Extrapolation and Risk-Return Trade-offs

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Abstract

This study investigates the impact of return extrapolation on risk-return trade-offs in both the aggregate time series and the cross section of stocks. We find that the relation between the market's expected return and expected variance is positive during periods with a low degree of extrapolation (DOX), as proposed by Cassella and Gulen (2018), but this relation is negative during high-DOX periods. In the cross section, we find that low-risk anomalies are significantly more pronounced following high-DOX periods and among firms with high firm-level DOX. These findings are consistent with the interpretation that, relative to a fully rational framework, the existence of extrapolators tends to raise equilibrium stock prices in response to a positive risk shock. The increase in stock prices is meant to sustain high perceived future returns to compensate for this enhanced risk and thus leads to overpricing and a weakening of the traditional risk-return trade-off.

JEL Classification: G11, G12, G40

Keywords: Extrapolation, Risk-return trade-off, Volatility

1 Introduction

One fundamental tenet in finance is the risk-return trade-off. Classic asset pricing theory typically implies a positive relationship between expected risk and expected returns in both the aggregate time series and the cross section. However, the existing empirical evidence is quite mixed. Some find a positive risk-return relation, while others suggest that the risk-return trade-off is weak or even negative.[1] This paper investigates how return extrapolation affects the traditional risk-return trade-off in both the aggregate time series and the firm-level cross section.

The intuition that return extrapolation influences the risk-return trade-off can be summarized as follows. The classic ICAPM model (Merton, 1973) predicts that when investors expect higher risk (e.g., higher expected volatility), they require a higher risk premium (i.e., expect higher returns). In particular, all else equal, in response to an increase in risk, the current stock price should be depressed to reflect the higher required rate of return in the future such that investors are compensated for bearing higher risk. However, in models with extrapolative investors (see, e.g., Barberis et al. (2015) and Barberis et al. (2018)), extrapolators' expected stock return is modeled as an exponentially decaying average of past realized stock returns. Thus, a high current stock price implies higher perceived future stock returns. As a result, when facing enhanced risk, extrapolators demand high expected returns in their perception, which tends to push up current equilibrium prices relative to the fully rational benchmark since a higher current return implies higher perceived future returns for extrapolators.[2] Thus, in a model featuring extrapolative investors, after an increase in risk, the current equilibrium stock price is not depressed as much as it is in the fully rational benchmark, in compensation for enhanced risk. As a result, during periods with enhanced risk, the current equilibrium price could be too high; when the price corrects to the fundamental in the subsequent period, a smaller and potentially negative return is realized in the subsequent period, weakening the standard positive risk-return trade-off.

We formalize the above economic intuition with a simple two-period model in Appendix

[1]For a positive relation, see French, Schwert, and Stambaugh (1987), Guo and Whitelaw (2006), Pástor, Sinha, and Swaminathan (2008), and Brandt and Wang (2010). For a weak or negative relation, see Black (1972), Black, Jensen, and Scholes (1972), Haugen and Heins (1975), Glosten, Jagannathan, and Runkle (1993), Ang et al. (2006), Baker, Bradley, and Wurgler (2011), Frazzini and Pedersen (2014), Moreira and Muir (2017), Shen, Yu, and Zhao (2017), and Lochstoer and Muir (2020).

[2]For example, in the recent GameStop episode, retail investors flooded the market to buy the stock despite its volatility spikes and a stock price well above the company's fundamental. This might have occurred because of retail investors' extrapolative belief.

A. In particular, the above argument has a few direct testable implications. First, during periods with a low degree of extrapolation (DOX), the market is relatively more rational, and the risk-return relation should be significantly positive. Second, during high-DOX periods, on the other hand, the risk-return relation should be significantly weakened by extrapolators. Third, the contemporaneous negative relation between innovations in expected variance and stock returns should also be weakened by extrapolators during high-DOX periods. Fourth, in response to enhanced risk, stocks might be relatively more overvalued following high-DOX periods relative to low-DOX periods. Thus, the interaction between volatility and DOX should predict subsequent earnings announcement returns, subsequent forecast errors, or both.

Using the DOX measure in Greenwood and Shleifer (2014) and Cassella and Gulen (2018), we first conduct an empirical investigation of the impact of extrapolation on the aggregate market's risk-return trade-off. In particular, we first dynamically estimate the degree of extrapolation in investors' beliefs using survey data. The recursively estimated DOX quantifies the degree to which investors extrapolate past returns. Using this DOX measure and various measures on expected market volatility, we find supportive evidence for all of the above four implications for the aggregate stock market. More specifically, we find that a one-standard-deviation increase in variance is associated with a 2.17% increase in monthly expected excess returns during low-DOX periods, whereas a one-standard-deviation increase in variance is associated with a 0.60% decrease in monthly expected excess returns during high-DOX periods. Similarly, we confirm that following high-DOX periods, the contemporaneous negative relationship between innovations in expected variance and stock returns is significantly weakened. For example, a one-standard-deviation increase in expected variance is associated with a 4.81% decrease in contemporaneous monthly excess returns during low-DOX periods, whereas a one-standard-deviation increase in expected variance is associated with only a 1.50% decrease in contemporaneous monthly excess returns during high-DOX periods. Lastly, we find that the interaction between expected volatility and DOX can significantly and negatively predict aggregate earnings announcement returns and aggregate earnings forecast errors.

In addition, we verify that our main finding remains valid under a battery of robustness checks. First, we show that our result is robust to using alternative volatility models. In particular, we conduct our empirical tests using the rolling window model (RW), GARCH, asymmetric GARCH, GRJ-GARCH models, and the mixed data sampling approach (MIDAS). Earlier studies find that these models typically produce different patterns regarding the aggregate market's risk-return trade-off. However, our results are remarkably consistent across all of these widely used volatility models, highlighting the robust role of DOX in the mean-variance relation. Second, our results are also robust using alternative regression specifications including treating DOX as a continuous variable and after controlling for many other economic forces such as investor sentiment, business cycle variables, and time variation in effective risk aversion. Third, additional tests suggest that other behavioral biases such as misguided risk perception appear unable to fully account for our documented weakened mean-variance relation during high-DOX periods. Finally, using survey data for stock market indices in other G7 countries, we find that DOX largely retains its significant role in affecting the aggregate mean-variance relation outside of the United States.

More important, we also provide consistent evidence based on the cross section of stocks. For example, a one-standard-deviation increase in DOX leads to a 0.41% increase in the monthly returns of the average low-risk anomaly, which is a long-short strategy based on total volatility, beta, and idiosyncratic volatility. Thus, the low-risk anomalies are much more pronounced during high-DOX periods. In addition, using analyst return forecasts, we can construct firm-level DOX. We then explore the effect of firm-level DOX on the low-risk anomalies. We find that low-risk anomalies in the cross section of stocks are significantly more pronounced among firms with high DOX than among firms with low DOX. More specifically, among firms with high DOX, the low-risk anomalies based on total volatility, beta, and idiosyncratic volatility earn a monthly Fama-French five-factor alpha of 0.82%, 0.62%, and 0.55%, respectively. On the other hand, among firms with low DOX, these corresponding values are only –0.14%, 0.25%, and –0.13%, respectively.

In terms of related studies, previous studies have found that empirical conclusions on the aggregate market's mean-variance relation tend to rely heavily on the conditional variance models selected, which leads to inconclusive overall evidence. For example, French, Schwert, and Stambaugh (1987), Baillie and Ramon (1990), Campbell and Hentschel (1992), Ghysels, Santa-Clara, and Valkanov (2005), Lundblad (2007), Guo and Whitelaw (2006), Brandt and Wang (2010), and Pástor, Sinha, and Swaminathan (2008) find a positive mean-variance relation. Campbell (1987), Nelson (1991), Whitelaw (1994), Lettau and Ludvigson (2010), and Brandt and Kang (2004) find a negative relation. Turner, Startz, and Nelson (1989), Glosten, Jagannathan, and Runkle (1993), Harvey (2001), and MacKinlay and Park (2004) find both a positive and a negative relation. Distinct from these studies, our empirical results are remarkably consistent across different conditional volatility models.

More recently, Yu and Yuan (2011) suggest that investor sentiment plays a significant role in weakening the aggregate market's mean-variance trade-off and the firm-level risk-return trade-off. However, sentiment is a broad concept, and the underlying mechanism remains unsettled. Motivated by evidence of return extrapolation prevalence, we aim to investigate whether this specific bias—return extrapolation—attenuates the mean-variance relation. More important, we show that the effect of DOX in the aggregate mean-variance relation remains robust after controlling for investor sentiment, and other economic forces. In another related study, Lochstoer and Muir (2020) suggest that investors have slow-moving beliefs about stock market volatility that lead to an initial underreaction to volatility shocks followed by a delayed overreaction, which helps explain a weak, or even negative, risk-return trade-off. Instead of relying on extrapolation on market volatility, we use return extrapolation, which has been widely confirmed in the data (see, e.g., Greenwood and Shleifer, 2014; Cassella and Gulen, 2018). In addition, we explicitly control for variance misperception, and the effect of return extrapolation on the mean-variance relation remains robust.

For the cross-sectional risk-return trade-off, prior studies tend to find a weak or even negative relation (i.e., the low-risk anomaly; see, e.g., Ang et al. (2006)). This paper adds to the literature that attempts to explain the cross-sectional low-risk anomalies. Previous studies have suggested that several forces are responsible for the low-risk anomalies, such as leverage aversion (Black, 1972; Asness, Frazzini, and Pedersen, 2012; Frazzini and Pedersen, 2014), benchmarked institutional investors (Brennan, 1993; Baker, Bradley, and Wurgler, 2011), money illusion (Cohen, Polk, and Vuolteenaho, 2005), disagreement (Hong and Sraer, 2016), investor sentiment (Shen, Yu, and Zhao, 2017), risk seeking in loss domain (Wang, Yan, and Yu, 2017), and arbitrage asymmetry (Stambaugh, Yu, and Yuan, 2015). This paper shows that the DOX effect on low-risk anomalies remains robust after controlling for these important economic forces such as inflation, aggregate analyst dispersion, TED spread, and sentiment. Moreover, our study contributes to the empirical literature on the weakened risk-return trade-off in the time series and the cross section within a unified framework.

Lastly, our study is also related to the huge emerging literature on the effect of extrapolation on asset prices. Early studies find that extrapolation plays a significant role in many asset pricing phenomena, including stock bubbles (Barberis et al., 2015, 2018), long-term return reversion (Barberis et al., 2015; Cassella and Gulen, 2018), short-term momentum (Pan, Su, and Yu, 2021), credit cycles (Bordalo et al., 2018), and the equity premium puzzle (Hirshleifer, Li, and Yu, 2015; Jin and Sui, 2022). We add to this literature by showing that extrapolation also plays a critical role in a fundamental principle in finance, the risk-return trade-off. Thus, this simple behavioral bias can account for a large set of stylized facts in asset markets.

The remainder of this paper is organized as follows. Section 2 develops the hypothesis with a two-period model. Section 3 presents the empirical findings. Section 4 provides extensions and robustness checks. Section 5 provides, cross sectional evidence. Section 6 concludes. The appendices contain detailed descriptions of the data and additional results.

2 Hypothesis Development

In this paper, we argue that investor extrapolation attenuates the mean-variance relation, especially during high-DOX periods. This argument is based on the following logic.

First, the classic fully rational ICAPM model (e.g., Merton, 1973) predicts that when investors expect higher risk, they require a higher risk premium. That is, there should be a positive mean-variance relation for the aggregate stock market. Thus, in response to an increase in risk (e.g., expected variance) and all else equal, the current stock price should be depressed to reflect a higher required rate of return in the future such that investors are compensated for bearing higher risk. That is, the contemporaneous relation between innovation in expected variance and stock returns should be negative.

Now consider models with both rational investors and extrapolative investors (e.g., Barberis et al., 2015, 2018), in which extrapolators' expected stock returns are typically modeled as an exponentially decaying average of past realized stock returns. Thus, for extrapolators, a higher current stock price implies higher perceived future stock returns. Hence, when facing enhanced risk (e.g., expected market variance), extrapolators demand a high expected return in their perception since they are risk-averse agents. This effect tends to push up current equilibrium prices relative to their fully rational benchmark since a higher current return implies higher perceived future returns for extrapolators. As a result, in a model featuring a significant fraction of extrapolative investors, after an increase in expected risk, the current equilibrium stock price is not depressed as much as it is in the fully rational benchmark.[3] Consequently, during periods with enhanced risk, the current equilibrium price is too high relative to its rational benchmark. When the price corrects

[3]In the extreme case in which the market is populated mostly by extrapolators, an increase in the expected variance could potentially lead to an increase in the current equilibrium price, since a higher current equilibrium price implies a higher perceived risk premium, as demanded by these extrapolators.

to the fundamental in the subsequent period, a smaller and potentially negative return is realized in the subsequent period, weakening the standard positive risk-return trade-off as obtained in rational models.[4] Obviously, when there is a large fraction of extrapolators in the market, or when DOX is high, the equilibrium overpricing after a variance shock should be stronger, further alleviating the mean-variance relation.

Thus, if the degree of extrapolation varies over time, we then reach our first hypothesis.

Hypothesis 1: The mean-variance relation should be stronger following low-DOX periods than following high-DOX periods.

In addition, when DOX is low, the market is closer to its rational benchmark, and thus the mean-variance relation should be positive. Thus, we arrive at our second hypothesis.

Hypothesis 2: The mean-variance relation should be significant and positive following low-DOX periods.

Moreover, our intermediate argument leading to the influence of DOX in the mean-variance relation also implies the following third hypothesis regarding the effect of DOX on the contemporaneous relation between shocks to the conditional variance and stock returns.

Hypothesis 3: The contemporaneous negative relation between innovations in the conditional variance and stock returns should be weakened during high-DOX periods.

As argued earlier, when there is a large fraction of extrapolators in the market, or when DOX is high, the equilibrium overpricing after a variance shock should be stronger, further alleviating the mean-variance relation. As a result, when this large amount of overpricing during high-DOX periods gets corrected, especially during subsequent earnings announcement periods when more information is released, we should expect that the interaction between volatility and DOX should predict subsequent earnings announcement returns. In addition, so far, for the ease of illustrative purposes, we have assumed a return extrapolation, as in Barberis et al. (2015, 2018) and Jin and Sui (2022). One can easily imagine that investors might also extrapolate on firm fundamentals, as in Hirshleifer, Li, and Yu (2015), Greenwood, Hanson, and Jin (2021), and Nagel and Xu (2021). As a result,

[4]By a similar argument, during periods with reduced risk, the current equilibrium price is too low relative to its rational benchmark. That is, there is overpricing in the stock market during enhanced risk periods. When the price corrects to the fundamental in the subsequent period, a larger return is realized in the subsequent period, again undermining the standard mean-variance relation.

it is also possible that the interaction between volatility and DOX can predict subsequent earnings forecast errors. Thus, we arrive at our fourth hypothesis.

Hypothesis 4: The interaction between volatility and DOX should negatively predict subsequent earnings announcement returns and subsequent forecast errors.

In Appendix A, we provide a simple two-period theoretical model that formalizes the intuition discussed in this section.

3 Main Empirical Results

In this section, we test our four hypotheses that were developed in the previous section. In particular, we investigate whether extrapolation bias indeed attenuates the positive relationship between aggregate expected returns and variance. We begin our empirical analysis by describing the construction of the main variables.

3.1 Construction of Key Variables

The key variable in our empirical analysis is DOX. We follow the literature (Greenwood and Shleifer, 2014; Cassella and Gulen, 2018; Cassella et al., 2020) in constructing DOX by estimating a nonlinear least squares regression with exponentially decayed weights:

$$X_t = a + b \cdot \sum_{s=0}^{59} w_s \cdot Ret_{t-3s-3:t-3s} + \varepsilon_t^X, \quad w_s = \frac{\lambda^s}{\sum_{k=0}^{59} \lambda^k}, \quad 0 < \lambda < 1, \tag{1}$$

where X_t is the expectation of future returns from surveys conducted in month-t, $Ret_{t-3s-3:t-3s} = \sum_{j=0}^{2} Ret_{t-3s-j}$ is the s-lagged quarterly market return, and w_s is the exponentially decayed weight. We then define the corresponding DOX as $1 - \lambda$, which captures the extent to which investors place higher weight on more recent returns. Specifically, we obtain the monthly average of consensus expectations (bullish minus bearish) from the Survey of Individual Investors from the American Association (AA) and the Investor Intelligence Survey (II) from Datastream database.[5] We follow French, Schwert, and Stambaugh (1987) and Lochstoer and Muir (2020) and use the value-weighted S&P 500 return from the Center for Research in Security Prices (CRSP) as the market return. To

[5]AA and II post updated weekly survey results on each Thursday, usually four times a month.

assign a DOX value to month s, we first obtain parameter estimates for months $s-12$ to $s-1$ based on the four window sizes: 24-month, 36-month, 48-month, and the expanding rolling window. We then obtain a set of 12 fitted errors for each moving window and calculate the mean squared forecast errors (MSFE). Finally, we aggregate the four separate estimates into a final DOX index using the inverse of the MSFE as weights (weights are normalized to sum to 1). Cassella and Gulen (2018) provide more details on the rolling methods. Our final sample starts in June 1974 to allow for a 12-month cross-validation period and ends in December 2019.

To calculate the conditional variance, we follow Yu and Yuan (2011) and employ two volatility models for our main analysis. The first method is the rolling window model (French, Schwert, and Stambaugh, 1987), which proxies the conditional variance for next month's return using the current month's realized variance:

$$\mathbb{E}_t\left(RV_{t+1}\right) = RV_t = \frac{22}{N_t}\sum_{d=1}^{N_t} r_{t-d}^2, \tag{2}$$

where r_{t-d} is the daily return in month t, day d, N_t is the number of trading days in month t, and 22 is the approximate number of trading days in one month.[6] Following French, Schwert, and Stambaugh (1987) and Yu and Yuan (2011), in this rolling window model, the innovation in expected variance $\Delta\mathbb{E}_t\left(RV_{t+1}\right)$ is simply the change in the realized variance:

$$\Delta\mathbb{E}_t\left(RV_{t+1}\right) = RV_{t+1} - \mathbb{E}_t\left(RV_{t+1}\right) = RV_{t+1} - RV_t. \tag{3}$$

The second approach uses GARCH models, which have been widely applied by the finance literature in volatility modeling (Bollerslev, 1986; Engle, 1990). Specifically, we impute the variance from the GARCH(1,1) model of the daily variance process h_t, that is,

$$r_{t+1} = \mu + \hat{\varepsilon}_{t+1} \text{ and } h_{t+1} = \omega + \alpha\hat{\varepsilon}_t^2 + \beta h_t, \tag{4}$$

where $\hat{\varepsilon}_t$ is the difference between the realized return and its conditional mean, r_{t+1} is the daily market return, and h_{t+1} is the conditional variance of the GARCH model. We then

[6] Our results remain robust if we demean daily returns by subtracting the monthly average return to calculate the realized variance.

calculate the monthly conditional variance and the innovations in conditional variance as

$$\mathbb{E}_t\left(RV_{t+1}\right) = \mathbb{E}_t\left(\sum_{d=1}^{22} h_{t+d}\right) \tag{5}$$

and

$$\begin{aligned} \Delta\mathbb{E}_t\left(RV_{t+1}\right) &= \mathbb{E}_{t+1}\left(RV_{t+2}\right) - \mathbb{E}_t\left(RV_{t+2}\right) \\ &= \mathbb{E}_{t+1}\left(\sum_{d=1}^{22} h_{t+1+d}\right) - \mathbb{E}_t\left(\sum_{d=23}^{44} h_{t+d}\right), \end{aligned} \tag{6}$$

where h_{t+d} is the conditional variance for date $t+d$.

Table 1 displays the summary statistics. Our sample spans from June 1974 to December 2019, a total of 547 months. Panel A of Table 1 shows that the average DOX value is 0.420 with a standard deviation of 0.228, which are close to the estimates by Cassella and Gulen (2018). Figure 1 also plots the estimated DOX over time. The graph indicates that DOX varies considerably throughout the sample period. This variation helps to uncover the time variation in risk-return trade-offs. Panel A of Table 1 also reports the summary statistics of several other control variables, which will be defined in more detail in Section 4 on robustness checks. In addition, Panel B shows that DOX is positively correlated with the Baker and Wurgler (2006) investor sentiment index. This is probably expected since it is likely that extrapolation is one of the specific behavioral biases that contribute to the movements in general investor sentiment.

3.2 DOX and Time-Series Mean-Variance Relation

We now investigate the key mean-variance relation, which is extensively studied by the literature, in the following equation:

$$R_{t+1} = a + b\mathbb{E}_t\left(RV_{t+1}\right) + \varepsilon_{t+1}, \tag{7}$$

where R_{t+1} is the excess market return in month $t+1$, and $\mathbb{E}_t\left(RV_{t+1}\right)$ is the conditional variance. To empirically assess our hypothesis, we follow Yu and Yuan (2011) and examine the following two-regime equation:

$$R_{t+1} = a_1 + b_1\mathbb{E}_t\left(RV_{t+1}\right) + a_2\text{High}_t + b_2\mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t + \varepsilon_{t+1}, \tag{8}$$

where High_t is the dummy variable that takes the value of one if the value of the DOX index in month t is greater than its median. The DOX dummy is defined in the same way as the sentiment dummy in Baker and Wurgler (2006) and Stambaugh, Yu, and Yuan (2012), a common practice in the literature.

Table 2 reports the average mean-variance relation and the relation conditional on the two DOX regimes. Columns (1) and (2) report the results for the rolling window conditional variance model, and columns (3) and (4) report the results for the conditional variance based on the GARCH(1,1) model. Column (1) indicates that the realized variance in the previous month (i.e., the expected future variance in the rolling window model) is on average negatively associated with the aggregate excess market return in the following month, a result contradicting leading rational asset pricing models, but consistent with the findings in earlier studies such as Glosten, Jagannathan, and Runkle (1993) and Lochstoer and Muir (2020). Using the GARCH(1,1) model to estimate the conditional variance yields a similar average negative mean-variance relation, as reported in column (3).

As argued earlier, our hypothesis implies that the mean-variance relationship should be different across different DOX regimes. We thus estimate the mean-variance relation separately for the two DOX regimes as in equation (8). As shown in columns (2) and (4) of Table 2, the coefficients on $\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$ are significantly negative with t-statistics of –3.893 and –3.513 for the rolling window model and GARCH(1,1) model, respectively. Thus, the mean-variance relation is significantly weaker during high-DOX periods than during low-DOX periods, consistent with hypothesis 1. In addition, although the mean-variance relation is on average negative, the mean-variance relation is significantly positive when the degree of extrapolation is low, with t statistics of 3.131 and 2.834 for the rolling window and GARCH(1,1) model, respectively. This finding lends support to our hypothesis 2. Moreover, these coefficients are not only statistically significant but also economically important. For example, a one-standard-deviation increase in the variance (i.e., in the rolling window model) is associated with a 2.17% increase in monthly expected excess returns during low-DOX periods, whereas a one-standard-deviation increase in the variance is associated with a 0.60% decrease in monthly expected excess returns during high-DOX periods. Lastly, the two-regime equation explains the expected return better than the one-regime equation, with the R^2 more than doubling from 1.2% to 3.1%. This could be useful for practitioners who use variance to time the market.

Figure 2 depicts the mean-variance relation with a binned scatter plot. The horizontal axis shows the realized variance. The values are grouped into 10 equal-sized bins, and for each bin, the vertical axis shows the mean values of excess market returns. The figure reveals a positive relationship between risk and return when DOX is low and a negative association between risk and return when DOX is high. In robustness analysis, we also verify that the two-regime pattern of the mean-variance relation remains unchanged after excluding the months with volatility spikes.

3.3 DOX and the Relation between Expected Variance Innovations and Returns

In this subsection, we examine how DOX affects the contemporaneous relation between realized returns and expected variance innovations; that is, we test our hypothesis 3. The test on this contemporaneous relation is essentially similar to the "indirect" test of the mean-variance trade-off by French, Schwert, and Stambaugh (1987), who find a contemporaneous negative relation between realized returns and expected variance innovations. It is also related to the well-known "volatility feedback effect," which emphasizes that investors require a lower price and therefore a high expected return in compensation for positive volatility innovations (French, Schwert, and Stambaugh, 1987; Campbell and Hentschel, 1992).[7] More important, we focus on how DOX affects this "volatility feedback effect," whereas earlier studies tend to focus on the unconditional effect.

In particular, the presence of extrapolators implies that prices do not drop enough in response to positive variance innovations, which therefore leads to overvaluation of stocks relative to their rational benchmark. Thus, the contemporaneous negative relation between realized returns and expected variance innovations should be weakened by DOX. The literature usually studies the following equation for the relation between contemporaneous returns and variance innovations:

$$R_{t+1} = c + d\mathbb{E}_t\left(RV_{t+1}\right) + e\Delta\mathbb{E}_t\left(RV_{t+1}\right) + \varepsilon_{t+1}, \tag{9}$$

where R_{t+1} is the aggregate excess market return in month $t+1$, and $\mathbb{E}_t\left(RV_{t+1}\right)$ is the conditional variance. To test the moderating effect of extrapolation bias, we examine the

[7] The contemporaneous negative relationship between volatility innovations and stock returns can also be explained by the "leverage effect." Black (1976) suggests that the "leverage effect" — falls in stock prices increase the firm's debt-to-equity ratio, making it more risky — explains the contemporaneous negative relation between risk and return. However, there is a technical difference between our test and the "volatility feedback effect" or the "leverage effect" when the rolling window model is not used. When other volatility models such as the GARCH model is used, the definition of volatility innovation and our expected variance innovation is different, and thus our tests are different from the standard "volatility feedback effect."

following two-regime equation:

$$
\begin{aligned}
R_{t+1} = c_1 + d_1 \mathbb{E}_t\left(RV_{t+1}\right) + e_1 \Delta \mathbb{E}_t\left(RV_{t+1}\right) \\
+ c_2 \mathrm{High}_t + d_2 \mathbb{E}_t\left(RV_{t+1}\right) \times \mathrm{High}_t + e_2 \Delta \mathbb{E}_t\left(RV_{t+1}\right) \times \mathrm{High}_t + \varepsilon_{t+1},
\end{aligned} \tag{10}
$$

where High_t is the dummy variable that takes the value of one if the value of the DOX index in month t is greater than its median.

Columns (1) and (3) of Table 3 indicate that the contemporaneous return-innovation relation is significantly negative in the low-DOX regime but turns rather flat in the high-DOX regime. Thus, the contemporaneous negative relation between innovations in the conditional variance and stock returns is significantly weakened during high-DOX periods, with t statistics of 4.665 and 2.879, respectively, for the rolling window model and the GARCH(1,1) model, lending support to hypothesis 3. In addition, during low-DOX periods, the negative return-innovation relation is statistically significantly negative, suggesting that the overall market is still quite averse to volatility in the low-DOX regime. Moreover, the economic magnitude of the DOX effect is also large. For example, a one-standard-deviation increase in the expected variance is associated with a 4.81% decrease in contemporaneous monthly excess returns during low-DOX periods, whereas a one-standard-deviation increase in the expected variance is associated with only a 1.50% decrease in contemporaneous monthly excess returns during high-DOX periods. Lastly, columns (2) and (4) show that adding the conditional variance does not change the effect of DOX on the contemporaneous return-innovation relation. Overall, such two-regime patterns in the return-innovation relation lend further support to the empirical conclusion in the previous section: extrapolation bias exerts significant influence in the aggregate risk-return trade-off.

3.4 DOX and Aggregate Earnings Announcement Returns/Forecast Errors

This subsection tests hypothesis 4 on the predictive power of the interaction between DOX and the variance on future earnings surprises by examining the relation between DOX, variances, earnings announcement returns, and analyst forecast errors.

We follow Arif and Lee (2014) and Li, Wang, and Yu (2021) in constructing the subsequent earning announcement return and subsequent analyst forecast errors, and then conduct further tests to better understand whether the effect of DOX originates from investor misperceptions. In particular, EAR_{q+1} is calculated as the value-weighted average firm-level earnings announcement return in quarter $q+1$. The firm-level earnings announcement return is the average cumulative stock return over trading window [-1, +1] around each of the firm's quarterly earnings announcement dates occurring in quarter $q+1$, and the weights are market capitalization by the end of quarter q. The value SUE_{q+1} is computed as the value-weighted average firm-level standardized unexpected earnings. Firm-level standardized unexpected earnings is the difference between the realized EPS and median EPS forecast (within 90 days) scaled by price at the end of the fiscal quarter and is winsorized at the 1% and 99% levels.

Table 4 reports the results of the quarterly predictive regressions of earnings announcement returns (EAR_{q+1}) and one-quarter-ahead analyst forecast errors (SUE_{q+1}) on the current value of the conditional variance, DOX dummy, and their interaction. If expectations were rational, at least SUE should be unpredictable. The results in Table 4 show that the conditional variance itself cannot predict either announcement returns or analyst forecast errors in both volatility models. In sharp contrast to this insignificance of unconditional predictive regressions, the results in columns (2) and (4) show that the conditional variance can predict both subsequent EAR and SUE during low-DOX periods, but this predictive power vanishes during high-DOX periods. In addition, the interaction effect between the conditional variance and DOX dummy is always significant, providing support to our hypothesis 4.

4 Robustness Checks

In this section, we consider additional robustness checks for our main result. First, we test the robustness by altering the regression specifications. Second, we investigate potential alternative behavioral explanations for the attenuated mean-variance relation. Finally, we test the mean-variance relation using survey data from other leading market indices.

4.1 Alternative Econometric Specifications

This section verifies that the effect of DOX is robust to alternative econometric model specifications. We proceed in the following six steps. First, we verify that the effect of DOX on the mean-variance relation is robust to alternative variance/volatility models. Previous studies suggest that positive and negative return shocks appear to result in volatility revisions in different directions (Engle, 1990; Glosten, Jagannathan, and Runkle, 1993). Following these lines, we reestimate the expected variance using the asymmetric GARCH model of Engle (1990) and the GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993). More specifically, the conditional variance from the asymmetric GARCH model is

$$h_{t+1} = \omega + \alpha\varepsilon_t^2 + \gamma\varepsilon_t + \beta h_t, \tag{11}$$

where negative values for γ indicate that negative shocks will increase volatility more than positive shocks will. The conditional variance from the GJR-GARCH model is as follows:

$$h_{t+1} = \omega + (\alpha + \gamma I_t)\,\varepsilon_t^2 + \beta h_t, \tag{12}$$

where I_t is the dummy variable that equals one for negative shocks, that is, when return innovation ε_t is negative.

In addition, a mixed data sampling approach (MIDAS) is also widely applied in volatility predictions in the literature. MIDAS can utilize the information at higher frequencies to forecast the low-frequency (monthly) variances. Thus, we calculate the conditional variance using MIDAS following Ghysels, Santa-Clara, and Valkanov (2005). The conditional variance from MIDAS with a normalized exponential Almon lag polynomial (Almon MIDAS) is as follows:

$$\mathbb{E}_t\left(RV_{t+1}\right) = 22\sum_{d=0}^{250} w_d \cdot r_{t-d}^2, \tag{13}$$

where weights

$$w_d = \frac{\exp(\phi_1 d + \phi_2 d^2)}{\sum_{i=0}^{250}\exp(\phi_1 i + \phi_2 i^2)}, \tag{14}$$

ϕ_1, and ϕ_2 are the parameters to be calibrated, and r_{t-d} is the daily return that is d-days ago. We use the daily variance in the previous 250 days to estimate the weight function and the conditional variance. In a similar vein, we also calculate MIDAS weights with a normalized beta probability density function specification (Beta MIDAS), that is,

$$w_d = \frac{x_d^{\theta_1-1}\left(1-x_d\right)^{\theta_2-1}}{\sum_{i=0}^{250} x_i^{\theta_1-1}\left(1-x_i\right)^{\theta_2-1}} + \theta_3, \tag{15}$$

where $x_d = 1/(d+1)$, and θ_1, θ_2, and θ_3 are the parameters to be calibrated.

Table 5 reports the mean-variance relations with these four alternative volatility models. Columns (1) and (2) report the results using the expected variance from the asymmetric

GARCH model of Engle (1990), and columns (3) and (4) report the results using expected variance from the GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993). Columns (5)-(6) of Table 5 report results using the expected variance from Almon MIDAS, and columns (7)-(8) report the results using the expected variance from Beta MIDAS. As we can see, the GARCH-based models tend to produce a negative mean-variance relation, whereas MIDAS-based approaches tend to produce a positive mean-variance relation, consistent with earlier findings on the unstable mean-variance relation using different volatility models. However, the coefficient estimates on $\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$ are always negatively significant and the coefficients on $\mathbb{E}_t(\text{RV}_{t+1})$ are always positively significant, leading consistent support to our hypotheses 1 and 2. In sum, although the average mean-variance relation is unstable across different volatility models, the effect of DOX on the mean-variance relation is remarkably consistent across different volatility models.

Moreover, in Table 6, we reexamine hypothesis 3 using these alternative models. The results indicate that the contemporaneous return-innovation relation is significantly negative in the low-DOX regime but turns rather flat in the high-DOX regime across all of these four alternative variance models. Thus, using these alternative models, we again obtain support for our hypothesis 3 that the contemporaneous negative relation between innovations in the conditional variance and stock returns is significantly weakened during high-DOX periods.

Second, it is possible that it takes more than one month for stock prices to revert to their fundamental value. We thus examine the effect of DOX using long-horizon predictive regressions. Table 7 reports the results using three-month and six-month excess market returns as the dependent variable. Again, the coefficient estimates on $\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$ are always negatively significant and the coefficients on $\mathbb{E}_t(\text{RV}_{t+1})$ are always positively significant across different specifications, indicating that the effect of DOX remains unaltered when we use future three-month and six-month excess returns.

Third, three months in our sample have an especially high realized variance exceeding 3% — October 1987, October 2008, and November 2008.[8] Thus, to make sure that our results are not being driven by these outliers, we exclude these three months from our sample and repeat the analysis in columns (1) and (2) of Table 8. As we can see, the results are quantitatively similar to those in Table 2. Since we cannot foresee this market turbulence in advance, we do not drop it in our main specification to avoid look-ahead bias. Nonetheless, we verify that the pattern observed in Figure 2 is not being completely driven by these few

[8]The first volatility spike corresponds to the "Black Monday" stock market crash in 1987, while the second and third ones occur during the Great Recession.

volatility spikes.

Fourth, although the standard procedure of calculating DOX does not require the estimated coefficient b of the nonlinear least squares regression in equation (1) to be positive (Cassella and Gulen, 2018), contrarian-return expectations would imply a negative b. Thus, to be more consistent with the model's interpretation of extrapolation, we restrict the coefficient b in equation (1) to be positive. The DOX index estimated from this method is highly correlated with the original one (corr = 0.94), suggesting that this positive constraint rarely binds. We then repeat the analysis using this DOX and display the results in columns (3) and (4). As can be seen, when using this alternative DOX, we obtain results qualitatively similar to those in Table 2.

Fifth, although using the DOX dummy variable to obtain two DOX regimes makes interpreting our results intuitive, we also use DOX as a continuous variable in our regression. The results are reported in columns (5) and (6). The coefficient on the interaction between the conditional variance and DOX is negative and marginally significant. This lower significance is partially expected given our model in Appendix A, which suggests that the mean-variance relation depends on DOX in a nonlinear way. In particular, our simple model in Appendix A shows that the mean-variance relation depends on 1/DOX (see, e.g., equation (A8)). Thus, a particular insightful robustness check is linking the mean-variance relation to the inverse of the DOX index ($1/\text{DOX}_t$). Columns (7) and (8) of Table 8 report the conditional mean-variance slope conditional on $1/\text{DOX}_t$. As shown, the coefficient estimates on $\mathbb{E}_t(\text{RV}_{t+1}) \times (1/\text{DOX}_t)$ are positively significant, more significant than the coefficients on DOX itself.

Taken together, the findings in the above analysis show that DOX plays an important role in determining the mean-variance relation, and this conclusion is robust across different econometric specifications.

4.2 Alternative Economic Explanations

This section explores whether other economic forces can explain the low relation between expected return and variance, and especially, whether our two-regime results are being driven by other well-known economic forces such as investor sentiment or misperception on volatility.

First, Yu and Yuan (2011) document the influence of investor sentiment on the market's mean-variance trade-off. In particular, they find that the stock market's expected excess return is positively related to the market's conditional variance in low-sentiment periods but unrelated to the variance in high-sentiment periods. Since DOX could be one of the underlying reasons leading to investor optimism or pessimism, it is important to rule out that the documented pattern in Table 2 is not being driven by the correlation between DOX and investor sentiment. Thus, in Table 9, we explicitly control for investor sentiment by including sentiment as one of our dependent variables. More specifically, we follow Yu and Yuan (2011) and measure investor sentiment for each month using the Baker and Wurgler (2006) sentiment index at the end of the previous year. We then classify a month as a high-sentiment month if the previous year's sentiment index is positive. Columns (1) and (3) of Table 9 show that the mean-variance relation is indeed weakened following the high-sentiment regime, consistent with the findings in Yu and Yuan (2011). However, Columns (2) and (4) show that the coefficient on the interaction between the conditional variance and the DOX dummy is always significant after controlling for the impact of sentiment. In fact, after controlling for the impact of DOX, the impact of sentiment on the mean-variance relation becomes insignificant under the rolling window conditional variance model.

Second, so far, we have used misperception in return to explain the weakened risk-return trade-off. It is also possible that misperception in risk (e.g., Lochstoer and Muir (2020)) can account for the weakened risk-return trade-off, and we want to make sure that our return-based misperception is not completely being driven by risk-based misperception. More specifically, Lochstoer and Muir (2020) suggest that investors have sticky expectations for volatility and perceive too high (low) risk after adverse (favorable) volatility shocks. Lochstoer and Muir (2020) show that sticky expectations for volatility can weaken the short-run risk-return trade-off.[9]

According to Lochstoer and Muir (2020), implied variance equals subjective expected variance up to a constant. Thus, we extend the method of Lochstoer and Muir (2020) and calculate the time-varying stickiness of variance expectations (SVE) as follows:

$$IV_t = a + b \cdot \sum_{s=0}^{11} \phi^s \cdot RV_{t-s} + \varepsilon_t, \quad b > 0, \quad 0 < \phi < 1, \tag{16}$$

[9] On the other hand, Pflueger, Siriwardane, and Sunderam (2020) and Atmaz (2020) suggest that investors expect future variance to be smaller (larger) following positive (negative) return shocks. According to this hypothesis, the investors perceive higher risk following negative return shocks, and require a higher expected return as compensation for bearing the risk, which reduces the current price more than the rational case and implies a stronger risk-return trade-off. Since this strong risk-return trade-off is inconsistent with the data, we do not consider this version of risk misperception.

where IV_t is the implied variance calculated using the CBOE S&P 100 Volatility Index (VXO) closing prices at month-t, and RV_{t-s} is the realized variance of the S&P 100 index in month $t-s$. We then define the corresponding SVE as the estimated value of ϕ, which captures the stickiness of variance expectations. Similar to the DOX estimation, we take the MSFE-weighted average of 24-, 36-, 48-, and expanding window estimates as the final SVE value. We use VXO instead of VIX since the series for VXO is longer available.

Panel A of Table 10 reports the results of controlling for states of SVE. Columns (1) and (2) use variance expectations from the rolling model, and columns (3) and (4) use those from the GARCH(1,1) model. As we can see, the SVE exerts no significant effect on the mean-variance relation. More important, we find that the effect of DOX on the mean-variance relation remains highly significant after controlling for the stickiness of variance expectations.

Second, as a further check on whether risk-based misperception drives our results, we use the price of volatile stocks (PVS) relative to price of the non-volatile stocks as a proxy for the time-varying underestimation of risk:

$$\text{PVS}_t = \overline{\left(\frac{B}{M}\right)}_{\text{low vol},t} - \overline{\left(\frac{B}{M}\right)}_{\text{high vol},t} \tag{17}$$

When market valuations are high, book-to-market ratios are low. Thus, PVS_t is high when the price of high-volatility stocks is high relative to low-volatility stocks. Pflueger, Siriwardane, and Sunderam (2020) suggest that PVS captures the subjective risk perception of economic agents. Importantly, PVS positively predicts revisions in risk expectations and therefore captures the degree to which investors underestimate volatility.

Column (1) in Panel B of Table 10 reports the mean-variance relation conditional on PVS regimes, and column (2) reports the results by further including DOX regimes as predictive variables. As can be seen in column (1), the coefficient estimate on $\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{PVS}}$ is insignificant, suggesting that risk underestimation fails to generate meaningful differences for the mean-variance relations in different PVS regimes. The result comes as no surprise as Merton (1980) suggests that estimating the variance of financial asset prices is statistically less challenging than forecasting the average returns. Thus, it is likely that investors are better at digesting volatility news and make fewer errors in volatility forecasting. Column (2) shows that the coefficient on $\mathbb{E}_t(\text{RV}_{t+1})$ changes sign after introducing DOX regimes as predictive variables, and the coefficient on the key interaction term $\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{PVS}}$

remains insignificant. More important, column (2) shows that there is still a large discrepancy in the mean-variance relation across different DOX regimes after controlling for PVS, consistent with the results in Table 2. Lastly, columns (3) and (4) show similar results when using the GARCH(1,1) volatility model.

Third, we now examine whether time variation in risk aversion can explain the average weak risk-return trade-off. According to Campbell and Cochrane (1999), the consumption surplus ratio captures the inverse of the effective risk aversion. Following Wachter (2006), we calculate the surplus ratio (SUR) as the smoothed average of real consumption growth over the past 40 quarters. Panel C of Table 10 repeats the exercise in earlier panels by conditioning on SUR regimes. We expect the mean-variance relation to be strong during low-SUR regimes since the effective risk aversion is higher. The results indicate that the effective risk aversion does not affect the mean-variance relation in a significant way. In addition, the sign on the interaction between the variance and SUR regimes is opposite to our prediction. Furthermore, controlling for the time-varying risk aversion does not affect the crucial role of DOX in the mean-variance relation.

Finally, Lustig and Adrien (2012) show that the risk-return trade-off is stronger during economic recessions than during economic expansions, probably because of different risk aversion coefficients across these two regimes. Thus, we control for a NBER recession dummy and report the results in Panel D of Table 10. We find that in our sample, there is no significant difference in the mean-variance relation across recessions and expansions. Moreover, controlling for the recession indicator does not alter any effect of DOX on the mean-variance relation. Overall, we find that investor sentiment, risk misperception, and time variation in risk aversion cannot explain the critical role of DOX in the mean-variance relation.

4.3 Evidence from International Markets

In this subsection, we attempt to test the effect of DOX in modifying risk-return trade-offs by employing data outside the US market. Using consensus forecast data from the ZEW Financial Markets Survey, we replicate the results for Germany, France, the United Kingdom (UK), Italy, and Japan. The ZEW Financial Markets Survey asks investors to forecast the future six-month direction of stock market index movements in six major developed markets (the US, Germany, France, the UK, Italy, and Japan). The selection of these markets is dictated by data availability and complete details for the construction of these DOX series are in Appendix B. In particular, the consensus forecast data are unavailable for Canada, another major developed market.

In Table 11, we repeat the mean-variance relation conditional on DOX for five major markets outside the US. For instance, Panel A reports the results for the German stock index. As shown in column (1), there is no significant relation between expected returns and variance. However, columns (1) to (3) indicate that the future realized return is positively related to the expected variance following low-DOX periods, but the relation turns negative following high-DOX periods, consistent with hypotheses 1 and 2. In addition, column (5) indicates that DOX also exerts a significant effect on the contemporaneous return-innovation relation in a way consistent with hypothesis 3. A similarly strong pattern obtains for France. The results for the UK and Italy are also consistent with our hypotheses, especially over longer horizons, albeit with weaker statistical significance.

However, DOX seems to have little statistical power in explaining the mean-variance relation in the Japan stock market, and with an opposite sign. This failure could potentially be a result of a shorter sample than the US counterpart or more noisy survey data in non-US data. Overall, using samples outside the US, we find reasonable support for our hypothesis that extrapolation bias attenuates the mean-variance relation during high-DOX periods.

In sum, the robustness tests in this section suggest that the DOX effect on the mean-variance relation is robust to alternative econometric models and to controlling for other economic forces that may attenuate the mean-variance relation. Nonetheless, we acknowledge that there may still exist other unknown forces that are not necessarily mutually exclusive and, if anything, may jointly contribute to the attenuated mean-variance relation together with our extrapolation channel.

5 Evidence from the Cross Section

Given the extensive literature on the mean-variance relation, we have so far only focused on the effect of DOX on this aggregate risk-return trade-off. In this section, we provide further corroborating evidence based on the cross-sectional risk-return tradeoff, another large and relatively more recent literature. Standard rational theory implies a positive risk-return trade-off in the cross section. In particular, the CAPM implies that the firm-level beta and expected return should be positively correlated. In addition, since investors cannot fully diversify away all the idiosyncratic volatility, typical models would also imply a positive relation between the firm-level expected return and firm-level volatility and idiosyncratic volatility. It is now well-known that the relation between various risk measures and expected returns are weak or even negative, especially after controlling for CAPM. This phenomenon is the called low-risk anomaly (see, e.g., Baker, Bradley, and Wurgler (2011), Ang et al. (2006), and Frazzini and Pedersen (2014)). Since extrapolators can weaken the volatility-feedback effect at both the aggregate and firm level, thus attenuating the mean-variance relation, the intuition on the influence of DOX on the time-series mean-variance relation can be extended to the cross-sectional risk-return trade-off.

In addition, Choi and Robertson (2020) find in surveys that retail investors view value stocks (which have lower valuations) as safer and expect lower returns, and high-momentum stocks (which have higher valuations) as riskier and expect higher returns. That is, extrapolators as proxied by individuals tend to positively associate current valuations with risks, although rational risk-return trade-offs suggest the opposite: high risks indicate high discount rates and low current valuations. We thus empirically investigate the DOX effect on the cross-sectional risk-return trade-off (i.e., the low-risk anomalies) in this section.

5.1 Time-Series Variation of Low-Risk Anomalies

First, we examine the time-series variation of the cross-sectional risk-return trade-off. To construct low-risk anomalies, we include common stocks (share codes 10 and 11) traded on NYSE, AMEX, and NASDAQ exchanges from June 1974 to December 2019, a horizon identical to our time-series analysis. Stock returns are adjusted for delistings, as in Shumway (1997). We calculate three low-risk anomalies. The first is based on total volatility (tv), calculated as the variance of daily raw stock returns in the previous month. The second is based on betting against beta ($beta$), calculated using a one-year rolling standard deviation of one-day log returns for volatilities and a five-year horizon for the correlation in overlapping three-day log returns following Frazzini and Pedersen (2014). The last is based on idiosyncratic volatility ($ivol$), calculated as the variance of the residuals from the Fama and French (1993) three-factor model using daily excess returns in the previous month (Ang et al., 2009). We also take the return average of these three anomalies as our average low-risk anomaly.

Table 12 reports the regression of low-risk anomaly returns on lagged DOX. As we can see, the coefficient b on lagged DOX is always positive for the long-short portfolio return spreads. For the average low-risk anomaly, this coefficient is marginally significant for the raw return spread, and significant for Carhart-four-factor and Fama-French five-factor-adjusted returns. The economic magnitude is also large. A one-standard deviation increase in DOX leads to about a 0.41% per month increase in the long-short return spreads of the average low-risk anomaly.

In addition, many studies have suggested possible forces responsible for the low-risk anomalies, such as leverage aversion, money illusion, disagreement, and investor sentiment. Thus, it is important to rule out the possibility that the effect of DOX on the low-risk anomalies is being driven by the forces proposed by these studies resulting from the correlation of DOX with these forces. For example, when investor sentiment or aggregate disagreement is high, there might be more extrapolators in the stock market. Indeed, the correlation between DOX and investor sentiment is about 26%. Thus, it is interesting to investigate whether DOX still has predictive power after controlling for these mechanisms.

To investigate this possibility, we examine the relation between low-risk anomalies and DOX by controlling for the Baker and Wurgler (2006) investor sentiment index (Shen, Yu, and Zhao, 2017), funding constraints (TED spread) of Frazzini and Pedersen (2014), the money illusion effect (inflation) of Cohen, Polk, and Vuolteenaho (2005), and the earnings forecast dispersion of Hong and Sraer (2016) and Yu (2011). The results are reported in Table 13, which shows that the predictive power of DOX for low-risk anomaly return spreads is only slightly weaker compared to Table 12. Moreover, for the average low-risk anomaly, the coefficient on DOX is more significant than that on inflation, TED, dispersion, and sentiment. Thus, our DOX channel at least provides incremental predictive power for low-risk anomalies.

5.2 Cross-sectional Heterogeneity of Low-Risk Anomalies

In this subsection, we investigate the cross-sectional heterogeneity of the cross-sectional risk-return trade-off. In particular, we first sort stocks into three groups based on firm-level DOX and then examine how the extent of low-risk anomalies varies with firm-level DOX across these three groups of firms.

To our knowledge, the firm-level DOX measurement remains uncommon in the literature because of data limitations. One exception is Da, Huang, and Jin (2021), who analyze firm-level extrapolative behavior using private ranking data from a crowdsourcing platform. Da, Huang, and Jin (2021) suggest that there is cross-sectional heterogeneity in firm-level DOX.

Inspired by Bouchaud et al. (2019), we use analyst return expectations to estimate firm-level DOX. We need the entire series to estimate the firm-level DOX as analysts infrequently issue target prices, and we acknowledge that the look-ahead bias of this empirical design is hard to avoid.[10] This might be less of a concern here since our purpose is to test our economic intuition on how DOX affects the risk-return trade-off rather than to improve the trading strategy related to low-risk anomalies.

We extract analyst target prices from Thompson Reuters IBES from 1999, when they becomes available, to 2019. Following prior literature (Brav and Lehavy, 2003; Da and Schaumburg, 2011; Cannon, 2021; Wang, 2021), we calculate the expected return over the next 12-month period as

$$Ret_{i,j,d}^{A,12m} = \frac{P_{i,j,d}^{A,12m}}{P_{i,d}} - 1, \tag{18}$$

where $P_{i,j,d}^{A,12m}$ is the unadjusted target price for firm i issued by analyst j on date d for the 12-month-ahead price, and $P_{i,d}$ is the closing price of the firm at date d (or the most recent closing price if the stock does not trade). We then take the arithmetic average of the analyst forecasts as the consensus forecast for each month.

Following the standard procedure for DOX calculation (Greenwood and Shleifer, 2014; Cassella and Gulen, 2018), we estimate the firm-level DOX as follows:

$$Ret_{i,t}^{A,12m} = a_i + b_i \sum_{s=0}^{7} w_{i,s} \cdot Ret_{i,t-3s-3:t-3s} + \epsilon_{i,t}, \quad w_{i,s} = \frac{\lambda_i^s}{\sum_{k=0}^{7} \lambda_i^k}, \quad 0 < \lambda_i < 1,$$

where $Ret_{i,t}^{A,12m}$ is the average expected raw return over the next 12-month period issued during month t by analysts, and $w_{i,s}$ is the exponential decay weight. We include lagged previous returns up to eight quarters following Da, Huang, and Jin (2021). Here, we implicitly assume that retail investors share common behavioral biases as analysts or follow analysts' forecasts. However, we should note that estimates of b_i can be negative as analysts are on average more contrarian (Wang, 2021; Bouchaud et al., 2019). Thus, to be consistent with the literature and our simple model's intuition in Appendix A, we define the firm-level DOX measure as $DOX_i = \text{sign}(b_i)(1 - \lambda_i)$, where negative values of b_i imply that stock i's investors are more contrarian, or equivalently, less extrapolative.

With our firm-level DOX measure, we first sort stocks into terciles based on the firm-level

[10] As analysts infrequently update target prices, the sample has insufficient observations if we estimate firm-level DOX with rolling window regressions.

DOX. Then, at the end of each month and within each DOX tercile, we further sort stocks into quintiles based on risk measures. The portfolios are rebalanced each month, and we calculate the value-weighted returns for each portfolio. Table 14 presents the average excess returns for portfolios that are double-sorted on firm-level DOX and total volatility (Panel A), beta (Panel B), and idiosyncratic volatility (Panel C), respectively. Panel A shows that the excess return spread for Q1 − Q5 portfolios based on total volatility is –0.03% per month and insignificant (t-stat = −0.06) among firms with the lowest firm-level DOX (T1). In contrast, the excess return spread is 0.78% per month (t-stat = 1.41) among the firms with the highest firm-level DOX (T3). The spread difference between the highest and lowest DOX states is 0.81% per month with a t-statistic of 2.41. This is consistent with our intuition that extrapolators weaken risk-return trade-offs. We observe similar DOX effects for the beta (Panel B) and idiosyncratic volatility (Panel C) strategies. Overall, we find that the low-risk anomalies are more pronounced among the firms with higher DOX values.

Table 15 displays CAPM, Carhart (1997) four-factor, and Fama and French (2015) five-factor alphas for the double-sorted portfolios. For example, Panel A shows that the monthly CAPM-alpha is 0.60% per month (t-stat = 1.36) for the low-DOX tercile (T1) and 1.39% per month (t-stat = 3.20) for the high-DOX tercile (T3). The difference in the CAPM-alphas for tercile 3 and tercile 1 is 0.79% per month (t-stat = 2.26). Similar results are obtained when sorting on beta and idiosyncratic volatility.

Although the above double-sorting approach is simple and intuitive, it cannot explicitly control for other variables that could influence returns. Since firm-level DOX is correlated with other stock characteristics—in particular, past stock returns—one might be concerned that the results in Tables 14 and 15 are driven by effects other than DOX. More important, Wang, Yan, and Yu (2017) find that capital gain overhang (CGO) can also affect the low-risk anomalies. In particular, they find that the low-risk anomalies are much more pronounced among firms with low CGO. The attribute this pattern to investors' risk-seeking behavior and mental accounting. It is possible that DOX and CGO are correlated. To address these concerns, we perform a series of Fama and MacBeth (1973) cross-sectional regressions, which allow us to conveniently control for additional variables. In particular, we follow Wang, Yan, and Yu (2017) and estimate monthly Fama-MacBeth cross-sectional regressions of stock returns on lagged variables in the following form (subscripts omitted for brevity):

$$\begin{aligned} R = \alpha &+ \beta_1 \text{DOX} + \beta_2 \text{PROXY} + \beta_3 \text{PROXY} \times \text{DOX} + \beta_4 \text{CGO} \\ &+ \beta_5 \text{PROXY} \times \text{CGO} + \beta_6 \text{LOGBM} + \beta_7 \text{LOGME} \\ &+ \beta_8 \text{MOM}(-1, 0) + \beta_9 \text{MOM}(-12, -1) + \beta_{10} \text{MOM}(-36, -12) \\ &+ \beta_{11} \text{TURNOVER} + \varepsilon, \end{aligned} \tag{19}$$

where R is the monthly stock return in month $t + 1$, DOX is the firm-level degree of extrapolation estimated from IBES data, PROXY is one of our three risk proxies (total volatility, beta, and idiosyncratic volatility) at the end of month t, CGO is the Grinblatt and Han (2005) capital gain overhang at the end of month t, LOGBM is the natural log of the book-to-market ratio at the end of month t, LOGME is the natural log of market equity at the end of month t, MOM$(-1, 0)$ is the stock return in month t, MOM$(-12, -1)$ is the stock return from the end of month $t - 12$ to the end of month $t - 1$, MOM$(-36, -12)$ is the stock return from the end of month $t - 36$ to the end of month $t - 12$, and TURNOVER is the share turnover rate in month t.

Table 16 presents the results of the Fama-MacBeth regression in equation (19). The benchmark regressions in columns (1), (3), and (5) show that the coefficients on the interaction between DOX and risk proxies are always significant and negative, confirming the above double-sorting results. In columns (2), (4), and (6), we include the list of traditional return predictors, such as firm size, book-to-market ratio, past returns, and share turnover, as well as the interaction term between CGO and risk proxies. The results again confirm the previous double-sorting analysis that the interaction term is always significant and negative for all risk measures, even after controlling for firm size, book-to-market ratio, past returns, share turnover, and, in particular, the interaction between CGO and risk proxies.

Figure 3 plots the the cumulative excess returns of the long-short total volatility strategy by terciles of the firm DOX value. In this figure, we observe that the long-short total volatility strategy among the highest DOX tercile (T3) significantly outperforms the one among the lowest DOX tercile (T1) over the sample period.

Overall, the results from both portfolio sorts and Fama-MacBeth regressions provide consistent evidence on the significant role of extrapolation in cross-sectional risk-return trade-offs. Thus, our intuition on how DOX affects the time-series mean-variance relation can be extended to the cross-sectional risk-return trade-offs. Nonetheless, data limitations mean that there must be a huge amount of estimation noise in firm-level DOX. Thus, we only view the above results as corroborating evidence, rather than main evidence in support of the role of DOX in risk-return trade-offs.

6 Conclusions

This study investigates the impact of time-varying return extrapolation on risk-return trade-offs, in both the aggregate time series and the firm-level cross section. We find that the relation between the market's expected return and the variance is positive following periods with low levels of degree of extrapolation, as measured by the DOX of Cassella and Gulen (2018), but turns negative following high levels of DOX. Further, we find that when sorting on firm DOX values, low-risk anomalies are more pronounced among firms with higher DOX. These findings are consistent with the interpretation that, relative to a fully rational framework, the existence of extrapolators tends to raise equilibrium stock prices in response to a positive expected risk/volatility shock. The increase in stock prices is meant to sustain high perceived future returns to compensate for this enhanced risk and thus leads to overpricing and a weakening of the traditional risk-return trade-off.

Earlier studies find that extrapolation plays a critical role in anomalies related to the value premium, long-term reversals, stock bubbles, and so on. Our results contribute to the literature on extrapolation by showing that it also plays a significant role in the fundamental risk-return trade-offs in finance. Several related issues are likely to be fruitful avenues for future research. First, one direction that might be worth a careful investigation is the subjective risk-return trade-off, given the findings in Choi and Robertson (2020), and an exploration of the link between the subjective and objective risk-return trade-off in a unified framework. Second, the impact of extrapolation bias on the household investing decision and social welfare resulting from the weakened risk-return trade-off induced by extrapolation might also be worthy of further exploration.

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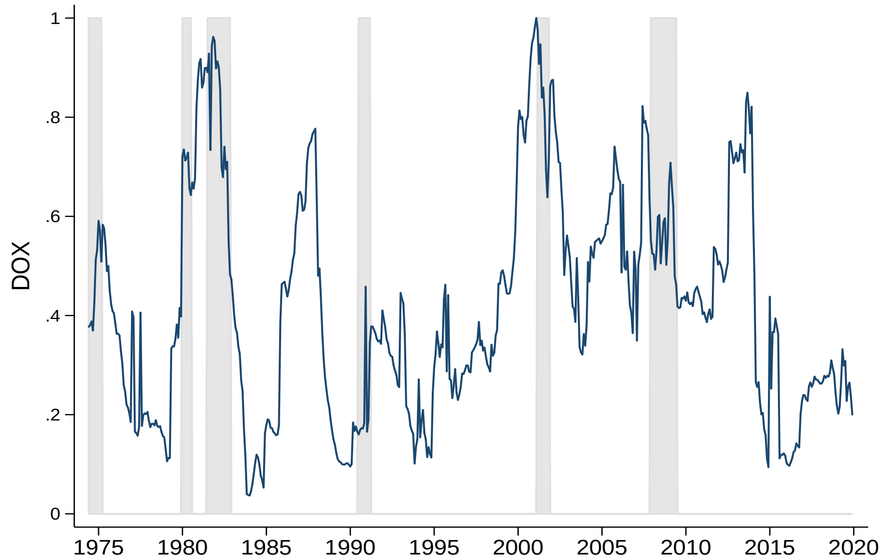
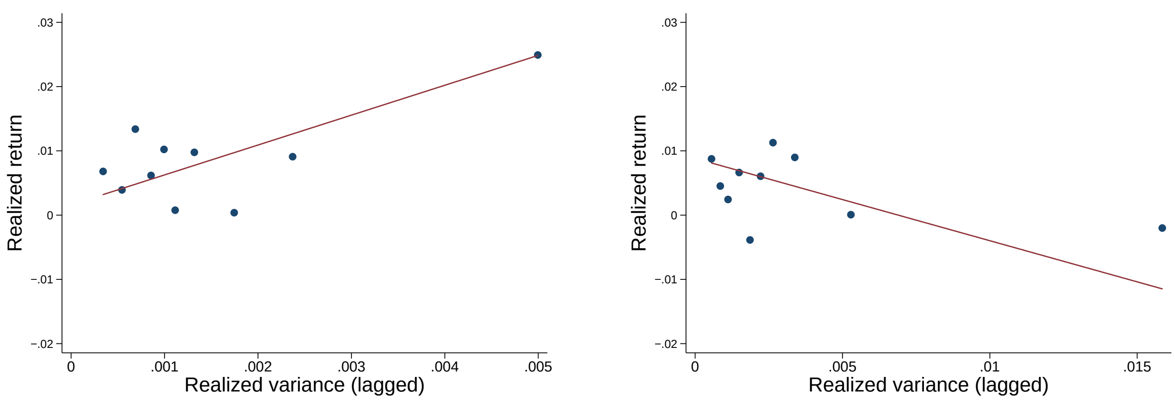


Figure 1: Time-series variation of DOX. The figure shows the time-series variation of degree of extrapolation (DOX) for the period June 1974 to December 2019. DOX is recursively estimated from investor survey data (II and AA). The shaded bars indicate NBER recessions.



Panel A: Mean-variance relation, Low DOX period

Panel B: Mean-variance relation, High DOX period

Figure 2: Conditional objective risk-return trade-offs. This figure shows the mean-variance relation conditional on DOX. Panel A plots realized excess market return against realized variance in the previous month following low-DOX periods. Panel B plots realized excess market return against realized variance in the previous month following high-DOX periods. Observations are split into 10 bins, based on percentiles of variance distribution.

Figure 3: Low volatility anomaly by terciles of firm DOX. The figure plots the cumulative excess returns of the long-short total volatility strategy by terciles of the firm-level degree of extrapolation (DOX). The firm-level DOX is estimated form IBES data. Total volatility is the variance of daily raw stock returns in the previous month. Portfolio returns are value weighted and span the period January 1999 to December 2019.

Table 1: Summary statistics

The table displays the summary statistics. Panel A reports the summary statistics of the key variables. DOX is degree of return extrapolation estimated from investor survey data following Cassella and Gulen (2018). Sentiment is the Baker and Wurgler (2006) investor sentiment index. PVS is the monthly Pflueger, Siriwardane, and Sunderam (2020) price of volatile stocks. R is the monthly value-weighted S&P 500 return in excess of one-month Treasury-bill rate (R^f). RV is the monthly realized variance. Monthly returns and variance are expressed in percentages. Panel B reports the pairwise correlations between the time series. The sample period of DOX is from June 1974 to December 2019. Sentiment is from Jeffrey Wurgler's website and ends in December 2018. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Summary Statistics						
	Obs	Mean	Std. Dev.	Median	Min	Max
DOX	547	0.420	0.228	0.387	0.037	1.000
Sentiment	535	0.045	0.875	0.071	-2.422	3.197
PVS	547	-0.326	0.443	-0.226	-3.029	0.489
R (%)	547	0.639	4.376	0.972	-22.180	16.301
R^f (%)	547	0.371	0.288	0.390	0.000	1.350
RV (%)	547	0.249	0.467	0.138	0.018	6.542
Panel B: Correlation						
	DOX	Sentiment	PVS	R	R^f	RV
DOX	1.000					
Sentiment	0.257***	1.000				
PVS	-0.037	0.413***	1.000			
R	-0.077	-0.045	-0.016	1.000		
R^f	0.061	0.110*	0.418***	-0.064	1.000	
RV	0.204***	-0.018	-0.219***	-0.333***	-0.074	1.000

Table 2: Excess returns and conditional variance: Effect of DOX

The table reports estimates of the regression

$$R_{t+1} = a_1 + b_1\mathbb{E}_t\left(RV_{t+1}\right) + a_2\text{High}_t + b_2\mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is excess market return (R_{t+1}), $\mathbb{E}_t(\text{RV}_{t+1})$ is the expected variance under rational expectation, and High_t is the dummy variable for the months with above-median DOX. We measure the expected variance under rational expectation as the realized variance in the previous month (RV_t) in columns (1)-(2) and as the conditional variance from the GARCH(1,1) model in columns (3)-(4). The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Dependent Variable: Excess Market Return (R_{t+1})			
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	–1.024***	4.650***	–1.060***	4.730***
	(–2.859)	(3.131)	(–2.705)	(2.834)
High_t		0.007*		0.008*
		(1.771)		(1.918)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		–5.930***		–6.022***
		(–3.893)		(–3.513)
Constant	0.009***	0.002	0.009***	0.000
	(5.374)	(0.553)	(5.346)	(0.117)
Obs	546	546	546	546
R^2	0.012	0.031	0.011	0.027

Table 3: Return-innovation relation: Effect of DOX

The table reports estimates of the regression

$$R_{t+1} = c_1 + d_1\mathbb{E}_t\left(RV_{t+1}\right) + e_1\Delta\mathbb{E}_t\left(RV_{t+1}\right) + c_2\text{High}_t + d_2\mathbb{E}_t\left(RV_{t+1}\right)\times\text{High}_t + e_2\Delta\mathbb{E}_t\left(RV_{t+1}\right)\times\text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is excess market return (R_{t+1}), $\mathbb{E}_t(\text{RV}_{t+1})$ is the expected variance under rational expectation, $\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$ is the monthly variance innovation, and High_t is the dummy variable for the months with above-median DOX. We measure the expected variance under rational expectation as the realized variance in the previous month (RV_t) in columns (1)-(2) and as the conditional variance from the GARCH(1,1) model in columns (3)-(4). Variance innovations are calculated from the simple rolling model for columns (1)-(2) and from the GARCH(1,1) model for columns (3)-(4). The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Dependent Variable: Excess Market Return (R_{t+1})			
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$	–10.337***	–10.294***	–10.496***	–10.104***
	(–5.854)	(–4.756)	(–4.092)	(–3.270)
High_t	–0.004	0.006	–0.002	0.009
	(–1.239)	(1.193)	(–0.519)	(1.633)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2}) \times \text{High}_t$	8.601***	7.090***	7.671***	6.166**
	(4.665)	(3.226)	(2.879)	(1.976)
$\mathbb{E}_t(\text{RV}_{t+1})$		0.090		1.007
		(0.035)		(0.359)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		–3.022		–3.695
		(–1.141)		(–1.273)
Constant	0.009***	0.008**	0.006***	0.004
	(5.058)	(2.173)	(3.052)	(0.998)
Obs	546	546	546	546
R^2	0.091	0.156	0.107	0.161

Table 4: Earnings surprises and conditional variance: Effect of DOX

The table reports estimates of the quarterly regression

$$Y_{q+1} = a_1 + b_1 \mathbb{E}_q\left(RV_{q+1}\right) + a_2 \text{High}_q + b_2 \mathbb{E}_q\left(RV_{q+1}\right) \times \text{High}_q + \varepsilon_{q+1},$$

where the dependent variable Y_{q+1} is the aggregate earning announcement returns (EAR_{q+1}) in Panel A or the standardized unexpected earnings (SUE_{q+1}) in Panel B, $\mathbb{E}_q(\text{RV}_{q+1})$ is the rationally expected variance measured by the realized variance in the last month of quarter q, and High_q is a dummy variable that takes the value of one if the last month of quarter q has a DOX value greater than the median. EAR_{q+1} is the value-weighted average firm-level earnings announcement return. The firm-level earnings announcement return is the average cumulative stock return over trading window [-1, +1] around each of the firm's quarterly earnings announcement dates occurring in quarter $q+1$ and the weights are market capitalization by the end of quarter q. SUE_{q+1} is the value-weighted average firm-level standard earnings surprise. The firm-level standard earnings surprise is the difference between realized EPS and median EPS forecast (within 90 days) scaled by price at the end of the fiscal quarter and is winsorized at the 1% and 99% levels. EAR is from 1974Q2 to 2019Q4, and SUE is from 1983Q4 to 2019Q4. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Panel A: Earning Announcement Returns (EAR_{q+1})			
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_q(\text{RV}_{q+1})$	0.210	1.264***	0.231	1.553***
	(0.663)	(6.485)	(0.601)	(5.411)
High_q		0.001		0.002
		(1.140)		(1.581)
$\mathbb{E}_q(\text{RV}_{q+1}) \times \text{High}_q$		–1.197***		–1.485***
		(–3.455)		(–3.346)
Constant	0.002**	0.001	0.002**	–0.000
	(2.561)	(0.805)	(1.992)	(–0.153)
Obs	182	182	182	182
R^2	0.008	0.040	0.008	0.041

	Panel B: Standardized Unexpected Earnings (SUE_{q+1})			
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_q(\text{RV}_{q+1})$	–0.035	0.250**	–0.027	0.308***
	(–0.587)	(2.050)	(–0.436)	(2.754)
High_q		0.001***		0.001***
		(2.647)		(2.906)
$\mathbb{E}_q(\text{RV}_{q+1}) \times \text{High}_q$		–0.343***		–0.397***
		(–2.740)		(–3.527)
Constant	–0.000	–0.001	–0.000	–0.001*
	(–0.176)	(–1.584)	(–0.214)	(–1.848)
Obs	145	145	145	145
R^2	0.006	0.091	0.003	0.084

Table 5: Robustness: Mean-variance relation (alternative volatility models)

The table reports estimates of the regression

$$R_{t+1} = a_1 + b_1\mathbb{E}_t\left(RV_{t+1}\right) + a_2\text{High}_t + b_2\mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is excess market return (R_{t+1}), $\mathbb{E}_t(RV_{t+1})$ is the expected variance under rational expectation, and High_t is the dummy variable for the months with above-median DOX. We measure the expected variance under rational expectation as the conditional variance from the asymmetric GARCH(1,1) model in columns (1)-(2), the conditional variance from the GJR-GARCH(1,1) model in columns (3)-(4), the conditional variance from MIDAS with normalized exponential Almon lag polynomial in columns (5)-(6), and the conditional variance from MIDAS with normalized beta density with a zero last lag in columns (7)-(8). The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Dependent Variable: Excess Market Return (R_{t+1})			
	Asymmetric GARCH		GJR-GARCH	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(RV_{t+1})$	–1.209**	5.435***	–1.098**	4.897***
	(–2.498)	(3.015)	(–2.342)	(2.960)
High_t		0.010**		0.009**
		(2.212)		(1.981)
$\mathbb{E}_t(RV_{t+1}) \times \text{High}_t$		–6.955***		–6.247***
		(–3.733)		(–3.623)
Constant	0.009***	–0.001	0.009***	0.000
	(5.366)	(–0.231)	(5.186)	(0.078)
Obs	546	546	546	546
R^2	0.010	0.028	0.012	0.029
	Almon MIDAS		Beta MIDAS	
	(5)	(6)	(7)	(8)
$\mathbb{E}_t(RV_{t+1})$	–0.930	6.629***	–0.972	6.540***
	(–1.293)	(4.274)	(–1.556)	(4.270)
High_t		0.013***		0.013***
		(2.762)		(2.882)
$\mathbb{E}_t(RV_{t+1}) \times \text{High}_t$		–8.031***		–7.943***
		(–4.816)		(–4.912)
Constant	0.009***	–0.004	0.009***	–0.004
	(4.371)	(–1.033)	(4.659)	(–1.171)
Obs	546	546	546	546
R^2	0.004	0.025	0.005	0.025

Table 6: Robustness: Return-innovation relation (alternative volatility models)

The table reports estimates of the regression

$$R_{t+1} = c_1 + d_1\mathbb{E}_t\left(RV_{t+1}\right) + e_1\Delta\mathbb{E}_t\left(RV_{t+1}\right) + c_2\text{High}_t + d_2\mathbb{E}_t\left(RV_{t+1}\right)\times\text{High}_t + e_2\Delta\mathbb{E}_t\left(RV_{t+1}\right)\times\text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is excess market return (R_{t+1}), $\mathbb{E}_t(RV_{t+1})$ is the expected variance under rational expectation, $\Delta\mathbb{E}_{t+1}(RV_{t+2})$ is the monthly variance innovation, and High_t is the dummy variable for the months with above-median DOX. We measure the expected variance under rational expectation as the conditional variance from the asymmetric GARCH(1,1) model in columns (1)-(2), the conditional variance from the GJR-GARCH(1,1) model in columns (3)-(4), the conditional variance from MIDAS with normalized exponential Almon lag polynomial in columns (5)-(6), and the conditional variance from MIDAS with normalized beta density with a zero last lag in columns (7)-(8). Variance innovations are calculated from the corresponding models accordingly. The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Dependent Variable: Excess Market Return (R_{t+1})			
	Asymmetric GARCH	GJR-GARCH	Almon MIDAS	Beta MIDAS
	(1)	(2)	(3)	(4)
$\Delta\mathbb{E}_{t+1}(RV_{t+2})$	–15.875***	–15.647***	–10.286***	–11.064***
	(–4.111)	(–3.972)	(–3.772)	(–4.025)
High_t	0.010*	0.007	0.013**	0.016***
	(1.717)	(1.500)	(2.222)	(2.643)
$\Delta\mathbb{E}_{t+1}(RV_{t+2}) \times \text{High}_t$	10.890***	11.472***	6.781**	7.589***
	(2.773)	(2.864)	(2.458)	(2.717)
$\mathbb{E}_t(RV_{t+1})$	0.946	–0.799	3.629	3.819
	(0.342)	(–0.359)	(1.300)	(1.420)
$\mathbb{E}_t(RV_{t+1}) \times \text{High}_t$	–3.589	–2.001	–5.767**	–6.086**
	(–1.233)	(–0.873)	(–1.966)	(–2.129)
Constant	0.004	0.007*	–0.001	–0.003
	(0.788)	(1.937)	(–0.315)	(–0.744)
Obs	546	546	546	546
R^2	0.222	0.239	0.165	0.168

Table 7: Robustness: Longer horizon predictability

The table reports estimates of the regression

$$R_{t+1:t+j} = a_1 + b_1 \mathbb{E}_t\left(RV_{t+1}\right) + a_2 \text{High}_t + b_2 \mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is j-month excess market return ($R_{t+1:t+j}$), $\mathbb{E}_t(\text{RV}_{t+1})$ is the expected variance under rational expectation, and High_t is the dummy variable for the months with above-median DOX. We measure the expected variance under rational expectation as the realized variance in the previous month (RV_t) in columns (1)-(2) and as the conditional variance from the GARCH(1,1) model in columns (3)-(4). The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Dependent Variable: Cumulative Excess Market Return ($R_{t+1:t+j}$)				
	Rolling		GARCH(1,1)	
Horizon	$t+1:t+3$	$t+1:t+6$	$t+1:t+3$	$t+1:t+6$
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	10.287**	18.111***	10.421***	18.456***
	(2.543)	(4.298)	(2.717)	(3.711)
High_t	0.011	0.010	0.014	0.015
	(0.945)	(0.471)	(1.124)	(0.668)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$	–11.574***	–18.080***	–11.716***	–18.465***
	(–2.721)	(–4.191)	(–2.844)	(–3.636)
Constant	0.009	0.021*	0.006	0.017
	(1.129)	(1.693)	(0.745)	(1.169)
Obs	546	546	546	546
R^2	0.029	0.037	0.024	0.031

Table 8: Robustness: Alternative specifications

The table reports regressions with alternative specifications. In columns (1) to (4), we report the estimates of the two-regime regression:

$$R_{t+1} = a_1 + b_1\mathbb{E}_t\left(RV_{t+1}\right) + a_2\text{High}_t + b_2\text{High}_t \times \mathbb{E}_t\left(RV_{t+1}\right) + \varepsilon_{t+1},$$

where the dependent variable R_{t+1} is excess market return in month $t+1$, the independent variable High_t is the dummy for months with DOX greater than median when excluding the three months with expected variance larger than 3% in columns (1) and (2), and the dummy for greater than median DOX values estimated from equation (1) when further restricting $b > 0$ in columns (3) and (4). In columns (5) to (8), we report the estimates of the regression

$$R_{t+1} = a_1 + b_1\mathbb{E}_t\left(RV_{t+1}\right) + a_2 Z_t + b_2\mathbb{E}_t\left(RV_{t+1}\right) \times Z_t + \varepsilon_{t+1},$$

where Z_t is the standardized value of DOX index in columns (5) and (6) and the standardized value of inverse DOX index ($1/\text{DOX}_t$) in columns (7) and (8). $\mathbb{E}_t(\text{RV}_{t+1})$ is the expected variance under rational expectation, proxied by previous month realized variance in columns (1), (3), (5), (7), and (9), and by conditional variance from the GARCH(1,1) model in columns (2), (4), (6), (8), and (10). The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Excluding Vol Spikes		DOX ($b > 0$)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	4.650***	4.730***	4.093**	4.227**
	(3.131)	(2.834)	(2.437)	(2.323)
High_t	0.008	0.009	0.005	0.006
	(1.611)	(1.495)	(1.264)	(1.373)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$	–6.232***	–6.239***	–5.377***	–5.498***
	(–3.164)	(–2.609)	(–3.115)	(–2.927)
Constant	0.002	0.000	0.003	0.002
	(0.553)	(0.117)	(0.946)	(0.500)
Obs	543	543	546	546
R^2	0.021	0.016	0.033	0.027
Z_t=	Linear DOX (DOX_t)		Inverse DOX ($1/\text{DOX}_t$)	
	(5)	(6)	(7)	(8)
$\mathbb{E}_t(\text{RV}_{t+1})$	0.130	0.058	0.516	0.657
	(0.151)	(0.064)	(0.699)	(0.747)
Z_t	–0.001	–0.001	–0.005***	–0.006***
	(–0.345)	(–0.346)	(–3.068)	(–2.784)
$\mathbb{E}_t(\text{RV}_{t+1}) \times Z_t$	–1.152*	–1.068*	2.653***	2.919**
	(–1.948)	(–1.843)	(2.675)	(2.328)
Constant	0.007***	0.007***	0.007***	0.006***
	(3.569)	(3.304)	(3.540)	(2.997)
Obs	546	546	546	546
R^2	0.022	0.019	0.019	0.018

Table 9: Robustness: Controlling for investor sentiment

The table reports estimates of the regression

$$R_{t+1} = a_1 + b_1 \mathbb{E}_t\left(RV_{t+1}\right) + a_2 \text{High}_t^S + a_3 \text{High}_t + b_2 \mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t^S + b_3 \mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is excess market return (R_{t+1}), $\mathbb{E}_t(\text{RV}_{t+1})$ is the expected variance under rational expectation, High^S is the dummy variable for the months with above-median Baker and Wurgler (2006) investor sentiment index, and High_t is the dummy variable for the months with above median DOX. We measure the expected variance under rational expectation as the realized variance in the previous month (RV_t) in columns (1)-(2) and as the conditional variance from the GARCH(1,1) model in columns (3)-(4). For each month, we assign the sentiment index using the value at end of previous year following Yu and Yuan (2011). The sample period is from June 1974 to December 2018. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Dependent Variable: Excess Market Return (R_{t+1})			
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	1.266	5.626***	2.047*	6.368***
	(1.043)	(3.189)	(1.814)	(3.457)
High_t^S	0.001	–0.000	0.003	0.002
	(0.197)	(–0.095)	(0.772)	(0.533)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^S$	–2.586**	–1.890	–3.428***	–2.827**
	(–2.053)	(–1.328)	(–3.003)	(–2.120)
High_t		0.006		0.006
		(1.277)		(1.296)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		–5.178***		–5.028***
		(–2.891)		(–2.729)
Constant	0.007**	0.002	0.005	–0.001
	(2.342)	(0.483)	(1.620)	(–0.266)
Obs	546	546	546	546
R^2	0.024	0.038	0.024	0.036

Table 10: Robustness: Alternative explanations

The table reports estimates of the regression

$$R_{t+1} = a_1 + b_1 \mathbb{E}_t\left(RV_{t+1}\right) + a_2 \text{High}_t^X + a_3 \text{High}_t + b_2 \mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t^X + b_3 \mathbb{E}_t\left(RV_{t+1}\right) \times \text{High}_t + \varepsilon_{t+1},$$

where the dependent variable is excess market return (R_{t+1}), $\mathbb{E}_t(\text{RV}_{t+1})$ is the expected variance under rational expectation, High^X is the dummy variable for the months with above median variable X in Panels A to C (or the indicator of NBER recessions in Panel D), and High_t is the dummy variable for the months with above-median DOX. We measure expected variance under rational expectation as the realized variance in the previous month (RV_t) in columns (1)-(2) and as the conditional variance from the GARCH(1,1) model in columns (3)-(4). Variable X is the stickiness of variance expectations (SVE) in Panel A, risk misperception measured by the Pflueger, Siriwardane, and Sunderam (2020) price of volatile stocks (PVS) in Panel B, and time-varying risk aversion measured by surplus consumption ratio following Wachter (2006) in Panel C. Panel D presents the results by controlling for economic recessions, where REC is an indicator variable for NBER recessions. Details on the construction of variables are in Appendix B. The full sample period is from June 1974 to December 2019, and SVE starts in January 1991. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Controlling for Stickiness of Variance Expectations (SVE)

	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	–0.845	4.584***	–0.597	4.566***
	(–1.439)	(2.911)	(–1.278)	(2.694)
$\text{High}_t^{\text{SVE}}$	0.005	0.005	0.006	0.007
	(0.991)	(0.916)	(1.110)	(1.140)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{SVE}}$	–0.357	–0.040	–0.815	–0.767
	(–0.323)	(–0.035)	(–0.681)	(–0.649)
$\text{High}_t^{\text{DOX}}$		0.003		0.003
		(0.673)		(0.665)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{DOX}}$		–5.821***		–5.352***
		(–3.815)		(–3.453)
Constant	0.007*	0.002	0.006	0.001
	(1.843)	(0.425)	(1.553)	(0.131)
Obs	347	347	347	347
R^2	0.018	0.049	0.017	0.041

Panel B: Controlling for Risk Misperception (PVS)				
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	–0.908	4.833***	–1.020	4.811***
	(–1.538)	(3.133)	(–1.617)	(2.787)
$\text{High}_t^{\text{PVS}}$	0.003	0.002	0.002	0.002
	(0.720)	(0.661)	(0.610)	(0.590)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{PVS}}$	–0.229	–0.377	–0.044	–0.179
	(–0.317)	(–0.702)	(–0.059)	(–0.310)
$\text{High}_t^{\text{DOX}}$		0.007*		0.008*
		(1.718)		(1.879)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{DOX}}$		–5.938***		–6.008***
		(–3.835)		(–3.473)
Constant	0.008***	0.000	0.008***	–0.001
	(2.825)	(0.119)	(2.868)	(–0.196)
Obs	546	546	546	546
R^2	0.013	0.032	0.011	0.027

Panel C: Controlling for Time-varying Risk Aversion (SUR)				
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\text{RV}_{t+1})$	–1.143**	4.353***	–1.216**	4.384***
	(–2.081)	(2.907)	(–2.017)	(2.637)
$\text{High}_t^{\text{SUR}}$	–0.006**	–0.005*	–0.006**	–0.006*
	(–2.080)	(–1.806)	(–2.042)	(–1.897)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{SUR}}$	0.154	0.255	0.240	0.419
	(0.247)	(0.564)	(0.357)	(0.813)
$\text{High}_t^{\text{DOX}}$		0.007*		0.008*
		(1.715)		(1.880)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t^{\text{DOX}}$		–5.778***		–5.896***
		(–3.736)		(–3.393)
Constant	0.012***	0.005	0.012***	0.004
	(5.763)	(1.323)	(5.857)	(0.949)
Obs	546	546	546	546
R^2	0.016	0.035	0.015	0.030

Panel D: Controlling for NBER Recessions (REC)				
	Rolling		GARCH(1,1)	
	(1)	(2)	(3)	(4)
$\mathbb{E}_t(\mathrm{RV}_{t+1})$	–0.852*	4.939***	–0.774	5.041***
	(–1.845)	(3.301)	(–1.543)	(2.949)
REC_t	–0.006	–0.010	–0.006	–0.009
	(–0.892)	(–1.332)	(–0.821)	(–1.193)
$\mathbb{E}_t(\mathrm{RV}_{t+1}) \times \mathrm{REC}_t$	–0.088	0.455	–0.299	0.230
	(–0.124)	(0.809)	(–0.399)	(0.395)
$\mathrm{High}_t^{\mathrm{DOX}}$		0.009**		0.010**
		(2.281)		(2.292)
$\mathbb{E}_t(\mathrm{RV}_{t+1}) \times \mathrm{High}_t^{\mathrm{DOX}}$		–6.329***		–6.324***
		(–4.065)		(–3.561)
Constant	0.009***	0.002	0.009***	0.000
	(6.124)	(0.518)	(5.668)	(0.052)
Obs	546	546	546	546
R^2	0.014	0.035	0.014	0.030

Table 11: DOX and risk-return trade-offs: International evidence

The table reports estimation results of mean-variance relation and return-innovation relation using samples from Germany (Panel A), France (Panel B), the UK (Panel C), Italy (Panel D), and Japan (Panel E). Specifically, we run regressions for each country k:

$$R^k_{t+1:t+j} = a^k_1 + b^k_1 \mathbb{E}_t\left(RV^k_{t+1}\right) + a^k_2 \text{High}^k_t + b^k_2 \mathbb{E}_t\left(RV^k_{t+1}\right) \times \text{High}^k_t + \varepsilon^k_{t+1},$$

$$R^k_{t+1:t+j} = c^k_1 + d^k_1 \mathbb{E}_t\left(RV^k_{t+1}\right) + e^k_1 \Delta\mathbb{E}_t\left(RV^k_{t+1}\right) + c^k_2 \text{High}^k_t + d^k_2 \mathbb{E}_t\left(RV^k_{t+1}\right) \times \text{High}^k_t + e^k_2 \Delta\mathbb{E}_t\left(RV^k_{t+1}\right) \times \text{High}^k_t + \varepsilon^k_{t+1},$$

where the dependent variable $R^k_{t+1:t+j}$ is the j-month excess market return for country k, $\mathbb{E}_t(\text{RV}^k_{t+1})$ is the expected variance under rational expectation measured by realized variance in the previous month (RV^k_t), $\Delta\mathbb{E}_{t+1}(\text{RV}^k_{t+2})$ is the variance innovation, and High^k_t is the dummy variable for the months with above-median DOX for country k. For each country k, we construct its DOX series using the consensus forecasts of stock market index movements from the ZEW and ICF surveys as the expected returns following Cassella and Gulen (2018). Details on the construction of DOX for these countries are in Appendix B. The sample period is from December 1996 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Panel A: Germany					
	Dependent Variable: Excess Market Return ($R_{t+1:t+j}$)				
Horizon	$t+1$	$t+1$	$t+2$	$t+1:t+3$	$t+1$
	(1)	(2)	(3)	(4)	(5)
$\mathbb{E}_t(\text{RV}_{t+1})$	0.116	3.284**	1.732**	6.855**	–1.787
	(0.178)	(2.025)	(2.191)	(2.527)	(–0.984)
High_t		0.008	0.004	0.022	–0.001
		(0.914)	(0.468)	(1.085)	(–0.150)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		–3.817**	–2.250**	–8.762**	–0.533
		(–2.012)	(–1.982)	(–2.541)	(–0.273)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$					–10.385***
					(–7.757)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})\times\text{High}_t$					5.054***
					(2.866)
Constant	0.005	–0.003	0.002	–0.002	0.016**
	(1.226)	(–0.416)	(0.325)	(–0.104)	(2.315)
Obs	276	276	275	276	276
R^2	0.000	0.026	0.010	0.041	0.264

Panel B: France					
	Dependent Variable: Excess Market Return ($R_{t+1:t+j}$)				
Horizon	$t+1$	$t+1$	$t+2$	$t:t+3$	$t+1$
	(1)	(2)	(3)	(4)	(5)
$\mathbb{E}_t(\text{RV}_{t+1})$	–0.316	1.661	2.892***	5.709***	–3.623***
	(–0.431)	(1.140)	(2.982)	(2.667)	(–3.013)
High_t		0.009	0.012	0.024	–0.005
		(1.084)	(1.467)	(1.196)	(–0.648)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		–2.542	–4.102***	–8.634***	1.374
		(–1.494)	(–4.065)	(–3.633)	(1.028)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$					–9.046***
					(–6.444)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})\times\text{High}_t$					4.319**
					(2.320)
Constant	0.005	–0.003	–0.006	–0.008	0.018***
	(1.141)	(–0.396)	(–0.987)	(–0.532)	(2.925)
Obs	276	276	275	276	276
R^2	0.001	0.014	0.037	0.053	0.252

Panel C: UK					
	Dependent Variable: Excess Market Return ($R_{t+1:t+j}$)				
Horizon	$t+1$	$t+1$	$t+2$	$t:t+3$	$t+1$
	(1)	(2)	(3)	(4)	(5)
$\mathbb{E}_t(\text{RV}_{t+1})$	0.012	0.938	0.729	2.242	–2.328***
	(0.022)	(0.900)	(0.562)	(1.223)	(–3.026)
High_t		0.004	0.002	0.017	0.001
		(0.941)	(0.413)	(1.291)	(0.217)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		–1.258	–0.967	–4.196**	0.364
		(–1.057)	(–0.759)	(–2.289)	(0.417)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$					–8.031***
					(–7.533)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})\times\text{High}_t$					3.353**
					(2.444)
Constant	0.000	–0.002	–0.001	–0.007	0.006**
	(0.124)	(–0.638)	(–0.373)	(–0.758)	(2.313)
Obs	276	276	275	276	276
R^2	0.000	0.004	0.002	0.019	0.249

Panel D: Italy					
	Dependent Variable: Excess Market Return ($R_{t+1:t+j}$)				
Horizon	$t+1$	$t+1$	$t+2$	$t:t+3$	$t+1$
	(1)	(2)	(3)	(4)	(5)
$\mathbb{E}_t(\text{RV}_{t+1})$	0.288	0.134	2.507***	2.174	–2.597*
	(0.395)	(0.127)	(3.045)	(1.015)	(–1.852)
High_t		–0.023**	–0.003	–0.036	–0.020**
		(–2.305)	(–0.300)	(–1.610)	(–2.391)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		0.403	–2.705***	–2.244	0.897
		(0.285)	(–2.979)	(–0.804)	(0.546)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$					–4.685***
					(–4.496)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})\times\text{High}_t$					–0.451
					(–0.302)
Constant	–0.000	0.011*	–0.003	0.016	0.022***
	(–0.014)	(1.823)	(–0.483)	(0.996)	(3.556)
Obs	276	276	275	276	276
R^2	0.001	0.028	0.034	0.052	0.193

Panel E: Japan					
	Dependent Variable: Excess Market Return ($R_{t+1:t+j}$)				
Horizon	$t+1$	$t+1$	$t+2$	$t:t+3$	$t+1$
	(1)	(2)	(3)	(4)	(5)
$\mathbb{E}_t(\text{RV}_{t+1})$	0.071	0.234	–0.079	–0.296	–1.658***
	(0.266)	(0.676)	(–0.249)	(–0.522)	(–6.368)
High_t		0.014	0.002	0.026	0.016**
		(1.549)	(0.238)	(1.114)	(2.145)
$\mathbb{E}_t(\text{RV}_{t+1}) \times \text{High}_t$		0.028	1.426	1.383	–1.484
		(0.022)	(1.395)	(0.636)	(–1.220)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})$					–2.898***
					(–10.078)
$\Delta\mathbb{E}_{t+1}(\text{RV}_{t+2})\times\text{High}_t$					–4.506**
					(–2.252)
Constant	0.002	–0.006	–0.001	–0.007	0.005
	(0.415)	(–1.013)	(–0.157)	(–0.398)	(1.021)
Obs	276	276	275	276	276
R^2	0.000	0.015	0.009	0.026	0.206

Table 12: Low-risk anomalies and DOX

The table reports estimates of b in the predictive regression

$$R_{j,t+1} = a + bDOX_t + cf_{t+1} + \varepsilon_{t+1},$$

where $R_{j,t+1}$ is the excess percentage return in month $t+1$ on the long leg, the short leg, or the difference, and f_{t+1} is either Carhart (1997) four-factor (columns (4) to (6)) or Fama and French (2015) five-factor (columns (7) to (9)). Total volatility (*tv*) is the portfolio that longs (shorts) stocks in the lowest (highest) tercile of the realized volatility in the previous month, beta (*beta*) is the portfolio that longs (shorts) stocks in the lowest (highest) tercile of betting-against-beta calculated following Frazzini and Pedersen (2014), and idiosyncratic volatility (*ivol*) is the portfolio that longs (shorts) stocks in the lowest (highest) tercile of idiosyncratic volatility per the Fama and French (1993) three-factor model. Also reported are returns on a strategy that equally combines these three low-risk strategies (*average*). Following Jensen, Kelly, and Pedersen (2021), we mitigate the impact of micro stocks by sorting stocks into characteristic terciles with breakpoints based on non-micro stocks (capitalization at the end of previous month greater than NYSE 20th percentile). Then the micro-cap stocks are distributed into the terciles based on the same characteristic breakpoints. Portfolio returns are value weighted. Independent variables are standardized to have zero mean and unit variance. The sample period is from June 1974 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses.

	Excess Return			Carhart-Adjusted			FF2015-Adjusted		
	Long (1)	Short (2)	Long-Short (3)	Long (4)	Short (5)	Long-Short (6)	Long (7)	Short (8)	Long-Short (9)
tv	–0.22	–0.67	0.45	0.08	–0.21	0.28	0.05	–0.13	0.18
	(–1.32)	(–1.71)	(1.54)	(1.55)	(–1.73)	(1.88)	(0.98)	(–2.03)	(2.01)
beta	–0.10	–0.48	0.38	0.16	–0.04	0.19	0.11	0.01	0.10
	(–0.62)	(–1.59)	(1.79)	(2.02)	(–0.56)	(1.73)	(1.84)	(0.10)	(0.89)
ivol	–0.23	–0.64	0.40	0.09	–0.20	0.29	0.07	–0.13	0.19
	(–1.35)	(–1.74)	(1.57)	(1.75)	(–2.01)	(2.11)	(1.68)	(–2.19)	(2.43)
average	–0.18	–0.60	0.41	0.11	–0.15	0.26	0.07	–0.08	0.16
	(–1.13)	(–1.71)	(1.66)	(1.98)	(–1.85)	(2.10)	(1.77)	(–1.57)	(2.03)

Table 13: Low-risk anomalies and DOX: Controlling for other factors

The table reports estimates of b and c in the predictive regression

$$R_{j,t+1} = a + bDOX_t + cz_t + df_{t+1} + \varepsilon_{t+1},$$

where $R_{j,t+1}$ is the excess percentage return in month t+1 on the long-short portfolio, z_t is the Baker and Wurgler (2006) sentiment (columns (1) and (2)), inflation rate (columns (3) and (4)), TED spread (columns (5) and (6)), or the Yu (2011) dispersion of analyst forecasts (columns (8) and (9)), and f_t is Fama and French (2015) five-factor. TED spread is the difference between London Interbank Offered Loan (LIBOR) and T-bill rates. Dispersion of analyst forecasts is measured as the value-weighted individual-stock long-term earnings growth rate from Thompson Reuters IBES data. Total volatility (*tv*) is the portfolio that longs (shorts) stocks in the lowest (highest) tercile of the realized volatility in the previous month, beta (*beta*) is the portfolio that longs (shorts) stocks in the lowest (highest) tercile of betting-against-beta calculated following Frazzini and Pedersen (2014), and idiosyncratic volatility (*ivol*) is the portfolio that longs (shorts) stocks in the lowest (highest) tercile of idiosyncratic volatility per the Fama and French (1993) three-factor model. Also reported are returns on a strategy that equally combines these three low-risk strategies (*average*). Following Jensen, Kelly, and Pedersen (2021), we mitigate the impact of micro stocks by sorting stocks into characteristic terciles with breakpoints based on non-micro stocks (capitalization at the end of previous month greater than NYSE 20th percentile). Then the micro-cap stocks are distributed into the terciles based on the same characteristic breakpoints. Portfolio returns are value weighted. Independent variables are standardized to have zero mean and unit variance. The sample period is from June 1974 to December 2019. Analysts' dispersion spans from January 1981 to December 2019. Newey-West 12-lag adjusted t-statistics are reported in parentheses.

$z =$	Sentiment		Inflation		TED Spread		Dispersion	
	b	c	b	c	b	c	b	c
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tv	0.15	0.12	0.18	–0.08	0.19	–0.05	0.16	0.10
	(1.44)	(1.15)	(2.01)	(–0.84)	(1.97)	(–0.41)	(1.52)	(1.14)
beta	0.14	–0.17	0.10	0.02	0.05	0.21	0.11	0.08
	(1.25)	(–1.42)	(0.90)	(0.20)	(0.51)	(1.74)	(0.91)	(0.73)
ivol	0.17	0.13	0.19	–0.03	0.20	–0.05	0.17	0.04
	(1.75)	(1.32)	(2.43)	(–0.35)	(2.37)	(–0.60)	(1.81)	(0.46)
average	0.15	0.03	0.16	–0.03	0.15	0.03	0.15	0.07
	(1.74)	(0.29)	(2.03)	(–0.34)	(1.85)	(0.34)	(1.62)	(0.94)

Table 14: Low-risk anomalies sorted by firm-level DOX

The table reports excess returns for value-weighted portfolios that are double-sorted on firm-level DOX and total volatility (Panel A), beta (Panel B), and idiosyncratic volatility (Panel C). Firm-level DOX is calculated from analyst expectations using the following nonlinear regression

$$Ret_{i,t}^{A,12m} = a_i + b_i \sum_{s=0}^{7} w_{i,s} \cdot Ret_{i,t-3s-3:t-3s} + \epsilon_{i,t}, \quad w_{i,s} = \frac{\lambda_i^s}{\sum_{k=0}^{7} \lambda_i^k}, \quad 0 < \lambda_i < 1,$$

where $Ret_{i,t}^{A,12m}$ is the average expected return over the next 12 months issued during month t by analysts, and $w_{i,s}$ is the exponential decay weight. The firm-level degree of extrapolation parameter is defined as $DOX_i = \text{sign}(b_i)(1 - \lambda_i)$. We first sort stocks into terciles of the firm-level DOX value. Within each tercile of the degree of extrapolation parameter, we sort firms into quintiles of total volatility, beta, and idiosyncratic volatility. Total volatility (tv) is the standard deviation of daily raw stock returns in the previous month. Beta ($beta$) is the betting against beta of Frazzini and Pedersen (2014), calculated using a one-year rolling standard deviation of one-day log returns for volatilities and a five-year horizon for the correlation in overlapping three-day log returns. Idiosyncratic volatility ($ivol$) is the standard deviation of the residuals from the Fama and French (1993) three-factor model using daily excess returns in the previous month. The portfolios are rebalanced each month. The sample is from January 1999 to December 2019 and includes stocks with analyst coverage to calculate firm-level DOX. Excess returns are reported in percentages, and Newey-West 12-lag adjusted t-statistics are reported in parentheses.

	Q1 (1)	Q2 (2)	Q3 (3)	Q4 (4)	Q5 (5)	Q1 − Q5 (6)
			Panel A: Total Volatility (tv)			
T1	0.63	0.62	0.81	0.64	0.66	-0.03
	(2.75)	(2.05)	(2.26)	(1.21)	(1.01)	(-0.06)
T2	0.71	0.42	0.54	0.15	0.31	0.39
	(3.13)	(1.34)	(1.31)	(0.26)	(0.39)	(0.57)
T3	0.66	0.45	0.48	0.61	-0.12	0.78
	(3.00)	(1.38)	(0.96)	(0.98)	(-0.18)	(1.41)
T3 − T1	0.03	-0.18	-0.33	-0.02	-0.78	0.81
	(0.25)	(-1.08)	(-1.60)	(-0.09)	(-2.80)	(2.41)
			Panel B: Beta ($beta$)			
T1	0.64	0.68	0.55	0.75	0.46	0.18
	(2.65)	(2.71)	(1.95)	(2.28)	(0.81)	(0.34)
T2	0.72	0.63	0.40	0.48	0.39	0.32
	(3.42)	(2.18)	(1.37)	(1.26)	(0.75)	(0.72)
T3	0.95	0.52	0.67	0.40	0.26	0.69
	(5.32)	(2.36)	(2.60)	(1.07)	(0.49)	(1.36)
T3 − T1	0.31	-0.16	0.12	-0.34	-0.20	0.51
	(1.70)	(-1.04)	(0.78)	(-2.29)	(-1.21)	(2.06)

	Q1 (1)	Q2 (2)	Q3 (3)	Q4 (4)	Q5 (5)	Q1 − Q5 (6)
			Panel C: Idiosyncratic Volatility (*ivol*)			
T1	0.71	0.54	0.53	0.84	0.77	-0.05
	(3.13)	(1.56)	(1.35)	(1.74)	(1.34)	(-0.12)
T2	0.61	0.65	0.12	0.51	0.23	0.38
	(2.25)	(2.30)	(0.25)	(0.88)	(0.32)	(0.62)
T3	0.51	0.45	0.66	0.50	-0.11	0.63
	(2.05)	(1.33)	(1.34)	(0.78)	(-0.18)	(1.36)
T3 − T1	-0.20	-0.09	0.13	-0.34	-0.88	0.68
	(-1.81)	(-0.57)	(0.49)	(-1.37)	(-3.13)	(2.02)

Table 15: Low-risk anomalies sorted by firm-level DOX: Alphas

The table reports the CAPM, Carhart (1997) four-factor, and Fama and French (2015) five-factor alphas for value-weighted portfolios, which are double-sorted on firm-level DOX and total volatility (Panels A to C), beta (Panels D to F), and idiosyncratic volatility (Panels G to I). We first sort firms into terciles of the firm-level DOX value. Within each tercile of the degree of extrapolation parameter, we sort firms into quintiles of total volatility, beta, and idiosyncratic volatility. The firm-level DOX is calculated from analyst expectations using the nonlinear regression

$$Ret_{i,t}^{A,12m} = a_i + b_i \sum_{s=0}^{7} w_{i,s} \cdot Ret_{i,t-3s-3:t-3s} + \epsilon_{i,t}, \quad w_{i,s} = \frac{\lambda_i^s}{\sum_{k=0}^{7} \lambda_i^k}, \quad 0 < \lambda_i < 1$$

where $Ret_{i,t}^{A,12m}$ is the average expected return over the next 12 months issued during month t by analysts, and $w_{i,s}$ is the exponential decay weight. The firm-level degree of extrapolation parameter is defined as $DOX_i = \text{sign}(b_i)(1 - \lambda_i)$. Total volatility ($tv$) is the standard deviation of daily raw stock returns in the previous month. Beta ($beta$) is the betting against beta of Frazzini and Pedersen (2014), calculated using a one-year rolling standard deviation of one-day log returns for volatilities and a five-year horizon for the correlation in overlapping three-day log returns. Idiosyncratic volatility ($ivol$) is the standard deviation of the residuals from the Fama and French (1993) three-factor model using daily excess returns in the previous month. The portfolios are rebalanced each month. The sample is from January 1999 to December 2019 and includes stocks with analyst coverage to calculate firm-level DOX. Alphas are reported in percentages per month, and Newey-West 12-lag adjusted t-statistics are reported in parentheses.

	Q1 (1)	Q2 (2)	Q3 (3)	Q4 (4)	Q5 (5)	Q1 − Q5 (6)
			Panel A: CAPM-Adjusted Returns to Total Volatility (tv)			
T1	0.28	0.11	0.14	-0.15	-0.32	0.60
	(2.33)	(1.35)	(0.92)	(-0.52)	(-0.94)	(1.36)
T2	0.41	-0.08	-0.10	-0.64	-0.71	1.12
	(3.02)	(-0.51)	(-0.55)	(-3.06)	(-1.66)	(2.18)
T3	0.30	-0.08	-0.22	-0.28	-1.09	1.39
	(2.06)	(-0.47)	(-1.06)	(-0.82)	(-3.24)	(3.20)
T3 − T1	0.02	-0.20	-0.36	-0.13	-0.77	0.79
	(0.16)	(-1.11)	(-1.92)	(-0.51)	(-2.60)	(2.26)
			Panel B: Carhart-Adjusted Returns to Total Volatility (tv)			
T1	0.24	0.13	0.21	-0.08	-0.21	0.45
	(2.99)	(1.45)	(1.25)	(-0.37)	(-0.85)	(1.53)
T2	0.37	-0.09	-0.06	-0.60	-0.63	1.00
	(3.44)	(-1.06)	(-0.42)	(-2.72)	(-1.73)	(2.52)
T3	0.24	-0.13	-0.24	-0.23	-1.01	1.25
	(2.16)	(-0.96)	(-1.17)	(-0.86)	(-3.16)	(3.23)
T3 − T1	-0.01	-0.26	-0.45	-0.16	-0.80	0.80
	(-0.06)	(-1.73)	(-1.92)	(-0.70)	(-2.50)	(2.14)

	Q1 (1)	Q2 (2)	Q3 (3)	Q4 (4)	Q5 (5)	Q5 − Q1 (6)
Panel C: FF2015-Adjusted Returns to Total Volatility (*tv*)						
T1	0.05	-0.03	0.22	0.11	0.19	-0.14
	(0.72)	(-0.29)	(1.25)	(0.53)	(0.85)	(-0.56)
T2	0.17	-0.23	-0.05	-0.32	-0.08	0.25
	(1.54)	(-2.11)	(-0.29)	(-1.84)	(-0.22)	(0.69)
T3	0.08	-0.27	-0.25	-0.06	-0.60	0.68
	(0.82)	(-2.08)	(-1.30)	(-0.20)	(-1.94)	(1.96)
T3 − T1	0.03	-0.25	-0.47	-0.16	-0.78	0.82
	(0.31)	(-1.61)	(-2.09)	(-0.58)	(-2.15)	(1.94)
Panel D: CAPM-Adjusted Returns to Beta (*beta*)						
T1	0.40	0.31	0.07	0.15	-0.43	0.83
	(2.06)	(2.23)	(0.39)	(0.96)	(-1.67)	(2.00)
T2	0.53	0.27	-0.03	-0.08	-0.45	0.98
	(2.83)	(1.18)	(-0.15)	(-0.37)	(-2.33)	(3.64)
T3	0.78	0.21	0.24	-0.23	-0.55	1.33
	(4.42)	(1.16)	(1.05)	(-1.06)	(-2.81)	(4.37)
T3 − T1	0.37	-0.10	0.17	-0.38	-0.12	0.50
	(2.17)	(-0.66)	(1.11)	(-2.32)	(-0.75)	(2.00)
Panel E: Carhart-Adjusted Returns to Beta (*beta*)						
T1	0.37	0.28	0.02	0.19	-0.31	0.68
	(2.49)	(2.29)	(0.13)	(1.34)	(-1.43)	(2.13)
T2	0.49	0.21	-0.06	-0.14	-0.36	0.85
	(3.72)	(1.33)	(-0.51)	(-0.92)	(-2.05)	(3.26)
T3	0.71	0.13	0.16	-0.28	-0.52	1.23
	(5.21)	(1.20)	(1.06)	(-2.04)	(-2.84)	(5.32)
T3 − T1	0.34	-0.15	0.14	-0.47	-0.21	0.55
	(1.95)	(-1.04)	(1.03)	(-3.10)	(-1.36)	(2.29)
Panel F: FF2015-Adjusted Returns to Beta (*beta*)						
T1	0.16	0.01	-0.28	-0.03	-0.10	0.25
	(1.09)	(0.07)	(-2.41)	(-0.21)	(-0.40)	(0.73)
T2	0.21	-0.07	-0.25	-0.23	-0.12	0.34
	(1.54)	(-0.43)	(-1.72)	(-1.42)	(-0.59)	(1.12)
T3	0.48	-0.11	-0.10	-0.46	-0.39	0.88
	(3.16)	(-1.09)	(-0.92)	(-3.38)	(-2.33)	(3.84)
T3 − T1	0.32	-0.11	0.18	-0.43	-0.30	0.62
	(1.86)	(-0.75)	(1.15)	(-2.93)	(-1.47)	(2.20)

	Q1 (1)	Q2 (2)	Q3 (3)	Q4 (4)	Q5 (5)	Q5 − Q1 (6)
Panel G: CAPM-Adjusted Returns to Idiosyncratic Volatility (*ivol*)						
T1	0.31	-0.02	-0.11	0.08	-0.16	0.47
	(2.85)	(-0.14)	(-0.87)	(0.37)	(-0.54)	(1.30)
T2	0.23	0.18	-0.55	-0.28	-0.77	1.00
	(1.42)	(1.17)	(-2.85)	(-1.23)	(-2.21)	(2.25)
T3	0.09	-0.13	-0.08	-0.37	-1.03	1.12
	(0.81)	(-0.82)	(-0.34)	(-1.04)	(-3.46)	(3.22)
T3 − T1	-0.22	-0.12	0.04	-0.45	-0.87	0.65
	(-2.03)	(-0.66)	(0.15)	(-2.06)	(-2.94)	(1.86)
Panel H: Carhart-Adjusted Returns to Idiosyncratic Volatility (*ivol*)						
T1	0.28	0.01	-0.04	0.15	-0.04	0.32
	(3.49)	(0.05)	(-0.37)	(0.90)	(-0.18)	(1.30)
T2	0.21	0.18	-0.52	-0.24	-0.65	0.86
	(1.75)	(1.95)	(-2.93)	(-1.12)	(-2.21)	(2.55)
T3	0.07	-0.18	-0.08	-0.28	-0.95	1.02
	(0.63)	(-1.41)	(-0.40)	(-1.03)	(-3.14)	(2.82)
T3 − T1	-0.21	-0.19	-0.05	-0.43	-0.91	0.70
	(-1.98)	(-1.40)	(-0.21)	(-2.12)	(-2.85)	(1.86)
Panel I: FF2015-Adjusted Returns to Idiosyncratic Volatility (*ivol*)						
T1	0.14	-0.08	-0.07	0.30	0.27	-0.13
	(1.71)	(-0.70)	(-0.57)	(1.77)	(1.10)	(-0.49)
T2	0.08	0.08	-0.53	-0.13	-0.18	0.26
	(0.70)	(0.69)	(-2.48)	(-0.63)	(-0.64)	(0.85)
T3	-0.06	-0.25	0.03	-0.16	-0.48	0.42
	(-0.66)	(-1.88)	(0.14)	(-0.55)	(-1.72)	(1.30)
T3 − T1	-0.20	-0.17	0.10	-0.46	-0.75	0.55
	(-2.07)	(-1.24)	(0.36)	(-1.88)	(-2.35)	(1.50)

Table 16: Low-risk anomalies and firm-level DOX: Fama-MacBeth regression
The table reports the results of the Fama-MacBeth regressions. Each month we run a cross-sectional regression of returns on lagged variables and report the time-series average of the coefficients in the regression

$$\begin{aligned} R = \alpha &+ \beta_1 \text{DOX} + \beta_2 \text{PROXY} + \beta_3 \text{PROXY} \times \text{DOX} + \beta_4 \text{CGO} \\ &+ \beta_5 \text{PROXY} \times \text{CGO} + \beta_6 \text{LOGBM} + \beta_7 \text{LOGME} \\ &+ \beta_8 \text{MOM}(-1,0) + \beta_9 \text{MOM}(-12,-1) + \beta_{10} \text{MOM}(-36,-12) \\ &+ \beta_{11} \text{TURNOVER} + \varepsilon, \end{aligned}$$

where R is monthly stock return in month $t+1$, DOX is the firm-level degree of extrapolation estimated from IBES data, PROXY is one of our three risk proxies (total volatility, beta, and idiosyncratic volatility) at the end of month t, CGO is the Grinblatt and Han (2005) capital gain overhang at the end of month t, LOGBM is the natural log of the book-to-market ratio at the end of month t, LOGME is the natural log of market equity at the end of month t, MOM$(-1,0)$ is the stock return in month t, MOM$(-12,-1)$ is the stock return from the end of month $t-12$ to the end of month $t-1$, MOM$(-36,-12)$ is the stock return from the end of month $t-36$ to the end of month $t-12$, and TURNOVER is the share turnover rate in month t. Total volatility (*tv*) is the standard deviation of daily raw stock returns in the previous month. Beta (*beta*) is the betting against beta of Frazzini and Pedersen (2014), calculated using a one-year rolling standard deviation of one-day log returns for volatilities and a five-year horizon for the correlation in overlapping three-day log returns. Idiosyncratic volatility (*ivol*) is the standard deviation of the residuals from the Fama and French (1993) three-factor model using daily excess returns in the previous month. The sample is from January 1999 to December 2019 and includes stocks with analyst coverage to calculate firm-level DOX. Newey-West 12-lag adjusted t-statistics are reported in parentheses.

PROXY=	Total Volatility		Beta		Idiosyncratic Volatility	
	(1)	(2)	(3)	(4)	(5)	(6)
PROXY × DOX	–15.35***	–14.67***	–0.23**	–0.23**	–15.79***	–15.20***
	(–5.39)	(–4.30)	(–2.01)	(–2.13)	(–5.47)	(–4.52)
PROXY	–8.78	–14.18**	–0.17	–0.11	–7.97	–12.57**
	(–1.08)	(–2.28)	(–0.42)	(–0.30)	(–0.98)	(–2.11)
DOX	0.26**	0.13	–0.03	–0.14	0.21**	0.07
	(2.30)	(1.10)	(–0.30)	(–1.16)	(2.00)	(0.71)
PROXY × CGO		0.83		0.44***		1.45
		(0.20)		(2.96)		(0.36)
CGO		–0.28		–0.58***		–0.29
		(–1.05)		(–2.63)		(–1.14)
LogBM		–0.06		–0.05		–0.06
		(–0.61)		(–0.57)		(–0.55)
LogME		–0.14**		–0.12		–0.13**
		(–2.50)		(–1.55)		(–2.56)
MOM(–1,0)		–0.01***		–0.01***		–0.01***
		(–2.81)		(–2.99)		(–2.80)
MOM(–12,–1)		0.00		0.00		0.00
		(0.79)		(0.82)		(0.78)
MOM(–36,–12)		–0.00		–0.00		–0.00
		(–1.27)		(–1.02)		(–1.25)
TURNOVER		–0.04		–0.07*		–0.06
		(–1.27)		(–1.95)		(–1.63)
Constant	1.27***	2.97***	1.36***	2.56**	1.19***	2.83***
	(4.36)	(3.51)	(4.50)	(2.50)	(4.11)	(3.46)
Obs	697985	697985	697985	697985	697985	697985

Appendix A. A Simple Two-Period Model

In this section, we formalize the intuition in the introduction with a two-period model. Consider an economy with two assets: one risk-free and one risky. Without loss of generality, we assume that the return of the risk-free asset is set to 0. The risky asset has a fixed supply of Q shares and pays dividend only at date 2. The dividend paid at date 2 has a stochastic variance: $D_2 \sim N(\mu, \exp(a + b\epsilon))$, where ϵ is standard normal and observable to economic agents at date 1 (but unobservable to an econometrician), and a and b are constants governing the variance process. There are two types of traders in the economy. Fraction N^e of the investors are extrapolators and fraction N^f are fundamental investors. All investors share the same CARA utility functions with the risk aversion coefficient γ.

At date 0, extrapolators own wealth W_0. At date 1, the extrapolator maximizes his subjective utility:

$$\max_{\pi_1} \mathbb{E}_1^X \left[-e^{-\gamma(W_1 + \pi_1(P_2 - P_1))} \right], \tag{A1}$$

where $\mathbb{E}_1^X(\cdot)$ denotes the extrapolator's subjective expectation at date 1, and π_1 is the number of shares invested in the risky asset at date 1.

We assume that the extrapolator forms beliefs about next-period's price change by extrapolating the past price changes:

$$\mathbb{E}_1^X[P_2 - P_1] = (1 - \theta)(P_1 - P_0) + \theta X_0, \tag{A2}$$

where $0 < \theta < 1$, P_0 is the asset price at date 0, and X_0 is the extrapolator's enthusiasm at date 0.

We assume that at date 1 both extrapolators and fundamental investors know the variance of the date 2 price change:

$$Var_1^X[P_2 - P_1] = Var_1[P_2 - P_1] = Var_1[D_2] = \exp(a + b\epsilon), \tag{A3}$$

where we have used the assumption that the price reverts to fundamental value D_2 at date 2. In fact, this assumption seems innocuous, as Bordalo et al. (2018) and Choi and Mertens (2019) suggest that extrapolators hold mistaken beliefs about the conditional mean returns but accurate beliefs about the variance.

The fundamental trader expects the price to revert to the fundamental value at date 2:

$$\mathbb{E}_1[P_2 - P_1] = \mathbb{E}_1[D_2 - P_1] = \mu - P_1. \tag{A4}$$

The market clearing condition requires that

$$N^f \frac{(\mu - P_1)}{\gamma \exp(a + b\epsilon)} + N^e \frac{(1-\theta)(P_1 - P_0) + \theta X_0}{\gamma \exp(a + b\epsilon)} = Q, \tag{A5}$$

which implies that the date 1 asset price is

$$P_1 = \frac{N^f \mu - N^e((1-\theta)P_0 - \theta X_0)}{N^f - N^e(1-\theta)} + \frac{\gamma Q}{N^e(1-\theta) - N^f} \exp(a + b\epsilon). \tag{A6}$$

The above equation shows that extrapolators view today's high price (and large return) as compensation for expected risk.

At date 2, the fundamental value is revealed, and the price reverts to the fundamental. Thus, the date 2 price change is

$$\mathbb{E}_1[P_2 - P_1] = c_0 + \frac{\gamma Q}{N^f - N^e(1-\theta)} \exp(a + b\epsilon), \tag{A7}$$

where c_0 is a constant. Then, we can derive the conditional risk-return trade-off at date 1 from the view of an econometrician as

$$\begin{aligned} Cov_1(\mathbb{E}[P_2 - P_1], \exp(a + b\epsilon)) &= Cov_1(\frac{\gamma Q}{N^f - N^e(1-\theta)} \exp(a + b\epsilon), \exp(a + b\epsilon)) \\ &= \frac{\gamma Q}{N^f - N^e(1-\theta)} (\exp(2a + 2b^2) - \exp(2a + b^2)). \end{aligned} \tag{A8}$$

Thus, when extrapolators are sufficiently extrapolative (i.e., $N^e(1-\theta) > N^f$), the objective risk-return relationship is negative. In addition, we can also calculate the contemporaneous return-innovation relation:

$$Cov_1(P_1 - P_0, \epsilon) = \frac{\gamma Q}{N^e(1-\theta) - N^f} \exp(a + b^2/2) > 0, \text{ if } N^e(1-\theta) > N^f.$$

The above equation shows that the contemporaneous negative relation between innovations in conditional variance and stock returns is weakened during high-DOX periods.

From extrapolators' perspective, their conditional risk-return trade-off at date 1 is given by

$$
\begin{aligned}
Cov_1^X(\mathbb{E}^X[P_2 - P_1], \exp(a + b\epsilon)) &= Cov_1^X\left(\frac{\gamma Q(1-\theta)}{N^e(1-\theta) - N^f} \exp(a + b\epsilon), \exp(a + b\epsilon)\right) \\
&= \frac{\gamma Q(1-\theta)}{N^e(1-\theta) - N^f} (\exp(2a + 2b^2) - \exp(2a + b^2)).
\end{aligned} \tag{A9}
$$

The above equation shows how DOX can also affect the subjective risk-return trade-off. When extrapolators are sufficiently extrapolative, $N^e(1-\theta) > N^f$, extrapolators' subjective risk-return relationship is positive. Hence, high DOX may strengthen the subjective risk-return relation. Although our intuition is likely to survive in a dynamic setting with proper assumptions, we leave such a quantitative analysis for future research.

Appendix B. Definitions of Key Variables

CGO: Following Grinblatt and Han (2005), we calculate the reference price for each individual stock i as the weighted average price over the previous five years (260 weeks):

$$
P_{i,t}^{ref} = k^{-1} \sum_{n=1}^{260} \left(V_{i,t-n} \prod_{q=1}^{n-1} (1 - V_{i,t-n+q}) \right) P_{i,t-n}, \tag{A10}
$$

where V_t is turnover in week t, and k is a normalizing constant such that the weights on past prices sum to one. All ordinary common shares traded on the NYSE, AMEX, and NASDAQ exchanges are included. Following Gao and Ritter (2010), we scale down the NASDAQ volume to ensure comparability to the NYSE volume. For each week t, we measure capital gains overhang (CGO) as

$$
CGO_{i,t} = \frac{P_{i,t-1} - P_{i,t}^{ref}}{P_{i,t-1}} \tag{A11}
$$

We use the last week CGO in a month as the monthly CGO. Finally, we calculate the aggregate CGO by taking the value-weighted sum across all stocks.

PVS: We calculate the monthly price of volatile stocks (PVS) following Pflueger, Siriwardane, and Sunderam (2020). At the end of each month, we form book-to-market ratios for each stock. Stock return volatility is calculated using daily data in the previous two months. Then, we sort all stocks into quintiles based on the previous two months' volatility. PVS is calculated as

$$\text{PVS}_t = \overline{\left(\frac{B}{M}\right)}_{\text{low vol},t} - \overline{\left(\frac{B}{M}\right)}_{\text{high vol},t} \tag{A12}$$

Here, PVS is the difference between the book-to-market ratios of low-volatility stocks and high-volatility stocks. PVS positively predicts revisions in expected risk and measures the overvaluation of volatile stocks and therefore the underestimation of risk.

SUR: Following Wachter (2006) and He, Wang, and Yu (2020), we calculate the surplus ratio (SUR) as the smoothed average of consumption growth in the past 40 quarters. For each month, the most recent quarterly surplus ratio is used as the monthly measure.

DOX for other countries: To estimate the DOX time-series for markets outside the US, we used datasets from different sources. We obtained monthly historical equity index series from Global Financial Data (GFD).

The ZEW Financial Markets Survey is a monthly survey of around 300 financial market practitioners in Germany. Since December 1991, respondents have been asked to forecast the future six-month direction of stock market index movements in six major developed markets (the US, Germany, France, the UK, Italy, and Japan). Respondents can indicate "increase," "no change," or "decrease." The ZEW survey then defines the fraction of "increase" minus the fraction of "decrease" as "balance," a variable equivalent to the AA and II surveys' bullish-bearish (%) indicator. Please refer to Deaves et al. (2010) and Cassella and Gulen (2018) for more information.

Take France's DOX index for example. The daily CAC 40 price is obtained from Bloomberg. However, each valid DOX estimation requires the past 60 quarterly returns but CAC 40 starts in July 1987. To this end, we obtain the monthly returns of another French stock market index, France CAC All-Tradable Index from Global Financial Database (GFD) and link its returns to CAC 40. We proxy for the risk-free rate using the three-month interbank rates for France (converted to monthly), obtained from FRED Economic Data from the Federal Reserve Bank of St. Louis. We then recursively estimate the DOX series using "balance" as investors' subjective expectation. Our final DOX series starts in December 1996, such that different rolling windows have cross-validation periods of 12 months.

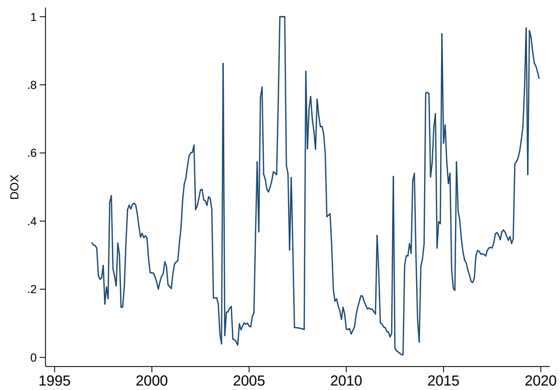
We follow the same procedure for the rest of the countries and detail the sources of data used to construct the international DOX series in Table A1. Note that for Japan, we use the principal component extracted from ZEW and ICF surveys to reduce noise in expectations.

Figure A1 plots the recursively estimated DOX series for these five countries. Each panel provides a separate series. These DOX series span from December 1996 to December 2019. As evidenced by Figure A1, there is significant heterogeneity across different markets' DOX levels. Notably, Japan's DOX index appears to be very low during the early part of the sample.

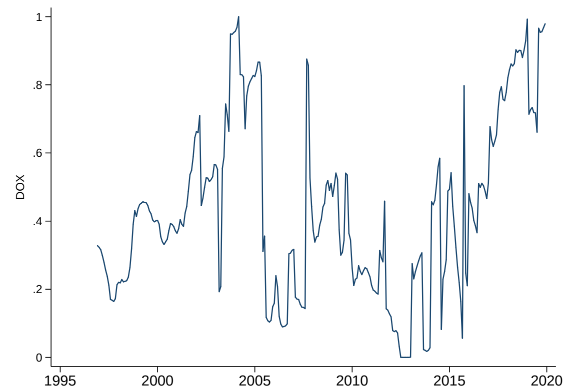
Table A1: Construction of international DOX series

This table reports the descriptions and sources of data used to construct DOX for countries other than the US. Ret^H denotes the return on the historical stock market index. FRED is the economic database from the Federal Reserve Bank of St. Louis. GFD is the Global Financial Database. ZEW is the Center for European Economic Research. The international Center for Finance (ICF) prepares the Japan One-Year Confidence Index.

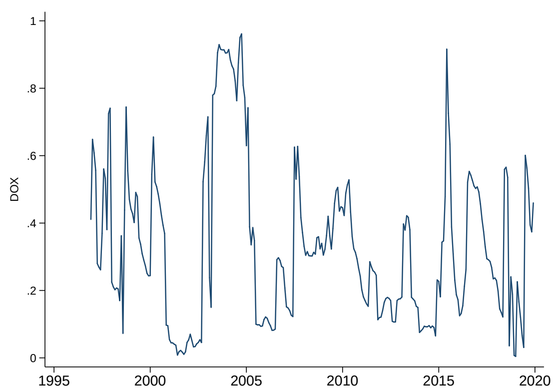
Nation	Variable	Description	Source
Germany	X	Increase - Decrease (%) in survey expectation	ZEW
	Ret	Monthly return of DAX	Bloomberg
	RV	Monthly realized variance of DAX	Bloomberg
	R^f	Three-month interbank rate as risk-free rate	FRED
France	X	Increase - Decrease (%) in survey expectation	ZEW
	Ret	Monthly return of CAC 40	Bloomberg
	RV	Monthly realized variance of CAC 40	Bloomberg
	Ret^H	Historical monthly market return (France CAC All-Tradable Index) before August 1987	GFD
	R^f	Three-month interbank rate as risk-free rate	FRED
UK	X	Increase - Decrease (%) in survey expectation	ZEW
	Ret	Monthly return of FTSE 100	Bloomberg
	RV	Monthly realized variance of FTSE 100	Bloomberg
	Ret^H	Historical monthly market return (UK FTSE All-Share Index) before January 1984	GFD
	R^f	Three-month interbank rate as risk-free rate	FRED
Italy	X	Increase - Decrease (%) in survey expectation	ZEW
	Ret	Monthly return of MIBtel	Bloomberg
	RV	Monthly realized variance of MIBtel from January 1998 and of MSCI Italy index before January 1998	Bloomberg
	Ret^H	Historical monthly market return (MSCI Italy index) before January 1998	Bloomberg
	R^f	Three-month interbank rate as risk-free rate	FRED
Japan	X	Increase - Decrease (%) in ZEW survey before June 2001 and the principal component of ZEW and ICF surveys after June 2001	ZEW/ICF
	Ret	Monthly return of Nikkei 225	Bloomberg
	RV	Monthly realized variance of Nikkei 225	Bloomberg
	R^f	Three-month certificates of deposit rate as risk-free rate	FRED



Panel A: Germany



Panel B: France



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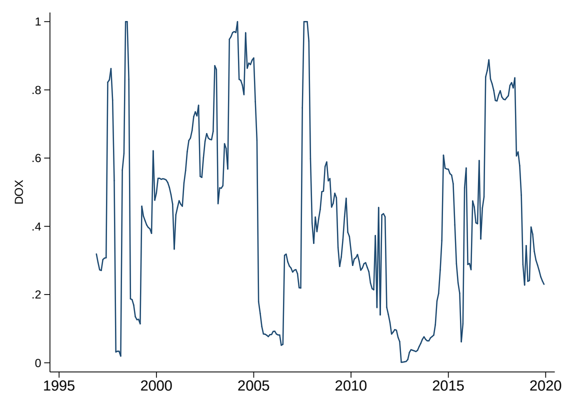
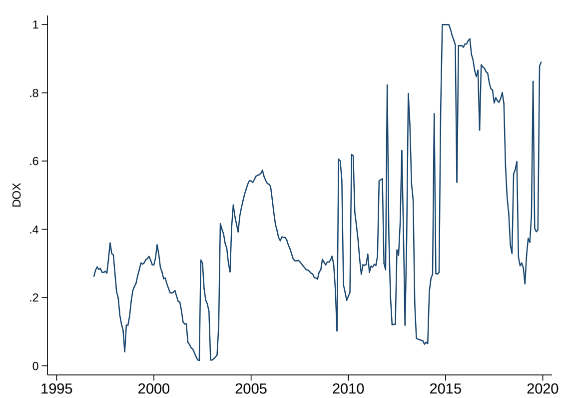


Panel D: Italy



Panel E: Japan

Figure A1: DOX for other countries. The figure shows the estimated DOX levels from the ZEW and ICF survey expectations. Panels A to E plot the estimated DOX levels for the stock market in Germany, France, the UK, Italy, and Japan, respectively. The sample period is from December 1996 to December 2019.