectly diversified away), our model result of the downward-sloping liquidation price (Lemma 1) and thereby other results will hold true. In reality, the number of banks is not infinite; therefore, e_i cannot possibly be diversified away even when e_i is independent across banks.

Then, there are φ units of assets in the system under fire sales.¹¹ Denote by l_i the liquidation (fire-sale) price of bank i 's assets.

2.1.3 Creditor Runs

Let us consider a typical bank i . If greater than $\frac{l_i}{F}$ proportion of its creditors decline to roll over their lending at $t = 1$, its liquidation value will not be sufficient to cover these creditors' claims, leading to its failure (we call this scenario a "creditor run"). Alternatively, one may think of l_i as the collateral value of the bank's assets. This means that the bank can raise at most l_i amount of cash at $t = 1$ by pledging its assets as collateral. If the demand for cash exceeds l_i at $t = 1$, the bank will fail.

Following the work of Rochet and Vives (2004), we use a simplified payoff structure of the creditor-run game. Specifically, as in Rochet and Vives (2004), we assume that each creditor of a bank is an institutional investor (a fund), run by its fund manager. A fund manager has the following compensation scheme. If the fund manager calls his fund's investment at $t = 1$, his payoff is a constant w_0 , or the face value 1 multiplied by proportion w_0 . This could be because a creditor who calls loans at $t = 1$ will either fully recover the face value of investment 1 (in the case of bank survival) or suffer a small loss (in the case of bank failure). If, instead, the fund manager holds the investment at $t = 1$, he will obtain compensation w conditional on his fund's investment not defaulting (i.e., the investment return is no less than R), where $w > w_0$. Compensation contingent on non-default captures the reality that "breaking the buck" has severe consequences.

A creditor's payoff depends crucially on the actions of other creditors of the same bank. Let λ denote the proportion of creditors of a bank who choose not to roll over (i.e., choose to call). Then, the payoff for a particular creditor is given in Table 1.

	Total calling proportion $\lambda \in [0, \frac{l_i}{F})$ (bank survives)	Total calling proportion $\lambda \in [\frac{l_i}{F}, 1]$ (bank fails)
Hold	$w \cdot \Phi \left(\frac{(1 - \frac{F}{l_i} \lambda) \theta_i - (1 - \lambda) FR}{(1 - \frac{F}{l_i} \lambda) \sigma_e} \right)$	0
Call	w_0	w_0

Table 1: Creditor-run payoff structure

If $\lambda \in [\frac{l_i}{F}, 1]$, a creditor run occurs and the bank fails at $t = 1$; its staying creditors get nothing and thus their fund manager's compensation is 0 because of the default. If $\lambda \in [0, \frac{l_i}{F})$, the

¹¹We will focus on the case of $\sigma_s \rightarrow 0$. In equilibrium, then, if a bank suffers a creditor run, it will liquidate its assets *entirely* (i.e., no partial liquidation).

bank must liquidate $\frac{F\lambda}{l_i}$ units of its assets to raise cash to pay its $F\lambda$ calling creditors. Thus, at $t = 2$, $1 - \frac{F\lambda}{l_i}$ units of assets will remain, the payoff distribution of which, conditional on θ_i , is $\left(1 - \frac{F\lambda}{l_i}\right) v_i \sim N\left(\left(1 - \frac{F\lambda}{l_i}\right) \theta_i, \left(1 - \frac{F\lambda}{l_i}\right)^2 \sigma_e^2\right)$. Since the number of staying creditors at $t = 2$ is $(1 - \lambda)F$, these creditors' total notional claim is $(1 - \lambda)FR$. Hence, the probability that the bank will not default to these creditors at $t = 2$ conditional on θ_i is $\Phi\left(\frac{\left(1 - \frac{F\lambda}{l_i}\right)\theta_i - (1 - \lambda)FR}{\left(1 - \frac{F\lambda}{l_i}\right)\sigma_e}\right)$, where $\Phi(\cdot)$ stands for the c.d.f. of the standard normal and $\phi(\cdot)$ denotes its p.d.f.

By introducing a third party, fund managers, we have a discrete-state payoff structure of the creditor-run game. This simplified structure captures the key feature of the creditor-run game: if the proportion of creditors of a bank calling is higher than $\frac{l_i}{F}$, the optimal strategy for an individual creditor is to also ‘call’; if, however, the proportion of creditors calling is less than $\frac{l_i}{F}$, the optimal strategy for an individual creditor is likely to also ‘hold’. The simplification in the payoff structure is a convenient way to deal with the fact that the property of global strategic complementarities fails to exist in a creditor-run game (see Goldstein and Pauzner (2005)).¹²

For a cleaner and simpler analysis, we follow Morris and Shin (2009) to simplify the payoff structure of the creditor-run game in Rochet and Vives (2004). Morris and Shin (2009) assume that “if there is not a run, new creditors will eventually be found and the balance sheet reverts to its initial state after the failed run.” Basically, they are assuming that “the partial liquidation of assets has no long-run effect” (in the language of Vives (2014a)). Concretely, if a bank has less than $\frac{l_i}{F}$ proportion of its creditors calling, partial liquidation will occur but the bank can still survive to $t = 2$, in which case Morris and Shin (2009) assume that the bank’s balance sheet reverts to its initial state. Essentially, after an *unsuccessful run*, the asset side of the balance sheet of the bank is restored to v_i and the liability side reverts to the total notional debt value FR claimed by F creditors. In short, the assumption of Morris and Shin (2009) gives the simplified payoff structure in Table 2.

	Total calling proportion $\lambda \in [0, \frac{l_i}{F})$ (bank survives)	Total calling proportion $\lambda \in [\frac{l_i}{F}, 1]$ (bank fails)
Hold	$w \cdot \Phi\left(\frac{\theta_i - FR}{\sigma_e}\right)$	0
Call	w_0	w_0

Table 2: Simplified creditor-run payoff structure

Notice that Table 2 is identical to Table 1 if we set $\lambda = 0$ for the payoff of holding in the case of $\lambda \in [0, \frac{l_i}{F})$ in Table 1. That is, as long as $\lambda \in [0, \frac{l_i}{F})$, the bank continues as if it had not experienced any withdrawals in Table 2.

¹²See also Dasgupta (2004). Relatedly, He and Xiong (2012) use a staggered debt structure.

In the main model, we use the simplified payoff function in Table 2. In Section 5, we will show that our model results are robust under the alternative payoff structure in Table 1.

2.2 Timeline

At $t = 0$, the liability side of the balance sheet of a bank is given by $(F, 1 - F)$ and the contract term $(1, R)$ with creditors is fixed. At $t = 1$, creditors move *first* by making their rollover decisions based on their private information s_i^h , and banks move *later* by conducting asset sales in the asset market based on the total withdrawals requested by their creditors, at which stage the asset prices $\{l_i\}$ are formed.

Importantly and realistically, when creditors make their rollover decisions, the asset prices $\{l_i\}$ will not have been formed yet and thus creditors' decisions cannot condition on the asset prices, unlike the setting in Angeletos and Werning (2006). Empirically, a bank's creditors, who hold a debt contract, hardly know the details of the business operation of the bank, including the selling price of its assets ex ante. Although creditors cannot perfectly observe the asset prices, a creditor's signal s_i^h is effectively a noisy signal about the asset price l_i , as will be shown later.

2.3 Equilibrium

We focus on the equilibrium at $t = 1$. At $t = 1$, creditors need to make their rollover decisions. We are interested in the equilibrium where every creditor uses a threshold (monotone) strategy. The strategy is given by

$$s_i^h \mapsto \begin{cases} \text{Call} & s_i^h < s^* \\ \text{Hold} & s_i^h \geq s^* \end{cases},$$

where s_i^h is the signal received by creditor h of bank i and s^* is the rollover threshold. Because banks are identical ex ante, we naturally consider the *symmetric* equilibrium in which creditors of all banks use a common strategy, i.e., the threshold s^* is not bank-specific.

We will show that *an upper dominance region* exists for the bank-run game in our model.¹³

2.3.1 Solving the Equilibrium

Formally, we define the equilibrium at $t = 1$ as follows.

Definition 1 *The equilibrium at $t = 1$ is characterized by the triplet $(s^*, \{l_i\}, \varphi)$, where s^* is the rollover threshold for creditors, l_i is the liquidation price of bank i 's assets, and φ is the total mass*

¹³An upper dominance region exists because in our model the interim liquidation value of a bank is fundamentals-dependent as in Rochet and Vives (2004) (as well as Goldstein (2005)). Goldstein and Pauzner (2005), following the original model of Diamond and Dybvig (1983), assume that the interim liquidation value of a bank is fundamentals-independent, so an upper dominance region does not exist in their model.

of banks under fire sales, such that 1) given creditors' rational expectations of φ , they set their rollover threshold as s^* ; 2) given the rollover threshold s^* , the mass of failed banks in the system is φ ; and 3) given the total fire sales φ , the equilibrium price of bank i 's assets in the asset market is l_i .

We derive the equilibrium in three steps.

Asset market in equilibrium The portfolio choice of an investor in the asset market at $t = 1$ is given by

$$\begin{aligned} \max_{\{q_i^j\}} \mathbb{E} \left[-\exp(-\gamma W^j) \mid \{x_i^j\}, \{l_i\} \right] \\ \text{s.t. } W^j = \int q_i^j (v_i - l_i) di, \end{aligned} \tag{1}$$

where q_i^j is the quantity of demand for asset i for investor j . As long as the risk of v_i (i.e., the term e_i) is correlated across i 's to some degree or the number of banks is finite, the risk of the portfolio payoff cannot be completely diversified away. For simplicity and without loss of generality, we have assumed that $e_i \equiv e$ is perfectly correlated across i 's.

For simplicity, we focus on the trading equilibrium in which $\{\theta_i\}$ is perfect information for investors. We can adopt either of the following two approaches to have such an equilibrium. First, we can follow the trading game (mechanism) in Vives (2014b) and Benhabib, Liu and Wang (2016) to focus on the *fully-revealing equilibrium* of the asset market, i.e., the equilibrium in which financial prices fully reveal the fundamentals of the trading assets in the spirit of Hayek (1945). In our context, it is the equilibrium in which $\{\theta_i\}$ is fully revealed to the investors through the financial prices. Alternatively, instead of $\{\theta_i\}$ being fully revealed through the financial prices, we can assume that the precision of investors' private signals approaches the limit $\sigma_x \rightarrow 0$ as in Morris and Shin (2004), just as the precision of creditors' private signals approaches the limit $\sigma_s \rightarrow 0$; this would also lead to $\{\theta_i\}$ being perfect information for investors. Note that this part of the model is not a focus of our paper, and we can adopt either alternative.

The first-order condition of (1) implies that $q_i^j = q_i$ for any j (i.e., a representative investor exists and demands q_i for asset i), and that

$$\int q_i di = \frac{\theta_i - l_i}{\gamma \sigma_e^2}.$$

Given that the total mass of banks suffering creditor runs is φ and that the total mass of investors is n , the market clearing condition dictates

$$n \int q_i di = \varphi.$$

Lemma 1 follows.

Lemma 1 *The liquidation price of bank i 's assets is given by $l_i = \theta_i - \varphi/k$, where $k \equiv n/(\gamma\sigma_e^2)$ and k measures market liquidity (i.e., market depth).*

The result of Lemma 1 is in the spirit of Grossman and Miller (1988). When the risk-averse market maker sector is forced to absorb more risky assets, the price of *every* risky asset is affected and reduced because of the limited risk-absorbing capacity of the market maker sector. It is worth noting that every bank is a price-taker in the liquidation market. As in the large literature, “market liquidity” is measured as *market depth*, k .

Creditor run in equilibrium for an individual bank Considering that l_i is fundamentals (θ_i)-dependent by Lemma 1, when θ_i is sufficiently high, bank i will survive even if every creditor of it withdraws. That is, an *upper dominance region* exists. Therefore, we only need to focus on threshold equilibria (see, e.g., Morris and Shin (2003) and Vives (2014a)). Note that since creditors rationally anticipate that l_i will be $l_i = \theta_i - \varphi/k$, the signal s_i^h about θ_i is also a signal about l_i .

Given that all other creditors of a bank use threshold s^* , the bank when realizing asset quality as θ_i has a $\lambda(\theta_i; s^*) = \Pr(\theta_i + \sigma_s \epsilon^h < s^*) = \Phi\left(\frac{s^* - \theta_i}{\sigma_s}\right)$ proportion of its creditors withdrawing. Moreover, the bank with realized asset quality θ_i will have its asset liquidation value as $l_i = \theta_i - \varphi/k$. Hence, by the nature of creditor runs, the threshold of the bank's failure, denoted by θ^* , is given by

$$\frac{\theta^* - \varphi/k}{F} = \Phi\left(\frac{s^* - \theta^*}{\sigma_s}\right). \quad (2)$$

That is, the bank fails if and only if $\theta_i < \theta^*$ and individual creditors rationally anticipate this. Given the bank's failure threshold θ^* , what is the optimal strategy of an individual creditor? He rolls over if and only if his signal is above threshold s^* , where s^* is given by the following indifference condition:

$$\int_{\theta^*}^{+\infty} w \Phi\left(\frac{\theta_i - FR}{\sigma_e}\right) \cdot \frac{1}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \phi\left(\frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^*\right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}}\right) d\theta_i = w_0. \quad (3)$$

When receiving the signal exactly equal to s^* , the fund manager of the investor has the expected payoff on the right-hand side (RHS) of (3) if he chooses to withdraw, and has the payoff on the left-hand side (LHS) of (3) if he chooses to roll over: when $\theta_i < \theta^*$, the bank will fail and thus the fund manager will get nothing; conditional on $\theta_i \geq \theta^*$, his expected compensation at $t = 2$ is $w \Phi\left(\frac{\theta_i - FR}{\sigma_e}\right)$ for a given θ_i .

We focus on the limiting case of signal precision: $\sigma_s \rightarrow 0$. Under the limiting case, we prove that the system of equations (2) and (3) is transformed into $\theta^* = s^*$ and

$$\underbrace{\frac{s^* - \varphi/k}{F}}_{\text{Illiquidity (coordination) risk}} \cdot \underbrace{\Phi\left(\frac{s^* - FR}{\sigma_e}\right)}_{\text{Insolvency risk}} = \frac{w_0}{w}. \quad (4)$$

The first term on the LHS of (4) characterizes the interim illiquidity risk and the second term gives the insolvency risk at the final date. The limit $\sigma_s \rightarrow 0$ also implies that in equilibrium all creditors of a bank are in the same position ex post, i.e., either all of them decide to roll over or none of them does so. This in turn implies that in equilibrium a bank either completely liquidates its assets or does not liquidate any fraction, i.e., there is no partial liquidation.

Bank failures in the system Recall that the asset quality distribution across banks in the system at $t = 1$ is $\theta_i \sim N(\mu_\theta, \sigma_\theta^2)$. Banks with realized asset quality $\theta_i \geq \theta^*$ survive at $t = 1$ while all others fail. Hence, by $\theta^* = s^*$ under $\sigma_s \rightarrow 0$, the total measure of failing banks in the system is given by

$$\varphi = \Phi\left(\frac{s^* - \mu_\theta}{\sigma_\theta}\right). \quad (5)$$

Lemma 2 *The creditor-run equilibrium in the system at $t = 1$, given by (s^*, φ) , solves the system of equations (4)-(5) under the limiting case of $\sigma_s \rightarrow 0$. Two-way feedback exists between liquidation prices (φ and thereby $\{l_i\}$) and the creditor-run threshold (s^*): $\frac{\partial s^*}{\partial \varphi} > 0$ in (4) and $\frac{\partial \varphi}{\partial s^*} > 0$ in (5).*

The two-way feedback highlighted in Lemma 2 is intuitive. When creditors run on banks with a higher threshold, more banks in the system will fail, resulting in a lower liquidation price for every bank. Creditors of a bank have *rational expectations* on this and thus have higher incentives to run in the first place.

2.3.2 Characterization of the Equilibrium

It is helpful to start by examining two benchmark cases. First, consider the case where the liquidation value l_i is exogenous. In this case, it is easy to show that the creditor-run equilibrium is given by

$$\underbrace{\frac{l_i}{F}}_{\text{Illiquidity (coordination) risk}} \cdot \underbrace{\Phi\left(\frac{s^* - FR}{\sigma_e}\right)}_{\text{Insolvency risk}} = \frac{w_0}{w}, \quad (6)$$

which clearly has a unique equilibrium. Proposition 1 follows.

Proposition 1 *(A single bank with an exogenous liquidation value) Under the limiting case*

of $\sigma_s \rightarrow 0$, when l_i is exogenous, the creditor-run equilibrium at $t = 1$, given by (6), has a unique (threshold) equilibrium.

Proof. See Appendix. ■

The uniqueness of equilibrium for runs on a single bank under global games has been well established in the literature. The result in Proposition 1 is essentially the result in Rochet and Vives (2004) and Goldstein and Pauzner (2005). (6) can be regarded as a hybrid of the two models.

Second, consider the case where a bank's liquidation value l_i depends on its own fundamentals θ_i , but not on other banks' situation; that is, φ is exogenous. In this case, the creditor-run equilibrium is given by (4) but with an exogenous φ .

Proposition 2 (A single bank with a fundamentals-dependent liquidation value) Under the limiting case of $\sigma_s \rightarrow 0$, when φ is exogenous, the creditor-run equilibrium at $t = 1$, given by (4), has a unique equilibrium.

In Proposition 2, our paper studies bank runs with a fundamentals-dependent liquidation value and a fundamentals-dependent payoff structure (i.e., both l_i and $w\Phi\left(\frac{\theta_i - FR}{\sigma_e}\right)$ depend on θ_i) and shows the uniqueness of the equilibrium under $\sigma_s \rightarrow 0$. Solving the equilibrium is nontrivial (see the proof of Lemma 2 in Appendix).¹⁴

We move on to study the equilibrium in Lemma 2, that is, there are many banks with a linkage through the asset market. Combining (4) and (5) yields one equation:

$$\left\{ \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) / k \right] \right\} \cdot \Phi \left(\frac{s^* - FR}{\sigma_e} \right) = \frac{w_0}{w}. \quad (7)$$

The equilibrium at $t = 1$ is fully characterized by equation (7). Mathematically, the first term on the LHS of (7) can be decreasing in s^* while the second term is increasing in s^* , so the function on the LHS with respect to s^* may be non-monotonic and thus multiple solutions to equation (7) are possible. Write the LHS of (7) as function $V(s^*; \mu_\theta, k)$. Figure 5 plots the equation $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ under a set of parameter values, where $\sigma_\theta = 0.4$, $F = 0.8$, $R = 1.1$, $\sigma_e = 1$ and $\frac{w_0}{w} = 0.5$. Given $\mu_\theta = 1.4$, when $0.54 \leq k \leq 0.65$, the equation admits three solutions; when $k < 0.54$ or $k > 0.65$, there is a unique solution. Given $k = 0.59$, when $1.38 \leq \mu_\theta \leq 1.5$, the equation admits three solutions; when $\mu_\theta < 1.38$ or $\mu_\theta > 1.5$, there is a unique solution.

¹⁴In the literature, Rochet and Vives (2004) use a fundamentals-dependent liquidation value but a fundamentals-independent payoff structure, while Goldstein and Pauzner (2005) use a fundamentals-dependent payoff structure but a fundamentals-independent liquidation value. Eisenbach (2017) assumes physical liquidation, where an asset's liquidation value does not depend on its fundamentals.

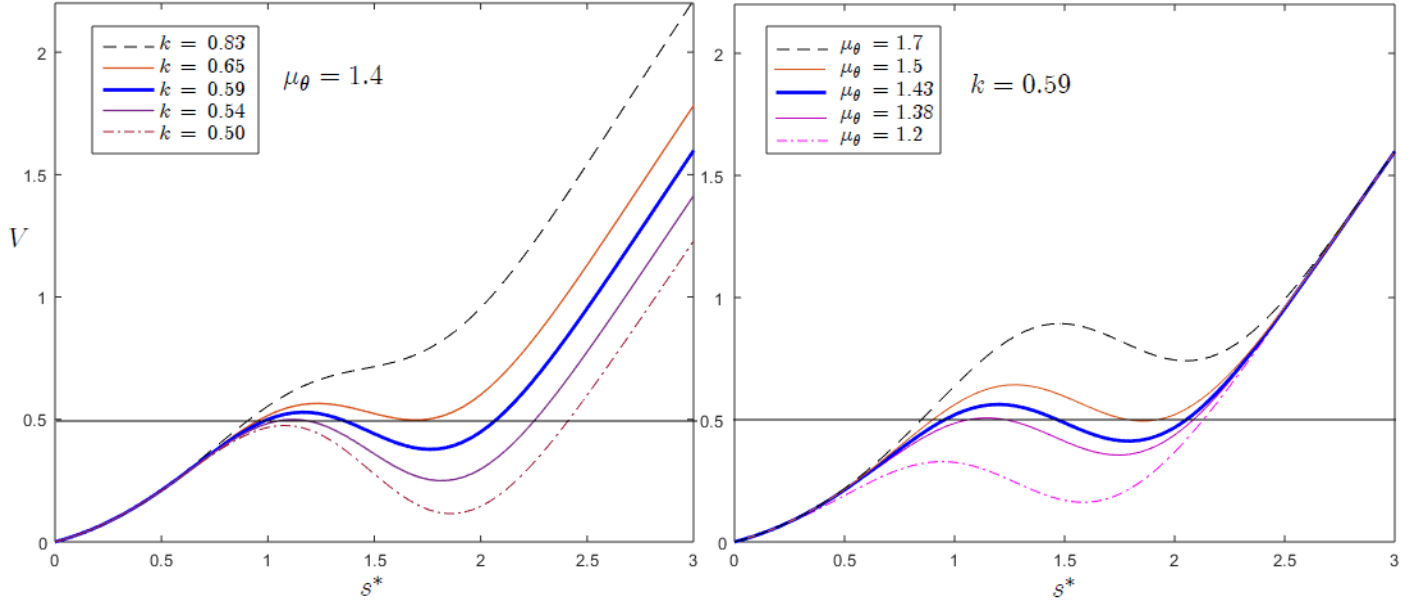


Figure 5: Equation $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$

Proposition 3 (Banking system with an asset market) Consider the limiting case of $\sigma_s \rightarrow 0$.

i) When k is high enough or μ_θ is low enough, the creditor run-asset market equilibrium in the system at $t = 1$, given by (4)-(5), is always unique.

ii) For a given sufficiently high μ_θ , when k is low enough or high enough, there is a unique equilibrium. When k is in an intermediate range, there exist multiple (typically three) equilibria where two of them are stable equilibria.

iii) For a given sufficiently low k , when μ_θ is low enough or high enough, the equilibrium is unique; when μ_θ is in an intermediate range, there exist multiple (typically three) equilibria where two of them are stable equilibria.

Moreover, at a stable equilibrium, we have the comparative statics $\frac{\partial s^*}{\partial \mu_\theta} < 0$ and $\frac{\partial s^*}{\partial k} < 0$.

Even under the limit $\sigma_s \rightarrow 0$, multiple equilibria can exist at the system level, which is in sharp contrast to the classical result of the bank-run game for a single bank. The intuition is the following. The presence of a common asset market gives rise to strategic complementarities among creditors of different banks, besides the complementarities among creditors of the same bank. That is, there is an increased degree of strategic complementarity among creditors in the system, which makes equilibrium multiplicity more likely. In fact, “a necessary condition for multiple equilibria is that strategic complementarity be strong enough; a sufficient condition is that strategic complementarity be strong at relevant points (candidate equilibria)” (Vives (2014a)). In other words, in our model, an individual creditor h ’s threshold s^{*h} becomes more sensitive to the actions s^* of other creditors. Indeed, the slope of the best response function can be $\frac{\partial s^{*h}}{\partial s^*} > 1$ at relevant points.

Figure 6 illustrates Proposition 3 based on the best response function. Rewrite (3) as

$$\int_{\theta_i=\theta^*(s^*)}^{+\infty} \Phi\left(\frac{\theta_i - FR}{\sigma_e}\right) \cdot \frac{1}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \phi\left(\frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s}\mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s}s^{*h}\right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}}\right) d\theta_i = \frac{w_0}{w}, \quad (8)$$

where s^{*h} denotes the threshold used by an individual creditor, s^* denotes the threshold used by other creditors within the bank as well as by creditors of other banks, and function $\theta^*(s^*)$ is given by the implicit function of (2) together with $\varphi = \Phi\left(\frac{\theta^* - \mu_\theta}{\sigma_\theta}\right)$. Write the LHS of (8) as function $\tilde{V}(s^{*h}; s^*, \sigma_s)$. Hence, the solution with respect to s^{*h} to equation $\tilde{V}(s^{*h}; s^*, \sigma_s) = \frac{w_0}{w}$ gives the best response function $s^{*h} = r(s^*; \mu_\theta, k)$. Note that $V(s^*) = \lim_{\sigma_s \rightarrow 0} \tilde{V}(s^{*h} = s^*, s^*, \sigma_s)$, given in (7). Consider the case where k is not too high and μ_θ is not too low as the cases of ii) and iii) in Proposition 3. Figure 6 plots the best response function, where different curves correspond to different μ'_θ s or k 's.¹⁵ Clearly, when μ_θ or k falls in different ranges, the nature of the equilibrium will be different.

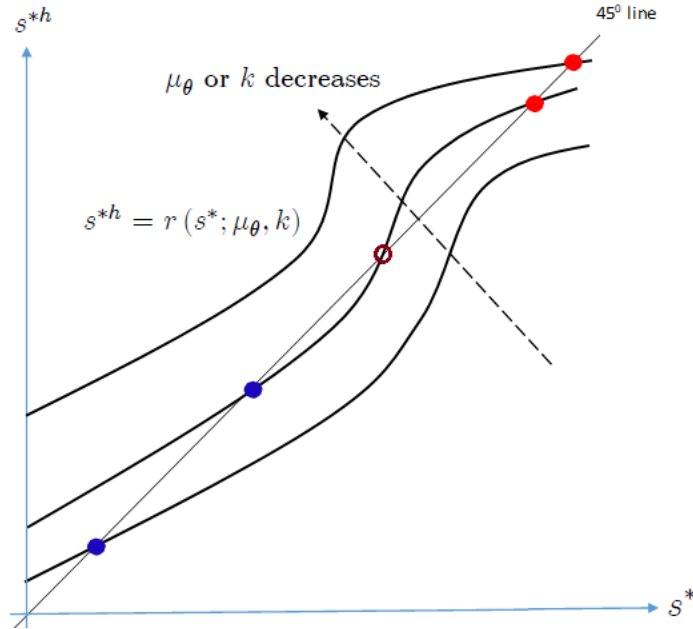


Figure 6: Best response function $s^{*h} = r(s^*; \mu_\theta, k)$

The result of multiple equilibria under diminishing noise of private signals in Proposition 3 is new in the literature. The contribution of this result can be better understood in the context of the literature. First, two-way feedback does not necessarily result in multiple (stable) equilibria. The earlier work of Liu (2016), who studies the feedback between interbank rates and creditor runs, shows that multiple equilibria exist only under a stronger condition that creditors' private signals are sufficiently noisy. Our present paper builds a tractable model framework to formalize the result

¹⁵The middle curve has three intersections with the 45° line. The middle intersection corresponds to an unstable equilibrium. The other two correspond to stable equilibria.

of multiple equilibria under a weak condition. Second, the existence of strategic complementarity within a group as well as between groups does not necessarily generate multiple equilibria (Goldstein (2005), Leonello (2017)). Our model featuring global games in a Walrasian economy gives equilibrium multiplicity. Third, we will show the difference between the case of a set of small banks and the case of a single large bank in terms of equilibrium multiplicity (Section 5.1), which can help to substantiate the non-triviality of the result. Indeed, the origin of multiple equilibria for a Diamond-Dybvig-type bank run model is externality (i.e., the early withdrawal of one creditor imposes a negative externality on other creditors). The same nature of externality applies for both cases,¹⁶ so it is not easy to see why the global games refinement can result in different results for these two cases.

The mechanism of equilibrium multiplicity in our model is different from that in Angeletos and Werning (2006) and Hellwig et al. (2006). In our model, the timeline is that creditors move *first* and banks move *later*. At the time when creditors make rollover decisions, they *do not* observe the liquidation prices. In Angeletos and Werning’s (2006) model, the endogenous price of a financial “derivative” in the first stage is observable by and serves as *a public signal* for agents who play the currency-crisis game in the second stage. Similarly, in Hellwig et al. (2006), the financial price — the interest rate — is a public signal, as well as having direct effects on the traders’ payoffs. The mechanism in our model is instead that the presence of an asset market with endogenous liquidation prices increases the degree of strategic complementarity among creditors in the system.

It is important to emphasize that multiple equilibria in our model are fundamental-based, rather than purely panic-based as in Diamond and Dybvig (1983).¹⁷ In Proposition 3, given the existence of multiple equilibria, creditors’ (threshold) strategy for each equilibrium conditions on fundamentals (signals), so at the system level there is always a proportion of banks with the lowest asset quality failing rather than all banks or none failing based on Diamond and Dybvig (1983). Also, whether multiple equilibria exist depends on fundamentals (i.e., the aggregate state μ_θ), which will be important for the dynamic model in Section 3. Therefore, the global games approach is crucial to obtaining our result of fundamental-based multiple equilibria. In this sense, the key advantage of the global games approach is that it links runs to fundamentals, not merely obtaining a unique equilibrium. In fact, Angeletos and Pavan (2013) show that as long as the equilibrium is fundamental-based, even with multiplicity of equilibria, the general policy analysis and comparative statics analysis go through across equilibria, generating conclusions that could not be obtained in the common-knowledge counterparts.

¹⁶For the case of a set of small banks, the only difference is that the externality is of “a larger scale”, across creditors of the same bank as well as across creditors of different banks.

¹⁷See discussions on panic-based versus fundamental-based runs in Gorton (1988), Allen and Gale (2009), and Goldstein and Razin (2015).

3 Dynamic Model

In this section, we extend the static model to the dynamic setting. We aim to use the simplest possible dynamic setting with minimum addition to the static model to deliver the key idea while maintaining the tractability of the model. We choose an OLG framework. In the OLG model, we will endogenize market liquidity k . The OLG model provides a dynamic equilibrium setting to study how market liquidity and creditor runs interact over time.

3.1 Setting

Agents As in the static model, in each period, there are three types of agents: bankers (bankowners), bank creditors (depositors), and investors in the asset market. However, investors in the current period are the surviving bankers from the last period. Specifically, bank creditors live for one period,¹⁸ while bankers live for two periods: when young they are bankers and when old they can become asset investors. That is, bankers experience an occupation change in their two-period life span.

In each period, a (young) banker borrows from bank creditors to finance his bank's investment as in the static model. In the next period, after "retiring" those (old) bankers can become investors in the secondary asset market as long as they did not go bankrupt from creditor runs when young.

Timeline In each period t , there are four stages.

Stage 1: A new generation of bankers and a new generation of bank creditors are born. The old generation of bankers, provided they did not go bankrupt in the last period, become investors in the secondary asset market.

Stages 2, 3 and 4: Stages 2, 3 and 4 correspond to $t = 0, 1, \text{ and } 2$ in the timeline of Section 2.2, respectively.

3.2 Equilibrium

In the dynamic model, we add subscript t to the notations in the static model (that is, X_t in the dynamic model corresponds to X in the static model). Based on the setting of the OLG model, we have

$$n_t = \bar{n} + (1 - \varphi_{t-1}),$$

where $\bar{n} > 0$ is a constant. Here we have made a technical assumption that there is a constant term \bar{n} in the expression of n_t ; this technical assumption is made to ensure that $k_t = 0$ is not a

¹⁸For simplicity and tractability, we assume that each generation of bank creditors survive for one period.

steady-state equilibrium, as will become clear later.¹⁹

In the dynamic economy, in each period t , the state variables are $(\mu_{\theta t}, \varphi_{t-1})$ while the equilibrium variables are $(k_t, s_t^*, \{l_{it}\}, \varphi_t)$. The equilibrium of the dynamic economy is given by the following joint equations (9a) to (9d):

$$\frac{s_t^* - \varphi_t/k_t}{F} \cdot \Phi\left(\frac{s_t^* - FR}{\sigma_e}\right) = \frac{w_0}{w} \quad (\text{creditor run for an individual bank}) \quad (9a)$$

$$l_{it} = \theta_{it} - \varphi_t/k_t \quad \text{with} \quad k_t = n_t / (\gamma \sigma_e^2) \quad (\text{asset market equilibrium}) \quad (9b)$$

$$\varphi_t = \Phi\left(\frac{s_t^* - \mu_{\theta t}}{\sigma_\theta}\right) \quad (\text{bank failures in the system}) \quad (9c)$$

$$n_t = \bar{n} + (1 - \varphi_{t-1}), \quad (\text{intertemporal investor flow}) \quad (9d)$$

where (9a), (9b), and (9c) correspond to (4), Lemma 1, and (5) in the static model. The only difference for the dynamic model is the additional equation (9d).

Lemma 3 *The equilibrium of the dynamic economy, given by $(k_t, s_t^*, \{l_{it}\}, \varphi_t)$, solves the system of equations (9a) to (9d) under the limiting case of $\sigma_s \rightarrow 0$.*

Based on Proposition 3, $\mu_{\theta t}$ and k_t are *substitutes* in terms of determining the nature of the equilibrium in period t . Lemma 4 follows.

Lemma 4 *Consider the equilibrium in period t . When $\mu_{\theta t}$ is low enough, there is always a unique equilibrium. For a given sufficiently high $\mu_{\theta t}$, there is a unique equilibrium when $k_t > \bar{k}$ or $k_t < \underline{k}$, and there are multiple (two) stable equilibria when $\underline{k} \leq k_t \leq \bar{k}$, where \bar{k} and \underline{k} are two thresholds, which satisfy $\underline{k} < \bar{k}$ and are both decreasing functions of $\mu_{\theta t}$.*

As φ_t is a function of k_t and $\mu_{\theta t}$, we can express k_{t+1} as a function of k_t and $\mu_{\theta t}$. Write this function as $k_{t+1} = g^+(k_t; \mu_{\theta t})$ for the “good” equilibrium and as $k_{t+1} = g^-(k_t; \mu_{\theta t})$ for the “bad” equilibrium, where functions $g^+(\cdot)$ and $g^-(\cdot)$ are both increasing functions and satisfy $g^+(k_t) > g^-(k_t)$ for any $k_t \in [\underline{k}, \bar{k}]$. For the case of a sufficiently low $\mu_{\theta t}$ at which there is always a unique equilibrium, to reduce notational clutter we still use g^+ and g^- but they actually coincide. Denote by $\underline{\mu}_\theta$ the upper boundary of $\mu_{\theta t}$ at which there is always a unique equilibrium. Proposition 4 follows.

Proposition 4 (Transitional dynamics) *The law of motion for market liquidity k_t is given by*

$$k_{t+1} = \begin{cases} g^+(k_t; \mu_{\theta t}) & \text{if } k_t \geq \underline{k}(\mu_{\theta t}) & (a) \\ g^-(k_t; \mu_{\theta t}) & \text{if } k_t \leq \bar{k}(\mu_{\theta t}). & (b) \end{cases} \quad (10)$$

¹⁹There are several ways to justify this assumption; for instance, \bar{n} can be interpreted as the government’s participation in providing market liquidity.

Based on (10), we can find the steady-state equilibrium.

Proposition 5 (Steady states) *Suppose that $\mu_{\theta t} = \mu_{\theta}$, a constant. If $\mu_{\theta} > \mu_{\theta}^{**}$ or $\mu_{\theta} < \mu_{\theta}^*$, there is a unique steady-state equilibrium; if $\mu_{\theta}^{**} \geq \mu_{\theta} \geq \mu_{\theta}^*$, there are two steady-state equilibria, where μ_{θ}^* and μ_{θ}^{**} are two thresholds and $\mu_{\theta}^{**} > \mu_{\theta}^*$.*

Figure 7 illustrates Proposition 5, where there are four regimes (i) to (iv).²⁰ Proposition 5 highlights possible runs-illiquidity traps with two steady-state equilibria in the dynamic economy. Importantly, transitions between the two steady states can be *asymmetric*. As shown in regime (ii), the economy can transit from the “good” steady state to the “bad” steady state if a change in creditors’ self-fulfilling beliefs driven by sunspots or sentiments suddenly switches the equilibrium to the lower branch of the equilibrium path. However, once the economy reaches the “bad” steady state, the economy is *trapped* there and there is no equilibrium path that can lead the economy back to the “good” steady state. In fact, in the “bad” steady state, market liquidity k_t is too low and creditors’ beliefs play only a limited role.

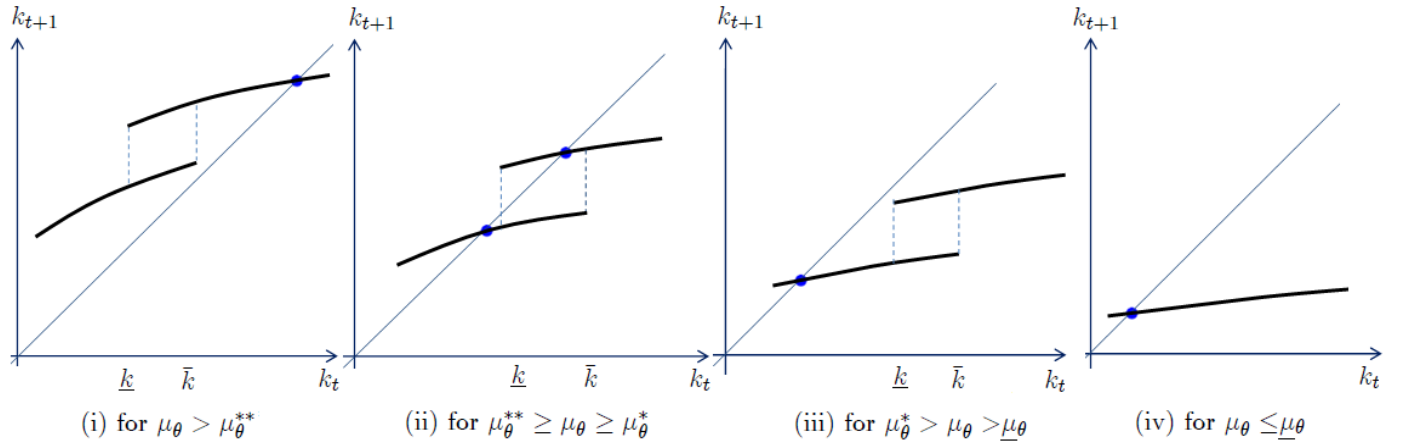


Figure 7: Possibly two steady-state equilibria

Next, we examine comparative statics for the dynamic model. Suppose that initially $\mu_{\theta t} = \mu_{\theta}$, which is constant and slightly higher than μ_{θ}^{**} , and suddenly a permanent negative shock hits $\mu_{\theta t}$. Proposition 6 gives the transitional dynamics under a permanent negative shock.

Proposition 6 (Transitional dynamics under a shock) *Suppose that initially $\mu_{\theta t} = \mu_{\theta} > \mu_{\theta}^{**}$ and the economy is in the steady state, and a negative permanent shock hits $\mu_{\theta t}$ resulting in $\mu_{\theta t} = \mu'_{\theta} < \mu_{\theta}$. If the shock is small enough such that $\mu'_{\theta} > \mu_{\theta}^{**}$, the transitional dynamics of k_t will always have a unique path given by 10(a). If the shock is medium-sized such that $\mu_{\theta}^* < \mu'_{\theta} < \mu_{\theta}^{**}$,*

²⁰ Even if only the “good” equilibrium is selected in each period (in which case the jump occurs at one unique point $k_t = \bar{k}$ rather than in an interval $[\underline{k}, \bar{k}]$), there can be two steady-state equilibria.

the transitional dynamics of k_t will have a unique path given by 10(a) at the beginning and then possibly experience a “sudden crash” by switching to the other path given by 10(b).

Figure 8 illustrates Proposition 6. For a small-sized shock, there is no regime change (i.e., remaining in regime (i) in Figure 7 after the shock). In contrast, for a medium-sized shock, a regime change occurs, from regime (i) to regime (ii) in Figure 7. Importantly, a medium-sized shock triggers a *two-stage crisis*. That is, the economy first declines gradually along a unique path and then it can suddenly experience a crash due to creditors’ self-fulfilling beliefs after the new second path emerges. This result helps to explain the empirical facts discussed at the beginning of the introduction. More specifically, by (9b) and (9c), we can find

$$\frac{\partial \varphi_t}{\partial k_t} < 0$$

$$\frac{\partial (\theta_{it} - l_{it})}{\partial k_t} = \frac{\partial (\varphi_t/k_t)}{\partial k_t} = \underbrace{\frac{1}{k_t} \frac{\partial \varphi_t}{\partial k_t}}_{\text{less supply (-)}} + \underbrace{\left(-\frac{1}{k_t^2} \varphi_t \right)}_{\text{more demand (-)}} < 0.$$

That is, after a negative shock to $\mu_{\theta t}$, as market liquidity k_t gradually declines, increasingly more creditor runs will occur ($\frac{\partial \varphi_t}{\partial k_t} < 0$) accompanied by increasingly larger fire-sale discounts ($\frac{\partial (\theta_{it} - l_{it})}{\partial k_t} < 0$). Note that there are two forces behind the widening discounts: less demand due to lower market liquidity and the resulting more supply due to more creditor runs. In particular, as market liquidity deteriorates further, the economy will eventually reach a tipping point with multiple equilibria starting to emerge. Then, market liquidity and liquidation prices could drop discontinuously concomitant with a jump in the number of financial institutions suffering runs.

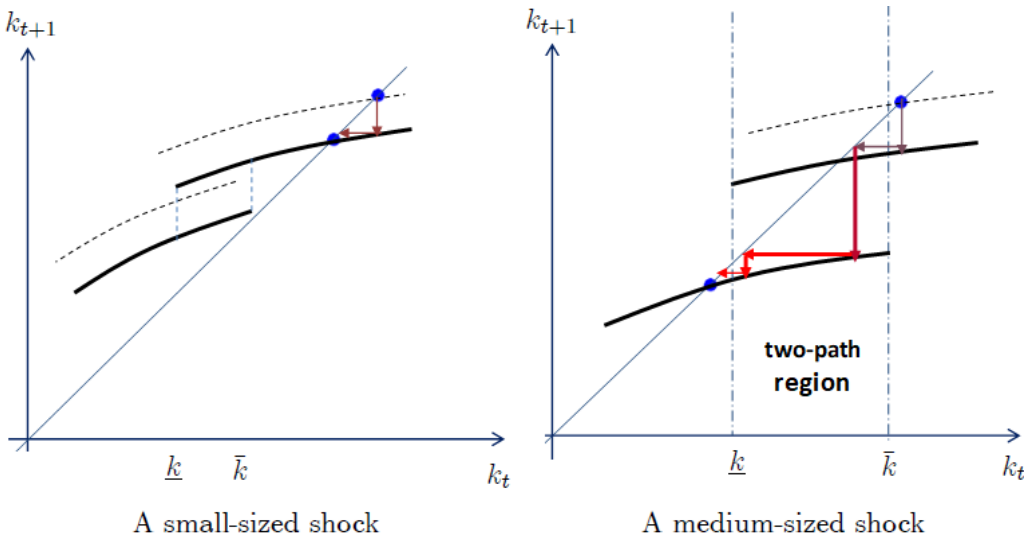


Figure 8: Transitional dynamics under a negative shock to $\mu_{\theta t}$

3.3 Discussions

For tractability and cleanness, we have chosen an OLG setup for the dynamic model. It is worth noting that for tractability a creditor run in our model’s equilibrium corresponds to the situation where a bank *completely* fails in the sense that all its creditors withdraw and its asset is completely liquidated. In reality, however, a creditor run is referred to in a broader sense that some creditors withdraw and the asset is partially liquidated. For example, the empirical study of Covitz et al. (2013) defines that an ABCP program experiences a run when it cannot roll over more than 10% of its maturing debt. The intuition of our model nevertheless goes through if a creditor run is defined in the broad sense. In the following, we interpret our OLG setup in two ways.

First, the old generation of bankers — the investors in the asset market — can be interpreted as other banks or financial institutions such as hedge funds. In this case, the secondary asset market in our dynamic model can be interpreted as an interbank market. In fact, financial institutions in the system can have *staggered* debt rollover or debt maturity dates, i.e., at a particular date some institutions need to roll over a large fraction of their debt while others do not. Therefore, at a particular date if those institutions that need to roll over experience runs (or large withdrawals by their creditors), at the next date they will be constrained and thus be less able to participate in providing market liquidity for other institutions, in the spirit of He and Krishnamurthy (2012) where low intermediary capital reduces the risk-bearing capacity of asset market investors.²¹ If asset market investors in our model are interpreted as hedge funds, the implication of our dynamic model is also consistent with the evidence documented by Mitchell and Pulvino (2012) that the spread of funding liquidity shocks from the broker-dealer sector to the hedge fund sector limits hedge funds’ capacity to provide market liquidity, which subsequently in turn causes problems for the broker-dealer sector.²²

Second, from the long-run perspective, the asset market in our model can correspond to the real side of the economy while the banking sector corresponds to the financial side of the economy. If the banking sector suffers runs more often, the performance of the real economy will be affected. The poor performance of the real economy will in turn mean lower market liquidity provided to the banking sector. That is, there is a vicious cycle between lower market liquidity and more bank runs. This situation resembles the episodes of the U.S. economy in the 19th century and early 20th century. Indeed, during that period, the banking system was plagued by runs while the growth of the real economy was frequently interrupted (see Figure 9).

²¹The evidence documented by Krishnamurthy et al. (2014) suggests that the problems in the repo market during the recent crisis looked “more likely a credit-crunch in which [lenders] tightened the terms for their borrowers”.

²²Diamond and Rajan (2005) (as well as Liu (2016)) study banking crises and aggregate liquidity shortages, showing that contagions can arise from a shrinking of the common pool of liquidity. Allen and Gale (2000) study financial contagion through interbank claims and assume that banks face exogenous idiosyncratic liquidity shocks (see also Bhattacharya and Gale (1987)).

U.S. RECESSIONS with BANKING PANICS

<u>Date</u>	<u>Major Recession (months)</u>	<u>Bank Panic or Suspension</u>	<u>Numerous Bank Failures</u>
1812	none: war-related	Aug 1814	
1819	1818–1821	Apr–May 1819	1819-1824 (not New England)
1837	1837–1838	May 1837	
1839	none	Oct 1839–Mar 1842	1839–1841 (state defaults)
1857	Jun 1857–Dec 1858 (18)	Oct 1857	
1861	none: war-related	Dec 1861	
1873	Oct 1873–Dec 1875 (27)	Sep 1873	
1893	Jan 1893–Jun 1894 (17)	Jun–Aug 1893	1890–1896 (agricultural)
1907	May 1907–Jun 1908 (13)	Oct 1907	
1914	none: war-related	Aug–Oct 1914 (no suspension)	
1920	Jan 1920–Jul 1921 (18)		1921–1930 (agricultural)
1929	Aug 1929–Mar 1933 (43)	Oct 1930–Mar 1933	1931–1933 (general)
2007	Dec 2007–June 2009 (18)	Aug 2007–Dec 2008	2008 (investment banks)

Figure 9: The history of U.S. recessions with banking panics (source: Hummel (2015))

4 Policy Implications

In this section, we analyze policy implications of our model. Our analysis mainly concerns the interaction of banks in the system from the dynamic perspective.²³

In our model, a creditor run is inefficient *ex post* because it makes the long-term illiquid assets prematurely liquidated, i.e., the assets with fundamental value θ_{it} are liquidated at the price of $l_{it} = \theta_{it} - \varphi_t/k_t$.²⁴ In the system, banks with realized asset quality $\theta_{it} < s_t^*$ face such inefficient liquidations. This gives room for government intervention.

4.1 Means of Intervention

Evidence In the recent crisis, the Federal Reserve adopted *unconventional* intervention measures to boost market liquidity/asset prices, such as creating emergency liquidity facilities for key credit markets and directly purchasing long-term securities (Bernanke (2009)).²⁵ For example, Duygan-Bum et al. (2013) document detailed evidence on a major unconventional intervention

²³Our analysis complements that of Vives (2014a) who offers extensive analyses of policy implications in the framework of bank runs with global games for a single bank (or a representative bank).

²⁴The short-term debt of a bank in our model may play a disciplining role (Calomiris and Kahn (1991), Diamond and Rajan (2001)). For example, without the threat of a short-term creditor run, the owner of a bank can take an (off-equilibrium) action which makes the bank asset more risky with a negative NPV.

²⁵For details, see https://www.federalreserve.gov/monetarypolicy/bst_crisisresponse.htm.

by the Federal Reserve, namely the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (henceforth AMLF). Duygan-Bum et al. (2013) write:

“[T]he AMLF was one of the most unusual facilities when viewed against traditional lender-of-last-resort operations by a central bank (i.e., discount window lending). For example, in a substantial departure from its “traditional” — recourse and overcollateralized — loans, the Federal Reserve accepted some credit risk under the AMLF by issuing nonrecourse loans to banks that purchased ABCP directly from money market funds and imposing no haircut on the collateral of these loans. ...the borrowing banks received from the AMLF nonrecourse loans that were collateralized by the ABCP purchased from MMMFs...”

Theory Our model framework provides a new angle to theoretically evaluate the unconventional interventions. Specifically, using our model framework, we study and compare two possible intervention measures: providing loans to investors (asset buyers) versus providing loans directly to banks. The former corresponds to the unconventional intervention of boosting market liquidity/asset prices while the latter corresponds to the conventional intervention of directly providing funding liquidity to banks (“lender of last resort”).

First, in period t , if market liquidity falls in the region of $k_t \in [\underline{k}, \bar{k}]$ as shown in Figures 6 and 7, multiple self-fulfilling equilibria exist. To eliminate the self-fulfilling inefficient equilibrium, the government can announce and commit to supporting asset prices in the asset market as long as the proportion of failing banks in the system exceeds φ_t^G (or equivalently the liquidity premium is higher than φ_t^G/k_t), where φ_t^G denotes the equilibrium φ_t for the “good” equilibrium. This announcement, with zero cost, would sufficiently change creditors’ expectation and thereby coordinate the agents to the efficient equilibrium.

Second, for a given selected stable equilibrium as in Figure 6, we show how the two different interventions can improve the ex post efficiency.

Supporting the asset market Suppose the government decides to give Q amount of liquidity support to the secondary asset market; that is, the government lends Q to the investors in the secondary market. This is equivalent to the government itself buying the assets but through the investors as the intermediary, by noting that the loans provided by the government to the asset buyers are nonrecourse loans that are collateralized by purchased assets only with no haircut as documented in Duygan-Bum et al. (2013). Then, the creditor run-asset market equilibrium under

the intervention is given by

$$\frac{l_{it}}{F} \cdot \Phi \left(\frac{s_t^* - FR}{\sigma_e} \right) = \frac{w_0}{w} \quad (\text{creditor run for an individual bank}) \quad (11a)$$

$$l_{it} = s_t^* - \varphi_t/k_t + \Delta \quad (\text{asset market equilibrium}) \quad (11b)$$

$$\varphi_t = \Phi \left(\frac{s_t^* - \mu\theta_t}{\sigma_\theta} \right) \quad (\text{bank failures in the system}) \quad (11c)$$

$$\Delta \cdot \varphi_t = Q. \quad (\text{government's liquidity support}) \quad (11d)$$

In (11a)-(11d), each unit of assets sold in the secondary market is essentially subsidized with Δ and the total subsidy is Q .²⁶ In other words, for each unit of assets that an investor purchases, that investor himself pays $\theta_{it} - \varphi_t/k$ while the government pays Δ . The government's intervention in the secondary market triggers a positive feedback loop: liquidity support Q increases Δ or l_{it} and thus reduces the running threshold s_t^* , which in turn reduces φ_t and thus further increases Δ , and so on. Importantly, the intervention in the secondary market will channel liquidity Q to the *troubled* banks only.

Supporting banks In contrast, the government can use the conventional measure of lending directly to the banks ("lender of last resort"). In this case, the government faces the information asymmetry problem, i.e., it does not know the situation of an individual bank. From the perspective of the banks, however, either all or none of them would want to borrow from the government (central bank).²⁷ Without loss of generality, we assume that the government distributes the Q amount of liquidity *evenly* among all banks. Then, the creditor run-asset market equilibrium under the intervention is given by

$$\frac{l_{it} + \Delta}{F} \cdot \Phi \left(\frac{s_t^* - FR}{\sigma_e} \right) = \frac{w_0}{w} \quad (\text{creditor run for an individual bank}) \quad (12a)$$

$$l_{it} = s_t^* - \varphi_t/k_t \quad (\text{asset market equilibrium}) \quad (12b)$$

$$\varphi_t = \Phi \left(\frac{s_t^* - \mu\theta_t}{\sigma_\theta} \right) \quad (\text{bank failures in the system}) \quad (12c)$$

$$\Delta \cdot \mathbf{1} = Q. \quad (\text{government's liquidity support}) \quad (12d)$$

In (12a)-(12d), each bank essentially receives some amount of additional cash, which will lower the running threshold s_t^* . However, in this case, all banks including those that do not suffer creditor runs ex post receive liquidity support. By noting the difference between (11d) and (12d), it is easy to show that s_t^* given by (11a) to (11d) is lower than s_t^* given by (12a) to (12d) under the same Q .

Corollary 1 *Supporting asset prices can not only break the runs-prices feedback along a stable*

²⁶We focus on the case where Q is not too large such that in equilibrium $\varphi_t/k_t \geq \Delta$ or $\theta_{it} \geq l_{it}$ for any i .

²⁷If the loans provided by the government to the banks are very cheap, all banks may want to borrow. On the other hand, obtaining liquidity support from the government may signal a bank's fundamental weakness to its creditors, which implies that no banks may want to initiate borrowing themselves (see Duygan-Bum et al. (2013)).

equilibrium but also eliminate self-fulfilling multiple equilibria.

i) Given the existence of multiple equilibria, the government's announcement and commitment to supporting asset prices can eliminate the inefficient equilibrium with zero cost.

ii) For a given selected stable equilibrium, supporting asset prices of the asset market can be more effective than providing funding liquidity to banks in reducing runs in the system, i.e., the equilibrium s_t^ is lower in (11a) to (11d) than in (12a) to (12d) under the same Q .*

Providing funding liquidity helps to mitigate the coordination problem among creditors within a bank, while supporting market liquidity/asset prices targets directly the coordination problem among creditors across different banks in the system. In recent decades, the banking system has shifted heavily toward the “market-based” business model and a large chunk of financial institutions have gotten involved in and linked to asset markets. Their business operations have become highly sensitive to changes in market liquidity. This means that during a crisis the intervention in asset markets may be more crucial nowadays than decades ago. Our model helps shed light on the theoretical foundation underlying the unconventional interventions in asset markets.

4.2 Timing of Intervention

Our dynamic model also has implications for the timing of intervention. The model suggests that intervention should not be left too late. If intervention is timely enough, the government's announcement with zero cost can be sufficient. However, if intervention is left too late, the actual support to the asset market will be necessary. The above implication can be illustrated in Figure 8. In the right panel of Figure 8, if intervention takes place before k_t falls below \underline{k} , the government's announcement can coordinate creditors to the upper branch of the equilibrium path and thus the “good” steady state can be reached. However, if interventions takes place after k_t falls below \underline{k} , the upper branch of the equilibrium path will have been closed off and the government's pure announcement will no longer work. In this case, the government has to first provide actual support to the asset market (for example, by lending to asset buyers) to lift market liquidity k_t above \underline{k} , and then adopt additional interventions such as making announcements and committing to support asset prices. This way, the economy can eventually reach the “good” steady state.

4.3 Temporary Shock versus Permanent Shock

The shock we discuss in Proposition 6 is a permanent shock, which justifies intervention. However, even if a shock is only temporary, intervention may be necessary. For example, in Figure 7(i), suppose the economy is originally in the steady state and a temporary shock results in k_t declining. (Because it is a temporary shock, the curves in Figure 7(i) do not move or shift). In this case,

without government intervention, the economy can still gradually return to the original steady state. However, if the shock is big enough resulting in the current k_t dropping below \underline{k} , it may take a long time for the economy to recover and return to the steady state; particularly, large inefficiency losses may occur in the periods during the recovery process (by nothing that the first stage of recovery is along the lower branch of the equilibrium path). To improve efficiency, the government can first support the asset market to lift market liquidity k_t above \underline{k} and then coordinate creditors to the upper branch of the equilibrium path. This way, the economy can return to the steady state much faster and also the efficiency in the periods during the recovery process is improved.

5 Model Robustness

In this section, we conduct two robustness analyses on our static model.

5.1 A Single Bank with a Downward-sloping Liquidation Price

Alternatively suppose there is only one bank, and when it liquidates its assets, it faces a downward-sloping demand curve. We show that in this case the creditor-run equilibrium is unique.

Specifically, consider a single bank in our main model.²⁸ Assume that the demand function of the bank’s assets (one unit in total) is $p = f(Q)$, where Q is the quantity of selling and p is the price. Let the revenue function of selling be $\pi(Q) = pQ = f(Q)Q$. Without loss of generality, it is reasonable to assume that $\pi'(\cdot) > 0$ for $Q \in [0, 1]$.

Then, in the payoff structure of Table 2, l_i is with the value $l_i = \pi(1)$. Intuitively, an individual creditor needs to think and ask “when my bank is going to fail”, and the answer is “in the worst-case scenario where the bank exhausts its asset sales” (that is $\pi(1)$). Figure 10 depicts the creditor-run payoff structure.

²⁸The setting for the single bank, other than the assumption of a downward-sloping asset demand curve, is the same as that in the main model. That is, the asset quality of the single bank has prior $\theta \sim N(\mu_\theta, \sigma_\theta^2)$, and the signal for creditor h of the bank (about asset quality θ) at $t = 1$ is $s^h = \theta + \sigma_s \epsilon^h$.

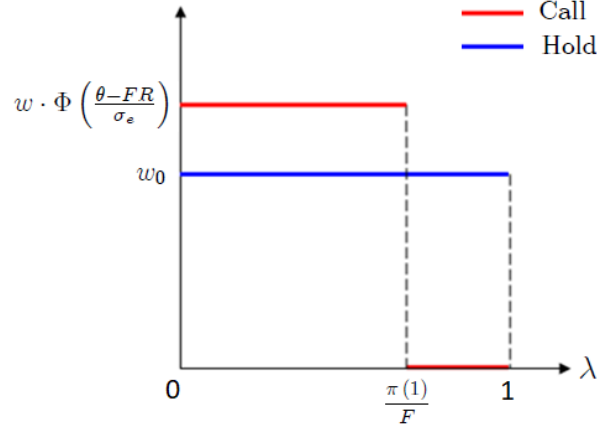


Figure 10: Creditor-run payoff structure for a single bank with a downward-sloping liquidation price

It is thus easy to find the creditor-run equilibrium for the single-bank case. (2) is replaced by

$$\frac{\pi(1)}{F} = \Phi\left(\frac{s^* - \theta^*}{\sigma_s}\right). \quad (13)$$

Equations (13) and (3) together gives the creditor-run equilibrium. Hence, like the case of the liquidation value l_i being exogenous in Proposition 1, the equilibrium, given by $\frac{\pi(1)}{F} \cdot \Phi\left(\frac{s^* - FR}{\sigma_e}\right) = \frac{w_0}{w}$ under the limit $\sigma_s \rightarrow 0$, is clearly unique.

In fact, a downward-sloping liquidation price (versus a constant one) may only result in a higher running threshold s^* of creditors, but it does not change the uniqueness of the threshold equilibrium. In the Appendix, we further confirm this to be the case for the original model of Goldstein and Pauzner (2005) under a downward-sloping liquidation price.

We explain why the case of a continuum of small banks is different from the case of a single large bank in terms of equilibrium multiplicity. In the continuum case, the liquidation price is a function of the fraction of banks suffering runs, which in turn depends on the running thresholds taken by creditors in those banks. The single-bank case does not have such a feature. We can intuitively illustrate that the strategic complementarity among creditors is stronger in the continuum case than in the single-bank case. Suppose that some creditors choose to “definitely” run (i.e., equivalent to setting threshold $s^* = +\infty$). We look at other creditors’ response (i.e., how they respond by choosing their own threshold s^{*i}). In the continuum case, some creditors’ definite run and thus their banks’ fire sales reduce the liquidation price l_i for a particular bank i , but the number of creditors of that particular bank i does not change — only one force at work. In contrast, in the single-bank case, the withdrawals of some creditors reduce the liquidation price for the remaining creditors but also reduce the number of remaining creditors in coordination — two opposite forces at work. Mathematically, recall that the term $\frac{l_i}{F}$ measures the coordination risk in (6). In the

former case, the coordination risk for creditors in a particular bank i corresponds to $\frac{l_i - \Delta_l}{F}$; in the latter case, the coordination risk for the remaining creditors in the single large bank corresponds to $\frac{l_i - \Delta_l}{F - n}$, where Δ_l is the liquidation price reduced due to fire sales caused by some creditors' "definite" run in the system and n is the number of creditors who choose to "definitely" run for the single-bank case. In other words, creditors are more sensitive (by choosing a higher response s^{*i}) to the actions of other creditors in the system in the former case than in the latter case. A stronger strategic complementarity makes multiple equilibria more likely (see, e.g., Vives (2014a)).

5.2 Alternative Creditor-run Payoff Structure

Suppose we use the payoff structure in Table 1 instead of the one in Table 2. Under this alternative payoff structure, we prove in the Appendix that (4) is replaced by

$$\int_0^{\frac{s^* - \varphi k}{F}} \Phi \left(\frac{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) s^* - (1 - \lambda) FR}{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e} \right) d\lambda = \frac{w_0}{w}. \quad (14)$$

Notice that if we set $\lambda = 0$ inside the integral, (14) becomes identical to (4). We prove in the Appendix that the following key property of (4) in Lemma 2 remains for (14):

$$\frac{\partial s^*}{\partial \varphi} > 0.$$

Therefore, under the alternative payoff structure, the results of the model change only quantitatively, not qualitatively.

6 Conclusion

This paper presents a dynamic global-games model of systemic bank runs. The model has two main features. First, it studies many banks sharing a common asset market and emphasizes the interaction among banks. The model formalizes the result that the increased scale of the coordination problem, which is among creditors of the same bank and among creditors of different banks, can give rise to fundamental-based multiple equilibria at the system level under a weak condition. Second, the model analytically characterizes the dynamics. The model shows how the coordination problem in the system in each period is affected by — and in turn affects — the timing-varying market liquidity. Our theory sheds light on the empirical facts regarding the evolution of a crisis and gives new policy implications. The main purpose of our paper is to illustrate the mechanism. Extending our model framework to a full-fledged DSGE model and quantifying the magnitude of the effect are left for future research.

Appendix

A Proofs

Proof of Lemma 1: Considering that the price l_i fully reveals the fundamentals θ_i (see the trading mechanism in Vives (2014b) and Benhabib, Liu and Wang (2016)), an investor does not rely on his private information in trading. Hence, all investors are basically the same (as a representative investor). Thus, the objective function of (1) can be transformed into one of maximizing

$$\begin{aligned} & \int q_i(\theta_i - l_i)di - \frac{1}{2}\gamma Var\left(e \int q_i di\right) \\ = & \int q_i(\theta_i - l_i)di - \frac{1}{2}\gamma\sigma_e^2\left(\int q_i di\right)^2. \end{aligned}$$

The first-order condition with respect to any q_i implies

$$(\theta_i - l_i)di - \gamma\sigma_e^2\left(\int q_i di\right) di = 0,$$

that is, $\int q_i di = \frac{\theta_i - l_i}{\gamma\sigma_e^2}$. Because $n \int q_i di = \varphi$ by the market clearing condition, we have $l_i = \theta_i - \varphi/k$, where $k \equiv n/(\gamma\sigma_e^2)$.

Proof of Lemma 2: Combining (2) and (3) gives

$$\int_{\theta_i=\theta^*}^{+\infty} \Phi\left(\frac{\theta_i - FR}{\sigma_e}\right) \cdot \frac{1}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \phi\left(\frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s}\mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s}s^*\right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}}\right) d\theta_i = \frac{w_0}{w}, \quad (\text{A.1})$$

where θ^* is given by the implicit function $\frac{\theta^* - \varphi/k}{F} = \Phi\left(\frac{s^* - \theta^*}{\sigma_s}\right)$ or $s^* = \theta^* + \sigma_s \Phi^{-1}\left(\frac{\theta^* - \varphi/k}{F}\right)$.

Write the LHS of (A.1) as $V(s^*; \sigma_s)$. We transform $V(s^*; \sigma_s)$ by changing variables to $z = \frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s}\mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s}s^*\right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}}$ and obtain

$$V(s^*; \sigma_s) = \int_{z=z_0}^{\infty} \Phi\left(\frac{\sqrt{\frac{1}{\tau_\theta + \tau_s}}z + \left(\frac{\tau_\theta}{\tau_\theta + \tau_s}\mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s}s^*\right) - FR}{\sigma_e}\right) \cdot \phi(z) dz,$$

where z_0 satisfies the joint equations

$$\begin{aligned} s^* &= \theta^* + \sigma_s \Phi^{-1} \left(\frac{\theta^* - \varphi/k}{F} \right) \Big|_{\theta^* = \theta_i} \\ z &= \frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \Big|_{z = z_0} \end{aligned} .$$

That is, z_0 satisfies

$$s^* = \theta_i + \sigma_s \Phi^{-1} \left(\frac{\theta_i - \varphi/k}{F} \right), \quad (\text{A.2})$$

with $\theta_i = \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^* \right) + z_0 \sqrt{\frac{1}{\tau_\theta + \tau_s}}$. Plugging the above expression of θ_i into (A.2), we can express z_0 by an implicit function

$$z_0 = \frac{\frac{\tau_\theta}{\tau_\theta + \tau_s} (s^* - \mu_\theta)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} - \frac{\sigma_s}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \Phi^{-1} \left(\frac{\left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^* \right) + z_0 \sqrt{\frac{1}{\tau_\theta + \tau_s}} - \varphi/k}{F} \right).$$

So it follows that

$$\lim_{\sigma_s \rightarrow 0} z_0 = -\Phi^{-1} \left(\frac{s^* - \varphi/k}{F} \right).$$

Thus, under the limit $\sigma_s \rightarrow 0$ for a given σ_θ , we have $\theta^* = s^*$ and

$$\begin{aligned} \lim_{\sigma_s \rightarrow 0} V(s^*; \sigma_s) &= \Phi \left(\frac{s^* - FR}{\sigma_e} \right) \cdot \int_{-\Phi^{-1} \left(\frac{s^* - \varphi/k}{F} \right)}^{\infty} \phi(z) dz \\ &= \Phi \left(\frac{s^* - FR}{\sigma_e} \right) \cdot \frac{s^* - \varphi/k}{F}. \end{aligned}$$

Hence, (4) is proved.

We also need to prove that a creditor rolls over when his signal is higher than s^* and otherwise withdraws. Denote by s^{*h} the threshold for an individual creditor h of a bank and by s^* the threshold for all other creditors. Rewrite the LHS of ((A.1) as

$$\tilde{V}(s^{*h}, s^*) = \int_{\theta_i = \theta^*(s^*)}^{+\infty} \Phi \left(\frac{\theta_i - FR}{\sigma_e} \right) \cdot \frac{1}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \phi \left(\frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^{*h} \right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \right) d\theta_i,$$

where θ^* is given by the implicit function $\frac{\theta^* - \varphi/k}{F} = \Phi \left(\frac{s^* - \theta^*}{\sigma_s} \right)$ or $s^* = \theta^* + \sigma_s \Phi^{-1} \left(\frac{\theta^* - \varphi/k}{F} \right)$. An individual creditor takes s^* or θ^* as given. Changing variables to $z = \frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}}$, we

obtain

$$\tilde{V}(s^{*h}, s^*) = \int_{z=\frac{\theta^* - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^{*h}\right) - FR}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}}}^{+\infty} \Phi \left(\frac{\sqrt{\frac{1}{\tau_\theta + \tau_s}} z + \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^{*h}\right) - FR}{\sigma_e} \right) \cdot \phi(z) dz.$$

If follows that $\partial \tilde{V}(s^{*h}, s^*) / \partial s^{*h} > 0$.

Next, we prove the comparative statics. It is straightforward to show that $\frac{\partial \varphi}{\partial s^*} > 0$ in (5). As for $\frac{\partial s^*}{\partial \varphi} > 0$ in (4), write the LHS of (4) as function $V(s^*, \varphi)$ and we have $\frac{\partial V}{\partial s^*} = \frac{1}{F} \Phi \left(\frac{s^* - FR}{\sigma_e} \right) + \frac{s^* - \varphi/k}{F} \phi \left(\frac{s^* - FR}{\sigma_e} \right) \frac{1}{\sigma_e} > 0$ and $\frac{\partial V}{\partial \varphi} = \frac{-1/k}{F} \Phi \left(\frac{s^* - FR}{\sigma_e} \right) < 0$, and thus $\frac{\partial s^*}{\partial \varphi} = -\frac{\partial V}{\partial \varphi} / \frac{\partial V}{\partial s^*} > 0$.

Finally, we show that a stable equilibrium corresponds to $\frac{\partial V(s^*)}{\partial s^*} > 0$ at the equilibrium point and an unstable equilibrium corresponds to $\frac{\partial V(s^*)}{\partial s^*} < 0$ at the equilibrium point. Based on the definition of function $\tilde{V}(s^{*h}; s^*, \sigma_s)$ in (8), it follows that $V(s^*) = \lim_{\sigma_s \rightarrow 0} \tilde{V}(s^{*h} = s^*, s^*, \sigma_s)$, given in (7). A symmetric equilibrium is given by $V(s^*) = \frac{w_0}{w}$. At a symmetric equilibrium, if $\frac{\partial V(s^*)}{\partial s^*} > 0$ is satisfied, it is a stable equilibrium because

$$\begin{aligned} & \frac{\partial V(s^*)}{\partial s^*} > 0 \\ \implies & \tilde{V}(s^{*h} + \Delta, s^* + \Delta) - \tilde{V}(s^{*h}, s^*) > 0 \text{ for a small } \Delta > 0 \\ \implies & \frac{\partial \tilde{V}}{\partial s^{*h}} + \frac{\partial \tilde{V}}{\partial s^*} > 0 \\ \implies & \frac{\partial s^{*h}}{\partial s^*} = -\frac{\partial V / \partial s^*}{\partial \tilde{V} / \partial s^{*h}} < 1, \end{aligned}$$

by noting that $\frac{\partial \tilde{V}}{\partial s^{*h}} > 0$ and $\frac{\partial \tilde{V}}{\partial s^*} < 0$. Similarly, at a symmetric equilibrium, if $\frac{\partial V(s^*)}{\partial s^*} < 0$ is satisfied, it is an unstable equilibrium.

Proof of Proposition 1: If the liquidation value l_i is exogenous, (2) is replaced by

$$\frac{l_i}{F} = \Phi \left(\frac{s^* - \theta^*}{\sigma_s} \right). \quad (\text{A.3})$$

Similar to the prove of Lemma 2, under the limit $\sigma_s \rightarrow 0$ for a given σ_θ , (A.3) and (3) together will give $\theta^* = s^*$ and (6).

Proof of Proposition 2: The proof is straightforward and hence omitted.

Proofs of Proposition 3: Write the LHS of (7) as function $V(s^*; \mu_\theta, k)$, where

$$V(s^*; \mu_\theta, k) = \left\{ \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) / k \right] \right\} \cdot \Phi \left(\frac{s^* - FR}{\sigma_e} \right). \quad (\text{A.4})$$

It is easy to show that

$$\begin{aligned} & \frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} \\ = & \frac{1}{F} \left[1 - \phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) \frac{1}{k\sigma_\theta} \right] \Phi \left(\frac{s^* - FR}{\sigma_e} \right) + \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) / k \right] \phi \left(\frac{s^* - FR}{\sigma_e} \right) \frac{1}{\sigma_e}, \end{aligned} \quad (\text{A.5})$$

$$\frac{\partial V(s^*; \mu_\theta, k)}{\partial \mu_\theta} = \left\{ \frac{1}{F} \left[\frac{1}{\sigma_\theta} \phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) / k \right] \right\} \cdot \Phi \left(\frac{s^* - FR}{\sigma_e} \right) > 0,$$

and

$$\frac{\partial V(s^*; \mu_\theta, k)}{\partial k} = \left\{ \frac{1}{F} \left[\Phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) / k^2 \right] \right\} \cdot \Phi \left(\frac{s^* - FR}{\sigma_e} \right) > 0.$$

By (A.5), when s^* is sufficiently higher or lower than μ_θ , $1 - \phi \left(\frac{s^* - \mu_\theta}{\sigma_\theta} \right) > 0$ and hence $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} > 0$.

When s^* is at an intermediate level close to μ_θ , it can be $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} < 0$ and thus $V(s^*; \mu_\theta, k)$ can be non-monotonic in s^* . Because there can be at most only one continuous interval around μ_θ in which $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} < 0$, the non-monotonic curve of $V(s^*; \mu_\theta, k)$ is “N”-shaped in s^* (i.e., increasing first and then decreasing and then increasing again). So $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ can admit one or (generically) three solutions with respect to s^* .

i) When k is high enough, $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} > 0$ also holds for any s^* , so $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ admits a unique solution with respect to s^* .

When μ_θ decreases, the curve of $V(s^*; \mu_\theta, k)$ shifts downward in Figure 4. When μ_θ is low enough, the curve of $V(s^*; \mu_\theta, k)$ intersects with the horizontal line $V = \frac{w_0}{w}$ only once for any k , so $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ admits a unique solution with respect to s^* .

ii) Consider a sufficiently high μ_θ such that the curve of $V(s^*; \mu_\theta, k)$ could intersect with the horizontal line $V = \frac{w_0}{w}$ more than once for some k . A decrease in k not only increases the curvature of $V(s^*; \mu_\theta, k)$ but also shifts the curve of $V(s^*; \mu_\theta, k)$ downward by $\frac{\partial V(s^*; \mu_\theta, k)}{\partial k} > 0$. Hence, when k is sufficiently high or sufficiently low, $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ admits a unique solution with respect to s^* . When k is not too high and not too low, $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ admits multiple (typically three) solutions with respect to s^* .

iii) Consider a sufficiently low k such that $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} < 0$ at some s^* (i.e., $V(s^*; \mu_\theta, k)$ is non-monotonic with respect to s^*). Considering that the curve of $V(s^*; \mu_\theta, k)$ shifts downward as μ_θ decreases, $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$ admits multiple (typically three) solutions with respect to s^* when μ_θ is not too high and not too low and a unique solution when μ_θ is sufficiently high or sufficiently

low.

iv) Based on the result in the proof of Lemma 2, at a stable equilibrium $\frac{\partial V(s^*)}{\partial s^*} > 0$. Because $\frac{\partial V(s^*; \mu_\theta, k)}{\partial \mu_\theta} > 0$ and $\frac{\partial V(s^*; \mu_\theta, k)}{\partial k} > 0$, it follows that $\frac{\partial s^*}{\partial \mu_\theta} = -\frac{\partial V / \partial \mu_\theta}{\partial V / \partial s^*} < 0$ and $\frac{\partial s^*}{\partial k} = -\frac{\partial V / \partial k}{\partial V / \partial s^*} < 0$.

Proof of Lemma 3: The proof is straightforward and hence omitted.

Proof of Lemma 4: Given the results of Proposition 3, we only need to prove that both \underline{k} and \bar{k} are decreasing functions of $\mu_{\theta t}$. At $k = \underline{k}$ or \bar{k} , the curve of $V(s^*; \mu_\theta, k)$ is tangent to the horizontal line $V = \frac{w_0}{w}$ in Figure 5 and so $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} = 0$. When μ_θ increases, $V(s^*; \mu_\theta, k)$ increases by $\frac{\partial V(s^*; \mu_\theta, k)}{\partial \mu_\theta} > 0$. To restore the equality of $V(s^*; \mu_\theta, k) = \frac{w_0}{w}$, k must decrease by considering that $\frac{\partial V(s^*; \mu_\theta, k)}{\partial k} > 0$ and $\frac{\partial V(s^*; \mu_\theta, k)}{\partial s^*} = 0$ at $k = \underline{k}$ or \bar{k} . That is, both \underline{k} and \bar{k} are decreasing functions of $\mu_{\theta t}$.

Proof of Proposition 4: As φ_t is a function of k_t and $\mu_{\theta t}$, we can express k_{t+1} as a function of k_t and $\mu_{\theta t}$. Based on the results of Proposition 3, at a stable equilibrium the comparative static result of $\frac{\partial s_t^*}{\partial k_t} < 0$ holds and therefore $\frac{\partial k_{t+1}}{\partial k_t} < 0$ is true. Because there are two stable equilibria, $g^+(\cdot)$ and $g^-(\cdot)$ are both increasing functions and satisfy $g^+(k_t) > g^-(k_t)$ for any $k_t \in [\underline{k}, \bar{k}]$.

Proof of Proposition 5: Suppose that $\mu_{\theta t} = \mu_\theta$, a constant. Because $\varphi_t \in (0, 1)$, it follows that $k_{t+1} \in \left(\frac{\bar{n}}{\gamma \sigma_e^2}, \frac{\bar{n}+1}{\gamma \sigma_e^2} \right)$ by (9a) to (9d).

First, we show that when μ_θ is sufficiently high, there is a unique steady-state equilibrium, that is, the function given by (10) has a unique intersection with the 45^0 line as shown in Figure 7(i). Based on Lemma 4, \bar{k} and \underline{k} are decreasing functions of $\mu_{\theta t}$. When $\mu_{\theta t}$ is sufficiently high, \bar{k} is close to zero. Considering that $k_{t+1} > \frac{\bar{n}}{\gamma \sigma_e^2}$, the point with coordinates $(k_t, k_{t+1}) = (\bar{k}, g^-(\bar{k}; \mu_{\theta t}))$ in Figure 7 is above the 45^0 line. So is the point with coordinates $(k_t, k_{t+1}) = (\underline{k}, g^+(\underline{k}; \mu_{\theta t}))$. On the other hand, considering $k_{t+1} < \frac{\bar{n}+1}{\gamma \sigma_e^2}$, the point with coordinates $(k_t, k_{t+1}) = (+\infty, g^+(k_t = +\infty; \mu_{\theta t}))$ is below the 45^0 line. That is, the upper branch of the function in Figure 7(i) has a unique intersection with the 45^0 line.

Second, we show that when μ_θ is sufficiently low, there is a unique steady-state equilibrium as shown in Figure 7(iii) or Figure 7(iv). Based on Lemma 4, \bar{k} and \underline{k} are decreasing functions of $\mu_{\theta t}$. When $\mu_{\theta t}$ is sufficiently low, \underline{k} is high and at the same time s_t^* is high relative to $\mu_{\theta t}$ and thus φ_t is high and k_{t+1} is low. Hence, the point with coordinates $(k_t, k_{t+1}) = (\underline{k}, g^+(\underline{k}; \mu_{\theta t}))$ is below the 45^0 line. So is the point with coordinates $(k_t, k_{t+1}) = (\bar{k}, g^-(\bar{k}; \mu_{\theta t}))$. On the other hand, considering that $k_{t+1} > \frac{\bar{n}}{\gamma \sigma_e^2}$, the point with coordinates $(k_t, k_{t+1}) = (0+, g^-(0+; \mu_{\theta t}))$ in Figure 7 is above the 45^0 line. That is, the lower branch of the function in Figure 7(iii) has a unique intersection with the 45^0 line.

Third, when μ_θ is in an intermediate range between the above two extreme cases, there are two steady-state equilibria as shown in Figure 7(ii).

Proof of Proposition 6: If the shock is small enough such that $\mu'_\theta > \mu_\theta^{**}$, there is no regime change (i.e., remaining in regime (i) in Figure 7 before and after the shock). If the shock is medium-sized such that $\mu_\theta^* < \mu'_\theta < \mu_\theta^{**}$, there is a regime change from (i) to (ii) in Figure 7. To have the pattern as in the right panel of Figure 8 (i.e., the upper steady-state k is between \bar{k} and \underline{k} while the lower steady-state k is below \underline{k}), the shock needs to be such that μ'_θ is slightly above and close to μ_θ^* .

Some results in Section 5.1: We show that the original model of Goldstein and Pauzner (2005) under a downward-sloping liquidation price has a unique threshold equilibrium. We use the same notations as in their paper. In their model, the liquidation price at the interim date is constant — the liquidation value is 1 per unit of assets. Hence, a patient agent's utility differential, between withdrawing in period 2 and withdrawing in period 1, is given by (see equation (3) on page 1304 of their paper):

$$v(\theta, n) = \begin{cases} p(\theta) u\left(\frac{1-nr_1}{1-n}R\right) - u(r_1) & \text{if } \frac{1}{r_1} \geq n \geq \lambda \\ 0 - \frac{1}{nr_1}u(r_1) & \text{if } 1 \geq n \geq \frac{1}{r_1} \end{cases}.$$

If the liquidation price is downward-sloping with the revenue function of selling being $\pi(Q) = pQ = f(Q)Q$ and $\pi'(\cdot) > 0$ for $Q \in [0, 1]$, the payoff function $v(\theta, n)$ in Goldstein and Pauzner (2005) is replaced by

$$v(\theta, n) = \begin{cases} p(\theta) u\left(\frac{1-\pi^{-1}(nr_1)}{1-n}R\right) - u(r_1) & \text{if } \frac{\pi(1)}{r_1} \geq n \geq \lambda \\ 0 - \frac{\pi(1)}{nr_1}u(r_1) & \text{if } 1 \geq n \geq \frac{\pi(1)}{r_1} \end{cases}. \quad (\text{A.6})$$

The new net payoff function $v(\theta, n)$ in (A.6) satisfies the *single-crossing property* as in Goldstein and Pauzner (2005). Hence there exists a unique threshold equilibrium. Figure A1 illustrates function $v(\theta, n)$ under a downward-sloping liquidation price with $f(Q = 0) = 1$ versus a constant liquidation price with $p = 1$ (see also Figure 2 on page 1305 of Goldstein and Pauzner (2005)).

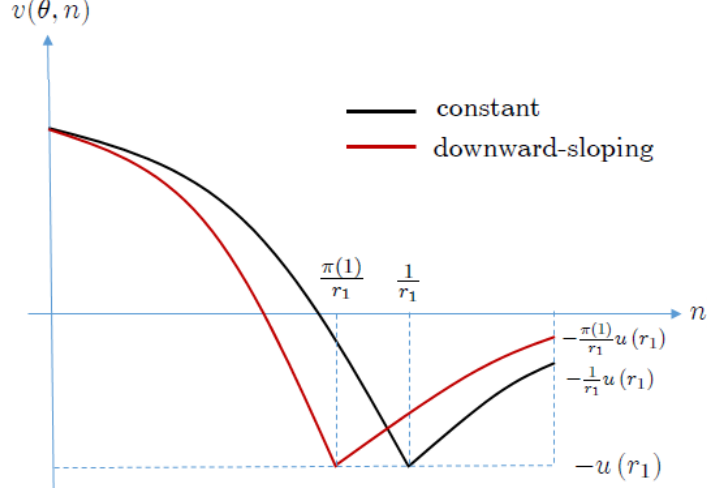


Figure A1: Payoff function $v(\theta, n)$ in Goldstein and Pauzner (2005) under a downward-sloping liquidation price versus a constant liquidation price

Proof in Section 5.2: Under the alternative payoff structure in Table 1, equation (3) is replaced by the following equation:

$$\int_{\theta^*}^{+\infty} w \cdot \Phi \left(\frac{\left[1 - \frac{F}{l_i} \Phi \left(\frac{s^* - \theta_i}{\sigma_s} \right)\right] \theta_i - \left[1 - \Phi \left(\frac{s^* - \theta_i}{\sigma_s} \right)\right] FR}{\left[1 - \frac{F}{l_i} \Phi \left(\frac{s^* - \theta_i}{\sigma_s} \right)\right] \sigma_e} \right) \frac{\phi \left(\frac{\theta_i - \left(\frac{\tau_\theta}{\tau_\theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} \right)}{\sqrt{\frac{1}{\tau_\theta + \tau_s}}} d\theta_i = w_0, \quad (\text{A.7})$$

by considering Table 1 and $\lambda(\theta_i; s^*) = \Pr(\theta_i + \sigma_s \epsilon^h < s^*) = \Phi \left(\frac{s^* - \theta_i}{\sigma_s} \right)$.

Combining (14) and (2) into one equation and using the same method as that in the proof of Lemma 1 of changing variables of integral, we can transform the combined equation for the limiting case of $\sigma_s \rightarrow 0$ into

$$\int_0^{\frac{s^* - \varphi k}{F}} \Phi \left(\frac{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) s^* - (1 - \lambda) FR}{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e} \right) d\lambda = \frac{w_0}{w}. \quad (\text{A.8})$$

Next, we prove that (A.8) retains the property $\frac{\partial s^*}{\partial \varphi} < 0$ of (4). Write the LHS of (A.8) as function $\Lambda(s^*, \varphi)$, where

$$\Lambda(s^*, \varphi) = \int_0^{\frac{s^* - \varphi k}{F}} \Phi \left(\frac{s^*}{\sigma_e} - \frac{(1 - \lambda) FR}{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e} \right) d\lambda.$$

We have

$$\frac{\partial \Lambda}{\partial s^*} = \int_0^{\frac{s^* - \varphi k}{F}} \phi \left(\frac{s^*}{\sigma_e} - \frac{(1 - \lambda) FR}{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e} \right) \left(\frac{1}{\sigma_e} + \frac{(1 - \lambda) FR}{\left[\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e\right]^2} F \lambda \sigma_e \frac{1}{(s^* - \varphi k)^2} \right) d\lambda > 0$$

and

$$\frac{\partial \Lambda}{\partial \varphi} = \int_0^{\frac{s^* - \varphi k}{F}} \phi \left(\frac{s^*}{\sigma_e} - \frac{(1 - \lambda) FR}{\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e} \right) \frac{(1 - \lambda) FR}{\left[\left(1 - \frac{F}{s^* - \varphi k} \lambda\right) \sigma_e\right]^2} F \lambda \sigma_e \frac{-\varphi}{(s^* - \varphi k)^2} d\lambda < 0.$$

Hence, by the implicit function theorem, it follows that

$$\frac{\partial s^*}{\partial \varphi} = -\frac{\partial \Lambda / \partial \varphi}{\partial \Lambda / \partial s^*} > 0.$$

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