The Convenience Yield, Inflation Expectations, and Public Debt Growth*

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Abstract

U.S. long-term treasury debt serves the important role of safe and liquid assets in the economy, hence carrying significant convenience yields. We present two new findings relating the convenience yield to inflation and government fiscal policy. First, the convenience yield of treasury debt is negatively correlated with inflation expectations. Second, inflation expectations predict future debt-to-GDP growth at different horizons. To explain these findings, we incorporate convenience yields into a staggered-price model with an active fiscal policy. The convenience yield for long-term debt is the discounted value of future convenience service flows, thus is negatively correlated with future debt supply. Furthermore, a government deficit shock leads to both higher debt in the future as well as higher expected inflation simultaneously. The model rationalizes the two empirical findings, and provides a natural framework to study the interactions among inflation, debt growth, and cost of borrowing, particularly the convenience yield component.

Keywords: Convenience yield; inflation expectation; fiscal theory of price level; debt-to-GDP ratio
JEL codes: E31; G12; E62; E63

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1 Introduction

It has been widely argued that investors and households have a special preference for safe and liquid assets. Those assets are often money-like claims either issued by the government, such as T-bills or T-bonds (or by the private sector). These money-like assets have higher prices compared with other assets with similar cash flow characteristics, known as the convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016). The size of the convenience yield varies significantly over time and affects the borrowing cost of the government (or similar private entities). Hence, it is important to understand what affects the size of the convenience yield.

In this paper, we present two new facts connecting the convenience yield, inflation expectations and government debt. First, we show that the convenience yield is negatively correlated with inflation expectations, despite the fact that the convenience yield is measured in real terms. Figure 1 plots the simple correlation between the convenience yield and the 10-year inflation expectation. Second, we find that higher inflation expectations predict higher U.S. public debt growth rate going forward, both at the one-year horizon as well as longer time horizons.

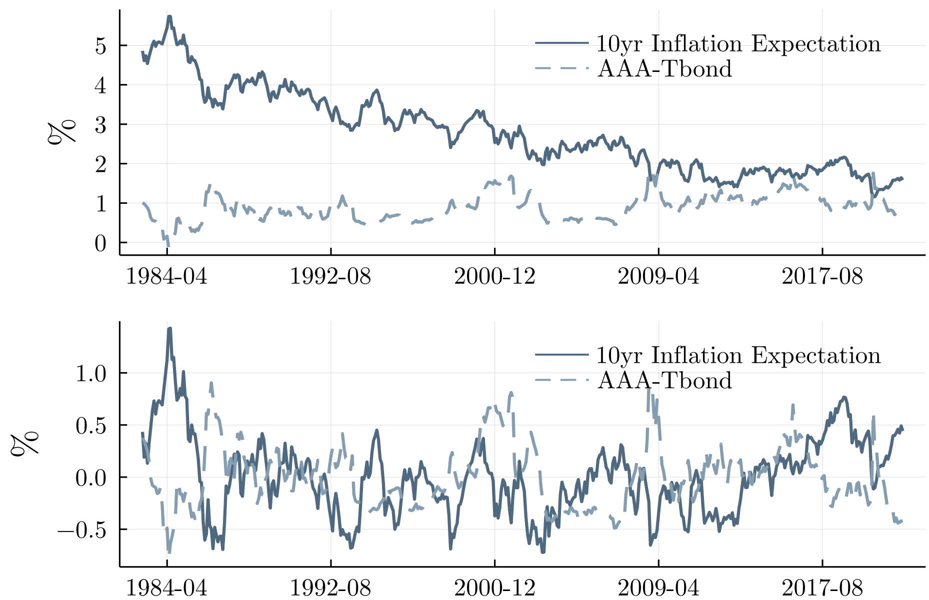

Figure 1: Negative Correlation between Inflation Expectations and AAA Spread

AAA-Tbond is the difference between Moody's Seasoned Aaa corporate bond yield and 20-year T-bond yield. Moody's Seasoned Aaa corporate bond yield is based on bonds with maturities 20 years and above. The 10-year inflation expectation series is from the Cleveland Fed.

We explain our two findings using a staggered-price model with active fiscal policy, with the additional feature that long-term government debt also provides convenience benefits. The convenience yield for any long-term asset is negatively correlated with future supply of government debt. Meanwhile, upon a government deficit shock, the government finances the deficit by both increasing borrowing from the market and partially inflating away the deficit. As a result, expected inflation rises at the same time as expected future debt increases, which leads to lower convenience yields today.

To the best of our knowledge, this is the first paper that studies the correlations between the convenience yield, inflation, and future debt growth. We also provide a unified explanation based on standard New Keynesian assumptions (with active fiscal policy in the language of Leeper (1991)). During the COVID-19 crisis, governments worldwide spent trillions of dollars to support the economies. Large amount of government spending is expected to raise future debt. At the same time, inflation expectations are steadily rising in U.S. and Europe. Given the current situation, it is particularly important to understand how government actions impact borrowing costs going forward. Our results shed light on this through the impact on the convenience yield.

We now describe our approaches in detail. Following Krishnamurthy and Vissing-Jorgensen (2012), we measure the convenience yield as the difference between AAA corporate bond rate and the T-bond rate. We also construct maturity-matched AAA-Treasury spread from ICE bond indices to rule out compounding factors related to term structure movements. We measure inflation expectations in two different ways: the baseline is the 10-year inflation swap rates available since July 2004. This is the most liquid market security on inflation expectation. To obtain longer time series data, we use the inflation expectation published by the Federal Reserve of Cleveland, which is estimated from a term structure model and is available since 1984.

We regress the change in convenience yield on change in inflation expectation, controlling for variables approximating the fundamental states of the economy, such as CBOE volatility index (VIX), growth rate of public debt, and the growth of industrial production. We perform most of our analyses at the monthly level. We find that one percentage point increase in long-run inflation expectations is associated with a 20 basis points reduction in the convenience yield for long-term assets. The estimate is statistically as well as economically significant, and is stable across different specifications. The qualitative result holds for different time horizons, and cannot be explained by either the default risk of treasury bonds or the default risk of corporate bonds.

Our result is robust to regressing the level of convenience yield on the level of inflation expectation. We verify the result holds when using option-adjusted spreads, Refcorp-Treasury spreads, Bond-CDS spreads, and Hodrick-Prescott (HP) filtered variables.[1] We also perform

[1]The Bond-CDS spread is defined as the corporate bond spread minus the yield of maturity-matched

robustness tests at the quarterly frequency, and find similar results.

Next, we show inflation expectations strongly predict future public debt growth. To include as long time horizon as possible, we use the inflation expectations from the Cleveland Fed as the baseline measurement. We regress detrended privately held debt-to-GDP growth rate on detrended inflation expectations, controlling for contemporaneous output growth, inflation risk, current debt-to-GDP ratio, and the effective Fed funds rate. We find that one percentage point increase in 10-year inflation expectations predicts a 10.3% higher growth in debt/GDP over the next year, a 34.7% higher growth over the next 5 years and 50.2% over the next decade. Our results remain significant after correcting for small sample bias, and regardless of whether we use market or par value of debt.

In the last section of the paper, we develop a model to rationalize our findings by introducing convenience yields into an otherwise standard staggered-price model under active fiscal policy. Following the literature, households derive utility from holding safe and liquid assets such as long-term government debt directly (e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Sunderam, 2014; Nagel, 2016). This value comes from either liquidity, use for transactions/collateral or special preference for extreme safe storage of value. The convenience yield on long-term government debt today is the discounted value of its convenience service in all future periods until maturity. Hence, the convenience yield today is decreasing in all future supply of government debt. Furthermore, under certain parameterization, we derive a closed form result linking the convenience yield today to current real debt-to-GDP ratio, contemporaneous demand shock, and the future path of expected government surpluses. Specifically, a negative government surplus shock predicts more debt issuance in the future, which reduces the convenience yield today. Lastly, in a wide range of models, a deficit shock which raises the real balance of government debt also yields a positive response in inflation and inflation expectations.

The positive response of inflation expectations to deficit shocks is a common result among a wide set of macroeconomic models with fiscal policy. Here we mainly focus on a particular active-fiscal model, adopted from Cochrane (2021a), and discuss other possibilities later. We assume a fiscal rule such that government surpluses positively respond to its debt value, so when debt increases, surplus automatically increases to pay the debt down. Therefore, with a deficit shock, surplus drops at first, and as debt rises, surplus eventually rises to pay back deficits. Cochrane (2021b) suggests that this policy rule gives more realistic government surplus process and more reasonable policy responses. Furthermore, (unexpected) inflation directly loads on innovations to surpluses, so upon a negative surplus shock, part of the government's debt is diluted away by higher inflation and inflation expectations. Such policy pattern is consistent with empirical findings in Berndt et al. (2012) and Cochrane (2021b).

credit default swaps (CDS). It allows us to take the default component out of corporate bonds spreads.

We generate a series of impulse responses to verify that our model can jointly explain the negative correlation between inflation expectations and the convenience yield, as well as inflation expectation's predictability of future debt-to-GDP growth rate.

We highlight that our explanation is not limited by the assumption of an active fiscal policy. In many New Keynesian models with standard active monetary policy (e.g., Ascari and Rankin, 2013; Kaplan et al., 2018; Woodford and Xie, 2022), when Ricardian equivalence breaks down,[2] it is also possible to generate such correlation patterns. To be more specific, consider a debt-financed lump-sum transfer to households in such a New Keynesian model. Such policy would raise output, inflation, as well as real debt balance in the future. As long as monetary policy does not fully offset the effect on inflation, we should see positive comovements between inflation and future debt supply. The key for our channel is that, upon a government deficit shock, the government both increases the borrowing and partially inflates away the deficit, either via intentional policy rules as we specify in our baseline model, or via the aggregate demand channels in those New Keynesian models.

Different from the existing literature that focuses on the long run trend of the convenience yield, we study its cyclical pattern and how it correlates with macroeconomic variables. In particular, we take a dynamic perspective where the convenience yield depends on expected future path of debt balances. The rest of the paper is organized as the following. The rest of this section reviews the literature. We discuss in detail sources of data and measurement in Section 2. We then show our two findings in Section 3 and Section 4, respectively. Section 5 presents the theoretical model and numerical analyses. Section 6 concludes.

1.1 Literature Review

Our paper contributes to the literature on safe assets and convenience yields (Longstaff, 2004; Bartolini et al., 2010; Coleman et al., 2011; Gorton et al., 2012; Gorton, 2017; Carlson and Wheelock, 2018). Krishnamurthy and Vissing-Jorgensen (2012) establish that the size of convenience yield is decreasing in the supply of treasuries relative to the size of the economy and Nagel (2016) looks at how the opportunity cost of holding money affects the convenience yield on all short-term near-money assets. Du et al. (2018) find the U.S. treasury bonds hold a premium internationally. Yet, a recent study He et al. (2021) provides evidence that such premium may be declining during the COVID-19 crisis. Recent literature also finds assets other than government debt may also provide convenience service (Mota, 2021; He and Song, 2021). Most papers in this literature focus on factors that contribute to the convenience yield

[2]Fiscal policies are non-Ricardian in a large set of New Keynesian models with a standard active monetary policy, such as New Keynesian over-lapping-generation models (e.g., Ascari and Rankin, 2013), models with bounded rationality (e.g, Gabaix, 2020; Xie, 2020; Woodford and Xie, 2022), and the class of heterogeneous agent New Keynesian (HANK) models (e.g., Kaplan et al., 2018).

in the current period, e.g. contemporaneous public debt supply (Krishnamurthy and Vissing-Jorgensen, 2012) and contemporaneous Fed funds rates (Nagel, 2016). Our paper instead takes a dynamic perspective – we focus on the convenience yield of long-term assets and study its forward-looking feature.

Our work is also closely related to the literature on government fiscal conditions and borrowing costs. Jiang et al. (2021a) study bond convenience yields in a currency union when countries have different fiscal conditions. Interestingly, on Eurozone bond yields, they find a larger fraction of variation is explained by convenience yields than by default spreads. This is direct evidence that convenience yield matters for government's borrowing cost. Berndt et al. (2012) argue that 9% of unexpected spending shocks is absorbed by debt revaluation, and 70% is absorbed by primary surplus. On the other hand, Jiang et al. (2021b) find neither future cash flows nor discount rates explain much variation in the debt-to-output ratio, indicating that debt-to-output ratio is highly persistent. Our mechanism does not rely on specific fiscal rules, hence does not contradict with any of these empirical patterns. Lastly, Elenev et al. (2021) explore how different monetary policies can reduce fiscal risks. Although we do not analyze the interactions between monetary and fiscal policies in this paper, the framework we adopt can be extended to study the interaction effects.

The model generates convenience yield for certain assets by assuming that households derive utility directly from holding them, similar to the common approach in the literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Sunderam, 2014; He and Song, 2021). Feenstra (1986) shows that it is equivalent to modeling a transaction cost in the budget constraint. The literature has also modeled safe asset as a result of coordination (He et al., 2019) or as information insensitive assets (Gorton and Pennacchi, 1990). We focus on the interaction of inflation expectation and price of a general group of safe assets, instead of why some assets are viewed as safe assets and others are not.

From the macro perspective, Caballero (2006) and Caballero and Farhi (2017) among others have studied the macroeconomic impact of safe asset shortages. Negro et al. (2017) argue that the low interest rates in the U.S. are primarily due to safety and liquidity premium. Christensen (2021) studies how unconventional monetary policy affects the convenience yield. But none have focused on the safe assets' connection with inflation. Our paper also builds upon the macro-finance literature that study the connection between bond return and inflation. This literature focuses on bond-stock return comovements that help to identify when inflation shock is associated with good fundamental news and when the opposite is true (e.g., Baele et al., 2010; David and Veronesi, 2013; Kang and Pflueger, 2015; Campbell et al., 2017, 2019). Our result holds before and after the bond-stock covariance sign switched, suggesting that the correlation between inflation and the convenience yield is not driven by economic fundamental movements.

Our paper is also closely connected with the literature on fiscal theories, particularly recent developments in fiscal theory of price level (e.g., Cochrane, 2021b,a). Jiang et al. (2019) propose valuation puzzles for US government debt and documents a bubble component, and Brunnermeier et al. (2020) further develop a theory of FTPL with a bubble term. In contrast, Cochrane (2021a) claims that the puzzle disappers by imposing an s-shaped response in primary surpluses to policy shocks. By utilizing the FTPL and introducing convenience benefits, our paper rationalizes the empirical observed relationship between the convenience yield and inflation expectations.

Lastly, the logic in our paper resembles the convenience yield and price backwardation in commodity pricing. Price backwardation refers to the fact the spot price of a storable commodity is higher than its near-future price. Working (1948), Brennan (1958) and Telser (1958) argue that producers and firms hold inventories because they derive convenience yield from holding physical stocks that at least partially offsets the inventory costs. Considine and Larson (2001) and Carter and Giha (2007) also show empirically such convenience yield constitutes a significant part of price backwardation. Convenience yield is inversely related to the supply of inventories, similar to our case, where convenience yield of treasuries is also inversely related to the expected future debt balance. However, there is major difference regarding the time horizon. Since the commodity literature has typically looked at the difference between spot price and near-future price, what matters is the contemporaneous supply. However, in the case of long-term treasury bonds, we argue that future expected debt supply also impacts current convenience yields.

2 Data

Before presenting our main results, we first introduce and discuss the data used in our empirical analysis. Details of the data construction as well as the sources for all variables can be found in Appendix A.

Measures of inflation expectations. We use the 10-year inflation swap rate as our baseline measure of inflation expectations for Fact 1 – inflation expectations are negatively correlated with the convenience yield. An inflation swap is a financial derivative in which one party pays a predetermined fixed rate cash flow while the other party pays a floating rate linked to the consumer price index (CPI). It provides a market-based estimation for the average inflation rate over the contract period. We choose 10-year inflation expectations as our baseline because it is the most commonly referred long-term inflation expectations, and 10-year inflation swaps are also one of the most liquid contracts in the inflation swap market. Our results are also robust at other horizons.

Inflation swap rates have many merits over alternative measures of inflation expectations, e.g., surveys or Treasury inflation-protected securities (TIPS): Inflation swaps are directly traded on financial markets, reflecting the real-time model-free inflation expectations by the market participants; it is highly liquid, thus is less prone to liquidity shocks than TIPS are (Haubrich et al., 2012; Fleming and Sporn, 2013); most importantly, unlike the break-even rates constructed from TIPS, the construction of this measure does not involve nominal yields, so it is immune from the concern that some unobserved variations in nominal yields drive both inflation expectations and the convenience yield.

However, the inflation swap rates have two limitations: First, the data series is relatively short. The inflation swap rates are only available since July 2004. This drawback is especially restrictive for our second analysis — predicting future debt growth at the 10-year horizon, leaving few observations in the sample. Second, when investors are risk-averse and inflation is stochastic, investors demand compensations for inflation risks, so the swap rate reflects not only average inflation expectations, but also the premium for the inflation risks. To overcome these limitations, we also use the inflation expectations estimated by the Federal Reserve Bank of Cleveland as an alternative measure. The Cleveland Fed utilizes the information in survey forecasts by Blue Chip and the Survey of Professional Forecastors, nominal yields, and inflation swaps to estimate an affine term-structure model. With a structure model, they are able to identify both the average inflation expectations and the inflation risk premium. Their inflation expectation series goes back to 1982 at the monthly frequency. Details of their model and estimation approach can be found in Haubrich et al. (2012).

Figure 2 plots the two measures of 10-year inflation expectations. These two series are both consistent in the long-run trends and the direction of short-run fluctuations, although they often differ in levels. Both the inflation swap rate and the Cleveland Fed estimates show the secular decline of long-run inflation expectations within the sample period until very recently, when inflation expectations picked up after the COVID-19 recession.

Measures of the convenience yield We use the AAA-Treasury spread as our baseline measurement of the convenience yield. As default risk for AAA bonds is very small, the literature has attributed the difference in the spreads to the convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012). Indeed, as shown in Table 1, the AAA-Treasury spread is on average roughly 88 basis points over our sample period. Due to the low historical default probability of AAA bonds, structural models often find it hard to explain this near 1 percentage-point spread with only credit risks (e.g., Huang and Huang, 2012; Feldhütter and Schaefer, 2018). We obtain the AAA yield from Moody's daily yield averages for Aaa

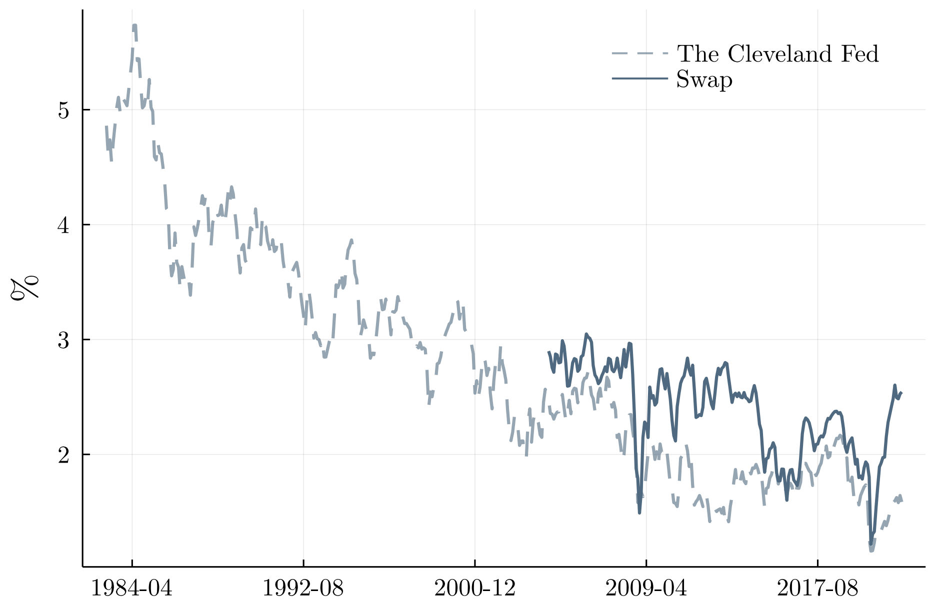

Figure 2: Two Measures of Inflation Expectations

This figure plots the two measures of 10-year inflation expectations in a monthly frequency. The black solid line indicates the data series constructed from inflation swaps, available since July 2004; the grey dashed line indicates the data series constructed by the Cleveland Fed, available since January 1982.

bonds,[3] and the Treasury yield is the market yield on U.S. Treasuries at 20-year constant maturity. Both sequences are retrieved via the FRED database.[4]

However small the default risk is in AAA bonds, default component is still a concern and could possibly correlate with inflation expectations. We address this concern about default risk using an alternative measure of the convenience yield: the Refcorp-Treasury spread. Refcorp stands for Resolution Funding Corporation, a government agency created by the Financial Institutions Reform, Recovery, and Enforcement act of 1989 (FIRREA). Different from other agency securities, Refcorps are backed by the Treasury and subject to the same taxation as Treasury bonds, so the Refcorp-Treasury spread is mostly (if not entirely) driven by convenience services carried on Treasury bonds. In Appendix B.2, we verify that our results hold when using the bond-CDS spread, which is also free of corporate default risks. We defer the discussion of data construction for the bond-CDS spread to the appendix.

[3]The formal release name is "Moody's Daily Corporate Bond Yield Averages." It is based on bonds with maturities 20 years and above.

[4]One may be concerned that the Moody's AAA yield index is constructed from bonds with maturities greater than 20 years so it has a longer duration than the 20-year Treasury. In Appendix B, we show our results are robust to using the 30-year Treasury as benchmark.

Public debt. Our measures of public debt are obtained from the Federal Reserve Bank of Dallas, which provides the market value and the par value of privately held government debt at the monthly frequency since 1942. Motivated by our model on the convenience yield, we use the market value of debt normalized by (nominal) GDP as the baseline measure for real debt burden. More discussions on this measures can be found in Section 4.

Control variables. The following variables are controlled in regressions when applicable: the industrial production, CBOE volatility index (VIX), public debt (measured in par value), the effective Fed funds rate, and the inflation risk premium. All the control variables are from the FRED Database except the inflation risk premium, which is estimated by Haubrich et al. (2012).

Descriptive statistics. Table 1 presents the summary statistics of the key variables, in levels and in first differences, all measured in annualized percentage points. $E\pi$ is our baseline measure of the 10-year inflation inflation swap rate, from July 2004 onwards. The 10-year inflation expectation averaged 2.39 percentage points throughout this sample period. Although the U.S. has not seen major inflation/deflation episodes in this century, there are still meaningful variations in the 10-year inflation expectation, as the standard deviation is 0.39 percentage points. The Cleveland Fed 10-year inflation expectation series dates back to as early as the 1980s, when the U.S. was just out of the Great Inflation and inflation expectations were as high as 5.74 percentage points. The average AAA-Tbond spread is 0.88 percentage points in the sample, and that for the Refcorp spread is smaller but still meaningful (0.48 percentage points). Both convenience yield measures peaked during the Great Recession, with 1.87 percentage points for AAA spread and 1.82 percentage points for the Refcorp spread.

3 Fact 1: Inflation Expectations Negatively Comove with the Convenience Yield

3.1 Baseline Results

We investigate the comovement of inflation expectations and the convenience yield by running the following time-series regression at the monthly frequency:

$$\Delta cy_t = \beta_0 + \beta \Delta E_t \pi_t^{t+10} + \beta_X \Delta X_t. \tag{3.1}$$

In our baseline design, we take first differences for each variable (denoted by Δ) to remove any potential trends. In Appendix B, we also use Hodrick-Prescott filter to remove the

Table 1: Summary Statistics

Variable	Time Period	Level				First Diff.			
		Mean	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
$E\pi$	04/07-21/09	2.39	0.39	1.22	3.05	-0.0	0.15	-0.64	0.52
$E\pi$ (Fed)	83/01-21/09	2.79	1.02	1.16	5.74	-0.01	0.12	-0.43	0.36
10yr Tbond	83/01-21/09	5.29	2.85	0.62	13.56	-0.02	0.29	-1.13	1.09
20yr Tbond	83/01-21/09	5.70	2.71	1.06	13.54	-0.02	0.28	-1.14	0.95
AAA	83/01-21/09	6.59	2.54	2.14	13.55	-0.02	0.23	-1.01	0.79
Refcorp	91/04-21/09	4.64	1.77	1.40	8.83	-0.02	0.26	-1.23	0.90
AAA-Tbond	83/01-21/09	0.88	0.33	-0.14	1.87	-0.0	0.13	-0.61	0.65
Refcorp-Tbond	91/04-21/09	0.48	0.24	0.15	1.82	-0.0	0.09	-0.31	0.51

Notes. This table reports summary statistics for key variables at the monthly frequency. Variables are measured in percentage points. Statistics in both levels and first differences are reported separately. For each variable, we report its available time period in our sample. $E\pi$ is our baseline measure of 10-year inflation expectations from inflation swaps. $E\pi$ (Fed) is the estimated 10-year expected inflation by the Cleveland Fed. 10yr (20yr) Tbond is the market yield on U.S. Treasuries at 10-year (20-year) constant maturity (DGS10/20 on FRED), AAA is the Moody's seasoned Aaa corporate bond yield (DAAA on FRED). Refcorp refers to the yield on RefCorp bonds with 10-year maturity. AAA-Tbond is the spread between AAA and 20yr Tbond, and Refcorp-Tbond refers to the spread between Refcorp and 10yr Tbond, to match maturities.

low-frequency movement and perform the regressions using the residuals. The results are robust.[5] cy_t denotes the convenience yield in month t. $E_t\pi_t^{t+10}$ is the 10-year average inflation expectations at month t. As mentioned before, we use inflation swaps for $E_t\pi_t$ in the baseline, and the Cleveland Fed estimates when a longer sample is needed.

X_t is a vector of control variables that generally fall into three categories. First, we control for cyclicality. Those include VIX index, industrial production, and an indicator for recessions. The VIX index is the implied volatility of S&P500 index options on CBOE, which is a widely used indicator of financial stress. Periods of financial market turmoils tend to coincide with high levels of the VIX index. Industrial production controls for the fundamental cyclical drivers of both inflation expectations and the convenience yield. The recession indicator equals one during recessions dated by the NBER business cycle dating committee and zero otherwise, to control for the opposite movement of the convenience yield and inflation expectations during recessions.

Second, we control for the inflation risk premium estimated by the Cleveland Fed (Haubrich et al., 2012). Periods with high inflation expectations often coincide with high inflation risks. Moreover, the fear for inflation risks is also directly priced in our baseline measure, the inflation swap rate. As inflation risks may affect corporate bond spreads via other channels, e.g., default probability (Kang and Pflueger, 2015), we use inflation risk premium as the proxy for the perceived inflation risks by investors.

Finally, we also control for other determinants of the convenience yield, i.e., the effective

[5] Our baseline results are also robust to regressions in levels.

Fed funds rate and the *contemporaneous* public debt supply. Krishnamurthy and Vissing-Jorgensen (2012) show that the convenience yield, which is essentially the price of convenience services on safe and liquid assets, is decreasing in the supply of public debt. [6]. Nagel (2016) further shows that at least in the short term, the elasticity of substitution between convenience services from Tbill and money is very high, so the effective Fed funds rate, as the opportunity cost of money (or the "convenience yield" of money), is highly correlated with the convenience yield of 3-month Tbills. By controlling for both the effective Fed funds rate and the contemporaneous debt supply in our regressions, we show that inflation expectations capture a new determinant of the convenience yield.

Our results are robust to further controls, such as the slope in the yield curve, included in Krishnamurthy and Vissing-Jorgensen (2012). However, as the slope is constructed from nominal yields, whose variations come largely from inflation expectations. Adding this as a control will make our estimates noisier.

Table 2 presents our baseline regression results. All columns are in first differences, as noted by Δ. Column (1) and (2) regress the 20-year Treasury yield and the AAA yield on 10-year inflation expectations, respectively. On average, one percentage point increase in 10-year inflation expectations is related with an 82 basis-points (bps) increase in the 20-year Treasury yield and a 47bps increase in AAA. These two columns address the potential concern of an extreme market segmentation: Only Treasuries get repriced when inflation expectations rise, but AAA yields do not, so the AAA-Treasury spread falls mechanically. Column (1) and (2) show that this is not the case. Both AAA and Treasuries respond to inflation expectations, but AAA yields are less sensitive.

Column (3) shows the effect on the convenience yield by regressing the AAA-Treasury spread on inflation expectations. Consistent with the pattern in Figure 1, unconditionally, one percentage point increase in 10-year inflation expectations is related to a reduction of 35bps in the AAA-Treasury spread. The effect is significant both economically and statistically. Column (4) controls for the relevant covariates. The coefficient in front of inflation expectations is stable and still highly significant.

A natural explanation to the negative correlation is the cyclicality or the "flight-to-safety" hypothesis. This argument suggests that during economic downturns, inflation expectations drop, and demand for Treasuries as safe assets increases in a flight-to-safety episode. Several pieces of evidence suggest that the flight-to-safety hypothesis is not the main driver of our results. First, whenever applicable, cyclicality and financial market sentiments are controlled for in regressions by industrial production growth and VIX. Second, we show in the next section that our results are robust when we measure the convenience yield using securities

[6] When as a control, we use the par value of public debt to avoid diluting the variation in inflation expectations.

Table 2: The AAA-Treasury Spread and Inflation Expectations

	Δ 20yr Tbond	Δ AAA	Δ AAA-Tbond			
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta E\pi$	0.82	0.47	-0.35	-0.18	-0.23	-0.27
	(0.13)	(0.13)	(0.08)	(0.07)	(0.06)	(0.06)
Measure	Swap	Swap	Swap	Swap	Fed	Fed
Controls	False	False	False	True	True	True
Sample	04/08-21/09	04/08-21/09	04/08-21/09	04/08-21/09	90/01-21/09	90/01-01/06
N	206	206	206	206	380	137
R^2	0.29	0.12	0.20	0.41	0.34	0.34

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In Column (1)-(4), inflation expectations is proxied by the 10-year inflation swap rate for the sample period from July 2004 to September 2021, and in (5)-(6) by the Cleveland Fed inflation expectations (Haubrich et al., 2012) for a longer sample period. Control variables in Column (4)-(6) include industrial production, VIX, the effective Fed funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Controls are also first differenced (or in growth rates) whenever applicable.

that are equally safe but differ in liquidity. Therefore, flight-to-safety cannot be the sole factor. Finally, as shown by Campbell et al. (2019), before 2001Q3, bond-stock correlation was actually *positive*, so long-term Treasuries were risky assets that *suffer* from a flight-to-safety along with stocks. If flight-to-safety is the cause, we shall see in the period when Treasuries were considered risky, our observed negative correlation is much dampened if not completely absent. However, this is not what we find, as suggested in Column (5) and (6). In these two columns, we switch to the Cleveland Fed estimates of inflation expectations to extend the sample back in last century. Column (5) presents the same regression in Column (4), with $\Delta E\pi$ proxied by the Cleveland Fed estimation. The sample starts from January 1990 when VIX became available. The point estimate is highly significant and comparable with that in Column (4). Column (6) restricts the sample only before June 2001, when Treasuries have positive betas on stocks. The point estimate using only the pre-2001 sample is comparable to that in the whole sample, and is also statistically significant, despite the smaller sample size.

Dynamic effects. To study the dynamic effect of the correlation, we employ a local projection design following Jordà (2005):

$$cy_{t+h} = \sum_{i=0}^{p} \beta_{i,h}(E\pi)_{t-i} + \sum_{j=0}^{p} \alpha_{j,h} cy_{t-j} + \beta_{X,h} X_t + \beta_0 + \beta_1 t, \tag{3.2}$$

where $E\pi_t$ is the 10-year inflation swap rate in period t and cy_{t+h} is the AAA-Treasury spread h-period ahead. The coefficient $\beta_{0,h}$ gives the dynamic responses in the convenience yield h-period ahead when there is a one-percentage-point increase in inflation expectations today. We include a linear trend t in the regression to avoid spurious correlation caused by common trends. The maximum lag p is chosen at two months according to the BIC criterion.

Figure 3.1 plots the estimated $\beta_{0,h}$ for horizons from 0 to 6 months together with the 95% confidence interval. The negative response in the convenience yield persists at least for 2 quarters: The estimated contemporaneous response $\beta_{0,0}$ is -0.53, and after 6 months the coefficient is still $\beta_{0,6} = -0.33$, larger than the estimate from the first-difference regression in Column (4) of Table 2. The long-lasting response in the convenience yield suggests that the disconnection between yields of AAA and Treasuries is unlikely due to market segmentation: As AAA bonds and Treasuries are both highly liquid, 6 months should be sufficient for capital to adjust.

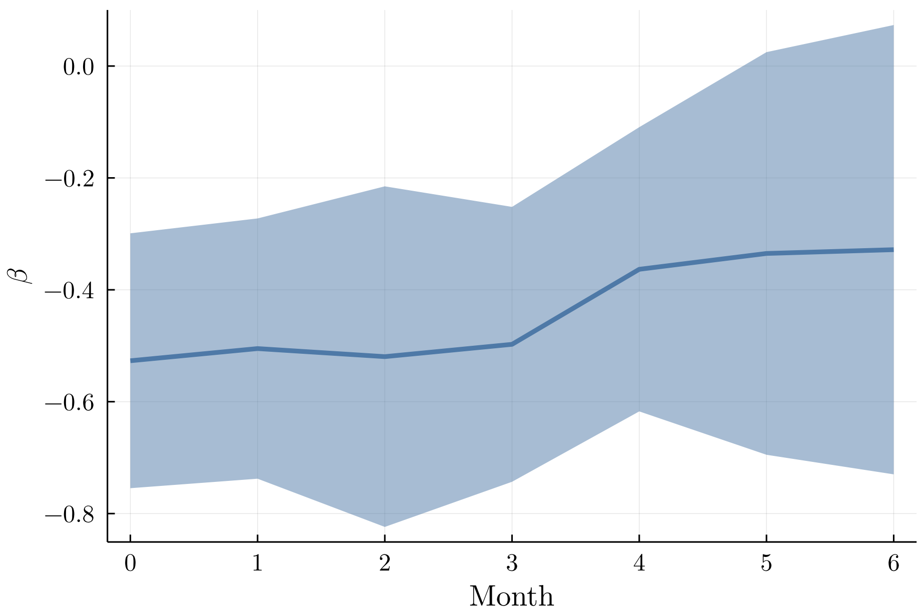

Figure 3: Local Projection of AAA-Tbond on Inflation Expectations

This figure plots the estimated $\beta_{0,h}$ from Eq. (3.2), i.e., the impulse response of cy_t, proxied by the AAA-Tbond spread, to a one-unit innovation to $E\pi$, proxied by the 10-year inflation swap rate. The shaded area represents 95% confidence interval with Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)).

3.2 Alternative Measures of the Convenience Yield

Though very small, the default risk of AAA corporate bonds is still a component of the AAA-Treasury spread. To show that our results are not solely driven by time-varying default risks, we turn to other default-risk-free measures of the convenience yield. We discuss the

Refcorp-Treasury spread in detail here, and leave other measures to the appendix.

Refcorp stands for Resolution Funding Corporation, a government agency created by the Financial Institutions Reform, Recovery, and Enforcement act of 1989 (FIRREA). Unlike other agency bonds that are often still subject to small credit risks, Refcorp is fully guaranteed by the Treasury under FIRREA, so they literally have the same credit risk as Treasury bonds. Refcorp bonds are also subject to the same taxation as Treasuries.[7] Therefore, the spread between Refcorp bonds and Treasuries should be mostly, if not all, attributed to the convenience premium on Treasuries.

Table 3 presents regressions of the Refcorp-Treasury spread on inflation expectations, following the same design as in Eq. (3.1). To match the horizon of inflation expectations, both the Refcorp and the Treasury have a maturity of 10 years. The estimated coefficients are highly significant across columns. In Column (1), we regress the Refcorp-Treasury spread univariately on inflation expectations measured by the inflation swap rate. The point estimate implies that a 1 percentage point change in inflation expectations translates into a reduction of 13bps (std. err. 4bps) in the Refcorp-Treasury spread, or 56% of its long-run standard deviation. Column (2) adds controls to the same regression. After controlling for covariates, the point estimate suggests a reduction of 10bps (or 35% of std.) in the Refcorp-Treasury spread for a one percentage point increase in inflation expectations. Column (3) extends the sample back to April 1993 using the Cleveland Fed inflation expectations. The results are similar as in Column (2).

During recessions, especially severe financial turmoils, investors may prefer to hold highly-liquid securities such as Treasuries rather than less-liquid but equally safe Refcorp bonds, increasing the convenience yield on Treasuries in a "flight-to-liquidity." In Column (4), we run the same regression using only the non-recession sample. The coefficient is stable and if anything, it becomes stronger.

Additional robust checks. In Appendix B, we perform further robustness checks and rule out several other potential theories. In B.2, we use bond-CDS spreads as an alternative default-risk-free measure of the convenience yield, and it is also negatively correlated with inflation expectations. Furthermore, our result is not driven by maturity differences in corporate bonds and treasury bonds. In B.1, we use treasury bonds with different maturities as the benchmark yield to make sure the variation is not driven by differences in time-to-maturity. We also use the optioned-adjusted spread to account for the callable options on corporate bonds. Finally, our results are stable when we filter out the low-frequency movement using the Hodrick-Prescott filter in B.3.

[7]See Longstaff (2004) for a more thorough discussion on the institutional details.

Table 3: The Refcorp-Treasury Spread and Inflation Expectation

	Δ Refcorp - TBond			
	(1)	(2)	(3)	(4)
$\Delta E\pi$	-0.13	-0.10	-0.16	-0.20
	(0.04)	(0.02)	(0.06)	(0.06)
Measure	Swap	Swap	Fed	Fed
Controls	False	True	True	True
Sample	04/08-21/09	04/08-21/09	91/04-21/09	No Recession
N	206	206	365	336
R^2	0.04	0.15	0.11	0.06

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In Column (1)-(2), inflation expectations are proxied by the 10-year inflation swap rate for the sample of 2004-2021, and in (3)-(4) by inflation expectations estimated by the Federal Reserve of Cleveland (Haubrich et al., 2012). Control variables for Column (2)-(4) include industrial production, VIX, the effective Fed funds rate, contemporaneous public debt supply, the inflation risk premium, and the recession indicator. Controls are also first differenced (or in growth rates) whenever applicable.

4 Fact 2: Inflation Expectations Positively Predict Future Government Debt Growth

We next provide evidence for the forecastability of future debt growth using inflation expectations. To include a longer sample size with sufficient variations in the debt-to-GDP ratio, we use inflation expectations from the Cleveland Fed, which dates back to 1982. Since GDP is reported quarterly, we base our analysis at the quarterly frequency in this section.[8]

Figure 4 plots the level of debt/GDP since 1982 and the ten-year forward cumulative growth rates, measured by the market value and the par value respectively. We only include privately held debt in both measures. During this period, the debt burden of the U.S. government has been steadily increasing, from only 20% of GDP in 1982 to more than 80% after the forceful fiscal response to the COVID-19 shock in 2020. The market-value measure shows slightly larger cyclical patterns than the par-value measure, reflecting time-varying discount rates through business cycles. Nevertheless, as made clear by Panel (b), the differences in 10-year growth rates are generally small. Therefore, over a long horizon such as 10 years, most of the variations in the market value are from the variations in the par value, i.e., from the active management of the fiscal authority.

In the remaining part of this section, we use the market value of privately held debt

[8] Our results are also robust at the monthly level where GDP is linearly interpolated in-between quarter ends.

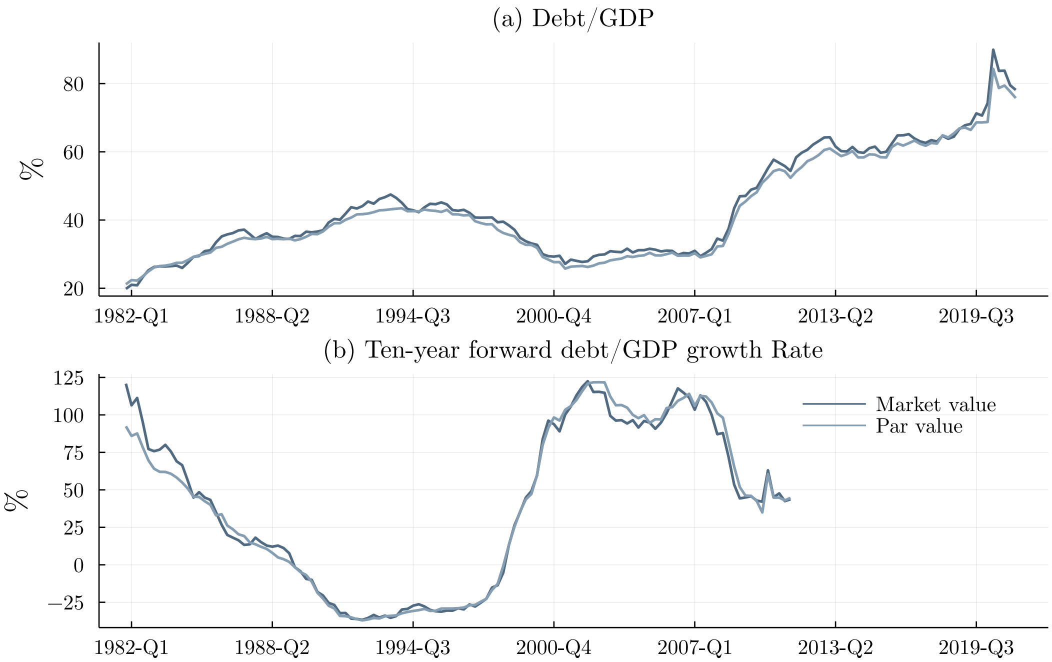

Figure 4: Debt/GDP and Foward Growth Rates

Panel (a) presents the raw series of the debt/GDP, where debt is measured by privately held public debt, both in market value and in par value. Panel (b) presents two measures of the ten-year forward growth rates.

divided by GDP as our baseline measure for debt-to-GDP ratio. This choice is motivated by the economics of the convenience yield: Households gain convenience services such as collateral values or regulatory reliefs from the market value of the public debt rather than the par value. The demand for such services scales with the level of economic activities, so we scale all the measures by GDP. One should note that there are indeed endogeneity concerns related to this measure, since the market value is determined by discount rates, which contain both the convenience yield and inflation. To formally address this concern, we measure the debt growth by the growth rate of par values, normalized by CPI. The results are presented in Appendix C.1 and are very similar.

Figure 5 plot together the inflation expectations (left y-axis) and debt/GDP growth (right y-axis), for one-year-ahead (Panel a) and ten-year-ahead (Panel b), respectively. All series are linearly detrended to avoid correlation caused by common trends. As shown in Panel (a), one-year inflation expectations predict the one-year growth rate of debt/GDP at a high frequency quite well. An increase in one-year inflation expectations is often accompanied by a higher debt/GDP growth one-year forward. Predicting the ten-year debt/GDP growth rate, on the other hand, is obviously much more demanding. However, although the short-term correlation is not as visible as that in Panel (a), over a longer horizon, periods when inflation expectations are high are often associated with higher debt/GDP growth in the next 10 years.

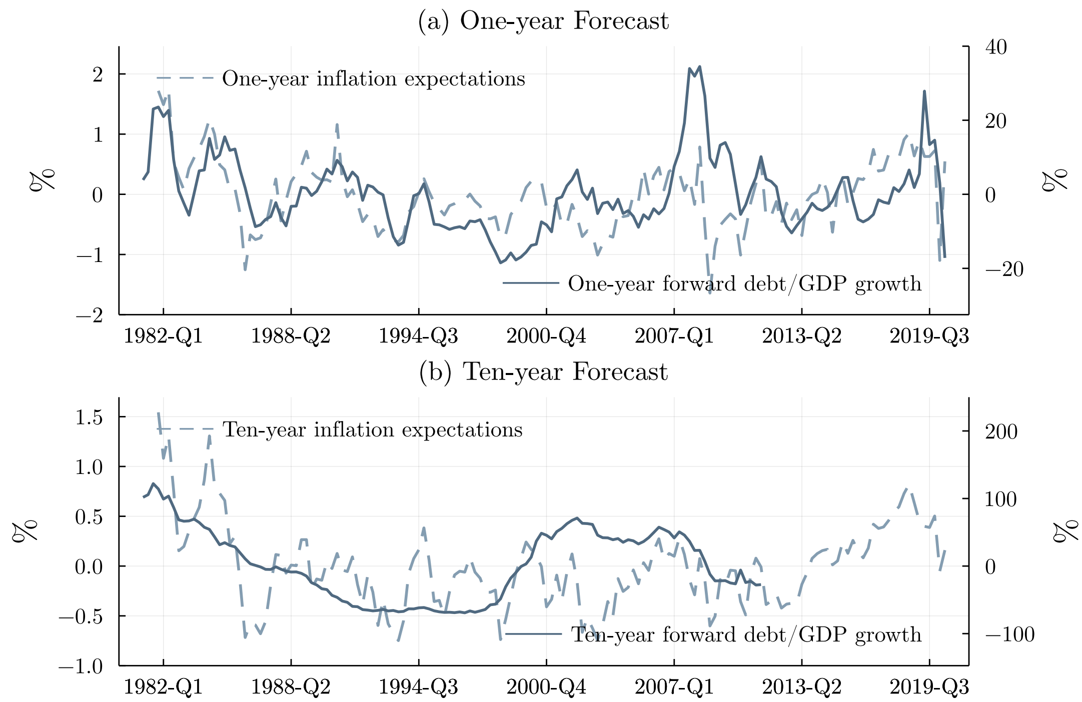

Figure 5: Inflation Expectations and Future Debt Growth

The light dotted lines plot the one-year (ten-year) inflation expectations at the end of each quarter, and the dark solid lines plot the one-year (ten-year) forward growth rate of the debt/GDP ratio. Debt is measured by the market value of privately held public debt. All sequences are linearly detrended to avoid common trends.

We now formally test the forecastability using regressions. The specification is as follows:

$$\Delta_t^{t+h}\left(\frac{V}{Y}\right) = \beta_h E_t \pi_t^{t+10} + \beta_X X_t + \epsilon_t, \tag{4.1}$$

where $\Delta_t^{t+h}(x_t)$ denotes the growth rate of x_t from t to $t+h$, $E_t\pi_t^{t+10}$ denotes 10-year inflation expectations,[9] and X_t denotes control variables, including contemporaneous output growth, inflation risk premium, the debt-to-GDP ratio, and the effective Fed funds rate.

Table 4 presents the coefficient β_h for horizons equal to 1 year, 5 years, and 10 years. Column (1)-(3) present the results of univariate regressions without the controls X_t. A one percentage point increase in 10-year inflation expectations predicts a 10.3% higher growth in debt/GDP over the next one year, a 34.7% higher growth over the next 5 years, and 50.2%

[9] We could match the horizons of inflation expectations to the debt/GDP ratio and the results are similar. We use 10-year inflation expectations to be consistent with previous sections.

over the next decade. All results are statistically significant ($p < 0.01$). Inflation expectations alone explain around 18% variation in debt/GDP growth across different horizons. Column (4)-(6) add in controls and the estimates are robust, and even slightly larger.

We further conduct a panel of robustness checks. We leave the full reports to the appendix and briefly summarize the main messages here. As pointed out by Stambaugh (1999) and Boudoukh et al. (2021), predictive regressions in a finite sample are subject to the small sample bias. We correct the bias using the formula by Boudoukh et al. (2021), and present the unbiased estimates in Appendix C.2. The differences are small. Appendix C.1 presents the results where debt growth is measured by the real growth rate of the par value of privately held debt. The results remain unchanged. Appendix C.3 uses the realized inflation as a proxy for inflation expectations to extend the sample back to 1947. Consistent with Cochrane (2021b), we find that the realized inflation also predicts future debt growth. However, when put in regressions together with inflation expectations, the realized inflation loses its predictive power while the coefficients on inflation expectations are not affected.

Table 4: Inflation Expectations Predict Future Debt/GDP Growth

	1yr	5yr	10yr	1yr	5yr	10yr
	(1)	(2)	(3)	(4)	(5)	(6)
$E\pi$	10.30	34.69	50.20	14.60	49.38	51.69
	(2.14)	(7.77)	(17.53)	(2.63)	(12.54)	(14.86)
Sample	81Q4-20Q2	81Q4-16Q2	81Q4-11Q2	81Q4-20Q2	81Q4-16Q2	81Q4-11Q2
Control	False	False	False	True	True	True
N	155	139	119	155	139	119
R^2	0.18	0.18	0.18	0.30	0.48	0.82

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. The predicted variables are future debt/GDP cumulative growth rates, where debt is measured by the market value of privately held public debt. Both variables are linearly detrended. Horizons are indicated in each column. The predictor variable is 10-year inflation expectations estimated by the Cleveland Fed (Haubrich et al., 2012). Column (1)-(3) performs the univariate prediction, and Column (4)-(6) control for output growth, the par value of debt to GDP, the inflation risk premium, and the effective Fed funds rate, all at time t.

5 A Model of the Convenience Yield and Inflation Expectations

To rationalize the relationship between the convenience yield on long-term government-issued bonds and long-run inflation expectations as showed in the empirical sections, we build a staggered-price DSGE model in which some assets such as Treasury securities also provide convenience benefits in the spirit of Krishnamurthy and Vissing-Jorgensen (2012). We regard assets with such a feature as convenience assets, and it is this feature that creates a wedge between the return on convenience and non-convenience assets, defined as the convenience yield.

We first show that the negative relationship between the convenience yield and inflation expectations holds generally, in the sense that it does not rely on the specific form of fiscal/monetary policy rules. That is — we derive a general expression of the convenience yield without specifying policy rules, and the expression implies that even in those New Keynesian models with a standard active monetary policy, as long as Ricardian equivalence breaks down,[10] the convenience yield is negatively correlated with inflation expectations.

We then close the model by assuming an active fiscal policy ("active" refers to the language in Leeper (1991)), following the approach developed in Cochrane (2021a). Given that our focus is the convenience yield on long-term Treasury securities, as suggested by Cochrane (2021b,a), this framework under active fiscal policy has the advantage of allowing for a more realistic government debt and surplus process and thus is able to produce more reasonable responses to policy shocks.

The economy consists of four types of agents: representative households, firms (owned by the households) that produce output, a fiscal authority (the government) that specifies fiscal policy – a rule of collecting primary surpluses which implicitly controls the supply of government bonds, and a central bank that sets the short-term riskless nominal interest rate. The only asset that provides convenience benefits is the long-term government-issued bonds.[11]

To model the long-term government bonds that provide convenience benefits, we follow the approach developed in Woodford (2001) and Carlstrom et al. (2017) by assuming only one type of bond issued by the government – perpetuities with coupon payments of 1, k, k^2, etc. Let Q_t^B denote the time-t price of a new issue. Given the time pattern of the perpetuity payment, the new issue price Q_t^B summarizes the prices at all maturities, e.g., kQ_t^B is the time-t price of the perpetuity issued in period $t-1$. The duration on these bonds is given by $h = (1-k)^{-1}$ and (gross) yield to maturity is given by $R_t^B = (Q_t^B)^{-1} + k$. The advantage of modeling such decaying coupon bond is that it abstracts from the heterogeneous maturities of outstanding government bonds and one only needs to track the total outstanding bonds rather than individual issues.

Let N_t denote the number of new perpetuities issued in time-t. In time-t, the government's

[10]For instance, the models in which Ricardian equivalence does not hold include New Keynesian overlapping generation (OLG) models, New Keynesian models with bounded rationality, and the class of HANK models.

[11]Short-term near-money assets could also provide convenience benefits (e.g., Greenwood et al., 2015; Nagel, 2016), but they are not the focus of this paper. Nevertheless, the way we model long-term convenience assets can also nest the analyses of short-term convenience asset as discussed in section 5.2.

nominal liability on past issues is given by

$$B_{t-1} = \sum_{j=1}^{\infty} k^{j-1} N_{t-j}.$$

Then we can write the newly issued debt in time-t as

$$N_t = B_t - kB_{t-1}.$$

Thus the budget constraint of government is given by

$$\frac{B_{t-1}}{P_t} = S_t + \frac{Q_t^B (B_t - kB_{t-1})}{P_t}, \tag{5.1}$$

where S_t is the real value of surplus and P_t is the aggregate price level of household consumption. The left hand side in equation (5.1) is the real coupon liability from all past issuances due in period t, which needs to be financed either through raising surplus or issuing new liabilities.[12] Here for simplicity, we abstract from government spending and distortionary taxes and assume that the government collects surpluses only through lump-sum taxes from households.

We can rewrite the government budget constraint (5.1) in a form of the real value of outstanding debt, i.e.,

$$\frac{V_{t-1}}{\Pi_t} \frac{1 + kQ_t^B}{Q_{t-1}^B} = S_t + V_t, \tag{5.2}$$

where $V_t \equiv Q_t^B B_t / P_t$ is the real (market) value of all outstanding debt and $\Pi_t = P_t / P_{t-1}$ is the inflation in time t.

5.1 Households

An infinitely-lived representative household maximizes the objective

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) + \mu(V_t)],$$

where C_t is the consumption and V_t is the real value of convenience assets (i.e., long-term government bond).

[12] We assume that the government is always capable of paying its liabilities, so that we can abstract from default issues.

The budget constraint of the household expressed in real terms is given by

$$\frac{Q_t^B(B_t - kB_{t-1})}{P_t} + \frac{Q_t^D(D_t - kD_{t-1})}{P_t} + \frac{L_t}{P_t} + C_t = Y_t + \frac{B_{t-1}}{P_t} + \frac{D_{t-1}}{P_t} + \frac{L_{t-1}R_{t-1}}{P_t} - S_t + \Phi_t,$$

where D_t is the holdings of perpetuities that have the same payment structure with government bonds but provide no convenience benefits, Q_t^D is the new issue price of this asset, L_t is the holdings of short-term riskless nominal bonds with an (nominal) interest rate R_t, Y_t is the household's real labor income, S_t is the real value of lump-sum taxation paid to the government, and Φ_t is the (real) lump-sum dividends transferred from firms to households. In particular, the only purpose of introducing D_t here is to calculate the convenience yield between the two long-term assets which share the same payment structure but one has convenience benefits while the other does not. We assume the net supply of L_t and D_t are zero.[13]

The household's first-order condition with respect to consumption yields the Euler equation

$$1 = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{R_t}{\Pi_{t+1}}]. \tag{5.3}$$

By applying first-order approximation, the household's first-order conditions with respect to real asset balances B_t and D_t yield

$$\log R_t = \frac{\mu_v(V_t)}{u_c(C_t)} + E_t \log(\frac{kQ_{t+1}^B + 1}{Q_t^B}), \tag{5.4}$$

and

$$\log R_t = E_t \log(\frac{kQ_{t+1}^D + 1}{Q_t^D}). \tag{5.5}$$

Details of deriving equations (5.4)-(5.5) can be found in Appendix D.1. The left-hand side of these two equations represents the marginal cost of holding one-period riskless nominal bonds (which is determined by the central bank), while the right hand side represents the marginal benefit, in terms of utility, of holding long-term assets with and without convenience benefits.

5.2 The Convenience Yield on Long-term Government Bonds

Before closing the equilibrium, we first derive an expression of the convenience yield on long-term government-issued assets that only originates from the optimality conditions of households and the government's budget constraint. Here in this section, though we have not introduced the supply side yet, we will utilize the market clearing condition that total

[13] Assets L_t and D_t are traded only among the households.

output Y_t of the economy equals households' total consumption C_t to simplify expressions. This must hold as a market clearing condition in this closed economy, once we further introduce the supply side in the following sections.

As conventional in this literature (e.g., Nagel, 2016), it is standard to derive a closed-form expression of the convenience yield by log-linearizing the equilibrium conditions around the steady state to a first-order approximation. We express the linearized equations in terms of log-deviations from the steady-state of the various variables by lowercase letters, using the notations below

$$i_t \equiv \log(R_t/\bar{R}), \qquad v_t \equiv \log(\frac{V_t}{Y_t}/\frac{\bar{V}}{\bar{Y}}), \qquad y_t \equiv \log(Y_t/\bar{Y}),$$

$$q_t^B \equiv \log(Q_t^B/\bar{Q}^B), \qquad q_t^D \equiv \log(Q_t^D/\bar{Q}^D), \qquad i_{t+1}^B \equiv \log(\frac{kQ_{t+1}^B + 1}{Q_t^B}/\frac{k\bar{Q}^B + 1}{\bar{Q}^B}).$$

Given the market clearing condition $Y_t = C_t$, by log-linearizing equation (5.4) and (5.5), we have

$$i_t = -\gamma\psi^{-1}[v_t + y_t] + \gamma\sigma^{-1}y_t + kE_t q_{t+1}^B - q_t^B, \tag{5.6}$$

and

$$i_t = kE_t q_{t+1}^D - q_t^D, \tag{5.7}$$

where $\psi \equiv -\mu_v(\bar{V})/(\mu_{vv}(\bar{V})\bar{V}) > 0$ is the inter-temporal elasticity of substitution (IES) of household convenience benefits, $\sigma \equiv -u_c(\bar{C})/(u_{cc}(\bar{C})\bar{C}) > 0$ is the inter-temporal elasticity of substitution of household expenditure, and $\gamma \equiv \mu_v(\bar{V})/u_c(\bar{C})$ measures the marginal benefits of holding one more unit of convenience assets relative to the marginal benefits of consumption at the steady state. Details of deriving expression (5.6) and (5.7) can be found in Appendix D.2.

Note that the nominal yields of long-term bonds with and without convenience benefits are given by

$$R_t^B = (Q_t^B)^{-1} + k, \qquad R_t^D = (Q_t^D)^{-1} + k,$$

respectively. By log-linearization, we then have

$$r_t^B = -(\bar{Q}^B)^{-1}q_t^B, \qquad r_t^D = -(\bar{Q}^D)^{-1}q_t^D, \tag{5.8}$$

where $r_t^B \equiv \log(R_t^B/\bar{R}^B)$ and $r_t^D \equiv \log(R_t^D/\bar{R}^D)$.[14]

[14]Here we have used the steady-state relationship $\bar{R}^B = (k\bar{Q}^B + 1)/\bar{Q}^B$ and $\bar{R}^D = (k\bar{Q}^D + 1)/\bar{Q}^D$, and assume $\bar{R}^B - 1$ and $\bar{R}^D - 1$ are at the scale of first order.

Thus by combining (5.6)-(5.8), the convenience yield $cy_t \equiv r_t^D - r_t^B$ is given by

$$cy_t = kE_t(cy_{t+1}) + (1-k)[-\gamma\psi^{-1}v_t + \gamma(\sigma^{-1} - \psi^{-1})y_t], \tag{5.9}$$

where we have imposed the assumption, to a first-order approximation, $1/\bar{Q}^B \approx 1/\bar{Q}^D$. Details of deriving (5.9) can be found in Appendix D.2. Since the decaying speed of perpetuity coupon payment satisfies $0 < k < 1$, (5.9) then implies Proposition 1.

Proposition 1 (Impact of government bond supply on long-term convenience yield) *The convenience yield on government-issued perpetuities is equal to the discounted sum of all current and expected future government real debt balances (scaled by aggregate output) plus the discounted sum of all current and expected future output gaps, i.e.,*

$$cy_t = (1-k)\sum_{j=0}^{\infty} k^j \gamma E_t[-\psi^{-1}v_{t+j} + (\sigma^{-1} - \psi^{-1})y_{t+j}]. \tag{5.10}$$

Proposition 1 can nest the analyses for both the long-term and short-term convenience yield simultaneously by simply varying the value of k. When k is close to one, (5.10) is equivalent to the long-term convenience yield modeled in Krishnamurthy and Vissing-Jorgensen (2012) but under the assumption of separable utilities for the convenience benefits and consumption. When $k = 0$, the perpetuities collapse to one-period riskless nominal bonds, and thus it corresponds to the analyses of short-term convenience yield in those models like Krishnamurthy and Vissing-Jorgensen (2012), Greenwood et al. (2015), and Nagel (2016).[15]

We can see from (5.10) in Proposition 1 that higher future government real debt balances (i.e., debt-to-GDP ratios) leads to a lower convenience yield on long-term government bonds. If the long-run inflation expectation positively predicts future debt-to-GDP ratio, which is documented as a stylized fact in Section 4, the long-run inflation expectation will then be negatively correlated with the long-term convenience yield, as found in Section 3, via the channel of future government real debt balances.

Next, to better understand the right hand side of the convenience yield expression (5.10) in Proposition 1, we further solve for the determination of future government real debt balances. For simplicity, assume that the IES of the convenience benefits $\mu(\cdot)$ and of consumption utility $u(\cdot)$ are both equal to one, i.e., $\sigma = \psi = 1$. Proposition 1 then implies the long-term convenience yield to be a function of only current and expected future government real debt balances.

[15] The expression of convenience yield in (5.10) with $k = 0$ corresponds to a special case in Nagel (2016) in which money and short-term Treasury securities are not substitutes.

Note that after log-linearization, the Euler equation (5.3) yields

$$y_t - u_t^d = E_t(y_{t+1} - u_{t+1}^d) - \sigma(i_t - E_t\pi_{t+1}), \tag{5.11}$$

where $u_{t+1}^d = \rho_d u_t^d + \epsilon_{t+1}^d$ denotes aggregate demand shock and ϵ_{t+1}^d is an i.i.d. innovation with mean zero,[16] and the government budget constraint (5.2) yields

$$\rho v_{t+1} = v_t + i_{t+1}^B - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t), \tag{5.12}$$

where i_{t+1}^B is the log-deviation of one-period nominal return on government-issued bonds and s_{t+1} is the scaled real primary surplus to output ratio, and $\rho \equiv e^{-\bar{i}^B} < 1$ is a constant.[17] Then by plugging (5.6) and (5.11) into (5.12), and also noting that $\sigma = \psi = 1$, we get the expected government real debt balance in any time $t+j+1$ satisfying the following inter-termporal evolution equation

$$E_t v_{t+j+1} = \frac{1}{\rho} E_t[(1+\gamma)v_{t+j} - s_{t+j+1} - (u_{t+j+1}^d - u_{t+j}^d)] \tag{5.13}$$

for any $j \geq 0$. Details of deriving (5.13) can be found in Appendix D.3.

Thus we can obtain the expected future real debt balance of the government $E_t v_{t+j+1}$ by iterating (5.13) from time $t+j+1$ to time t. It equals the summation of three terms: current debt-to-GDP ratio v_t, the expected future flows of government surpluses $\{E_t s_{t+n}\}_{n=1}^{j+1}$, and the contemporaneous demand shock u_t^d.[18] Then, by substituting v_{t+j+1} in the expression of the convenience yield (5.10), we turn Proposition 1 into Proposition 2.

Proposition 2 (Impact of government surpluses on long-term convenience yield) *Given the inter-temporal elasticities of substitution of convenience benefits and of consumption utility being one, i.e., $\sigma = \psi = 1$, the convenience yield on government-issued perpetuities is equal to the summation of current real debt balance, contemporaneous demand shock, and the discounted sum of all expected future government surpluses, i.e.,*

$$cy_t = \eta_v v_t + \eta_d u_t^d + \sum_{j=1}^{\infty} \eta_{s,j} E_t s_{t+j}, \tag{5.14}$$

[16] Here the demand shock u_t^d parameterizes disturbances to the urgency of consumption, i.e., $\log(u_c(C_t;\xi_t)/u_c(\bar{C};\bar{\xi})) = -\sigma^{-1}(c_t - u_t^d)$, where ξ_t is the exogenous preference shock of households.

[17] Details of deriving (5.12) follow the appendix of Cochrane (2021b).

[18] Details of the expression of $E_t v_{t+j+1}$ can be found in Appendix D.3.

where the constant coefficients are given by

$$\eta_v \equiv -\gamma(1-k)\sum_{j=0}^{\infty} k^j (\frac{1+\gamma}{\rho})^j < 0,$$

$$\eta_d \equiv -\gamma(1-k)\frac{1-\rho_d}{\rho\rho_d-(1+\gamma)}\sum_{j=1}^{\infty} k^j [\rho_d^j - (\frac{1+\gamma}{\rho})^j] < 0,$$

$$\eta_{s,j} \equiv \gamma\frac{1-k}{\rho}\sum_{n=j}^{\infty} k^n (\frac{1+\gamma}{\rho})^{n-1} > 0.$$

Proposition 2 indicates that the convenience yield on long-term government bonds is only affected by three parts: the current real value of debt-to-GDP, the contemporaneous demand shock, and future government surpluses. Conditional on current real debt-to-GDP ratio and the contemporaneous demand shock, an increase in expected future government surplus will increase the convenience yield. Then by controlling the current debt-to-GDP ratio and proxies for the current demand shock as in the empirical exercises in Section 3, the long-run inflation expectation must be correlated with the long-term convenience yield through changes in expected future government surpluses.

One may be concerned that the unit inter-temporal elasticity assumption in Proposition 2 is too restrictive. First, it is not unconventional to set the values of IES being one. For instance, Nagel (2016) adopts a log utility function of convenience benefits. Also, a large literature has estimated the IES of consumption with an estimate not far from one, but ranging from values smaller than one to larger than one.[19] Second, we relax this assumption after lay out the full structural model, and the numerical exercises show that a negative shock on government surpluses (so-called deficit shock) will result in a positive response in inflation and inflation expectation, as well as a negative response in the long-term convenience yield. Thus the association between long-run inflation expectation and long-term convenience yield via the channel of government surpluses generally holds.

Proposition 2 further suggests that if inflation expectation is negatively associated with the discounted sum of expected future flow of government surpluses, one would observe a negative relationship between inflation expectations and the long-term convenience yield on government-issued bonds, as documented in Section 3. This result is independent from any policy rules or the specification of supply side, since Proposition 2 only originates from the optimality problem of households and government budget constraint.

In the following sections, we illustrate the negative relationship between inflation expectation and expected future flow of government surpluses by closing the model with staggered-

[19] Among others, see Bansal and Yaron (2004), Gruber (2013), and Best et al. (2020).

price firm production and active fiscal policy following Cochrane (2021a). Note that this negative relationship holds more generally, not specific to the model assumption of an active fiscal policy. In those New Keynesian models in which Ricardian equivalence breaks down with a standard active monetary policy, a deficit shock in government surpluses will also lead to an increase in inflation and inflation expectations and a negative response in the long-term convenience yield.[20]

5.3 Production

To capture the dynamics of inflation and inflation expectation in a more realistic approach, we model firm production facing monopolistic competition and subject to sticky prices as in a standard New Keynesian model (see, e.g., Woodford, 2003, chap. 3.2). We assume a continuum of firms indexed by $j \in [0,1]$. Each firm produces a differentiated good, but follows the same production technology given by

$$Y_t(j) = A_t N_t(j)^{1-\phi},$$

where A_t represents the level of production technology and follows an exogenous process, and $N_t(j)$ is the labor input, and ϕ is a constant.

The representative household purchases differentiated outputs from firms for final consumption, following a CES aggregator

$$C_t \equiv \left(\int_0^1 C_t(j)^{1-\frac{1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution across differentiated firm outputs and $C_t(j)$ represents the quantity of good j consumed by the household in period t. Then the optimality problem of the household to maximize final consumption C_t for any given level of expenditures yields a demand function of C_j given by

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t \tag{5.15}$$

for all $j \in [0,1]$, where $P_t(j)$ is the price of good j.

Firms face the same demand schedule given by (5.15) and take the aggregate price level of consumption composite P_t and aggregate consumption C_t as given. Since the firms are owned by households, they transfer their profits, if there is any, to households via dividends.

Following Calvo (1983), we assume that each firm could reset its prices only with proba-

[20] Here we refer to the case in which the monetary policy is not set to fully offset the effects of shocks on government surpluses, such as when the monetary policy follows a Taylor-type rule.

bility $1-\lambda$ in any given period, independently from the time since its last adjustment. Then in aggregation, a fraction of $1-\lambda$ fraction of firms reset their prices in each period, while the rest have to keep their prices unchanged.

Together with the market clearing condition $C_t(j) = Y_t(j)$, the optimal pricing problem of firms yields the New Keynesian Phillips curve

$$\pi_t = \kappa(y_t - y_t^n) + \beta E_t \pi_{t+1}, \tag{5.16}$$

where parameter κ is the slope of Phillips curve and y_t^n is the natural level of output (a linear function of the productivity shock a_t). Here we have implicitly assumed a dis-utility of labor in the household's utility function as in the standard New Keynesian literature. It affects nothing else except showing up in deriving the Phillips curve. Since (5.16) is a standard result in the literature, we do not detail the derivation here.[21] One can refer to Woodford (2003, chap. 3.2), or Galí (2015, chap. 3), for more details.

5.4 Fiscal and Monetary Policies

The government sets the fiscal policy via a rule of collecting primary surpluses, while the central bank sets the rule of nominal interest rate on short-term riskless bonds. As commonly adopted in the monetary literature and in the literature of Fiscal Theory of Price Level (FTPL), we assume the fiscal and monetary policies following a simple linear form. That is

$$i_t = \theta_{i\pi}\pi_t + \theta_{iy}y_t + u_t^i \tag{5.17}$$

for the monetary policy, and for the fiscal policy, that is

$$s_{t+1} = \theta_{s\pi}\pi_{t+1} + \theta_{sy}y_{t+1} + \alpha v_t^* + u_{t+1}^s, \tag{5.18}$$

where the group of parameters θ captures the policy responsiveness to various endogenous variables. Both the exogenous monetary shock u_t^i and fiscal shock u_t^s follow AR(1) processes, i.e.,

$$u_{t+1}^i = \rho_i u_t^i + \epsilon_{t+1}^i, \qquad u_{t+1}^s = \rho_s u_t^s + \epsilon_{t+1}^s,$$

where ϵ_{t+1}^i and ϵ_{t+1}^s are i.i.d. innovations with mean zero.

The setup of fiscal rule (5.18) follows Cochrane (2021a). In particular, v_t^* in (5.18) represents the *target-level* of the real balance of government debt, which in equilibrium equals the

[21] We assume that an appropriate subsidy has been imposed so as to correct the markup due to monopolistic competition. Therefore, we do not consider inefficient cost-push shocks, and the equilibrium under flexible prices is efficient in this economy.

market value of government debt v_t.[22] Here $\alpha > 0$ captures to what extent the government respects its liability by collecting surpluses to pay its debt back. Together with the shock processes, this policy rule yields an s-shape response in surpluses to fiscal shocks: Upon a deficit shock u_t^s, surplus initially drops, and public debt value increases; the increased debt in turn drives up the surplus due to the fiscal policy rule, and as the original shock u_t^s dies out, surpluses will rise above the steady-state level to pay down the debt.

One may need to be careful about the concept of v_t^*, though it equals v_t in equilibrium. The target variable v_t^* differs from v_t in that v_t^* accumulates with the inflation targeted by the fiscal authority, but not arbitrary inflation, such as those arise in sunspot equilibria. This makes the fiscal policy become active in determining the price level. We briefly discuss the intuition below.[23] The law of motion for v_t^* is set to be the same as that for v_t in (5.12) except the realized inflation π_t being replaced by the inflation target π_t^*, i.e.,

$$\rho v_{t+1}^* = v_t^* + i_{t+1}^B - \pi_{t+1}^* - s_{t+1} - (y_{t+1} - y_t). \tag{5.19}$$

Moreover, the inflation target π_t^* directly responds to the innovation ϵ_{t+1}^s in surplus shock such that

$$\Delta E_{t+1}\pi_{t+1}^* = -\beta_s \epsilon_{t+1}^s, \tag{5.20}$$

where the operator $\Delta E_{t+1} \equiv E_{t+1} - E_t$ denotes the unexpected change in time $t+1$. Here the exogenous parameter $\beta_s > 0$ measures the unexpected change in the inflation target in response to fiscal shocks. In equilibrium, the realized inflation is consistent with the inflation target.[24] Therefore, with a positive value of β_s, a deficit shock leads to an increase in inflation and inflation expectations; furthermore, a small value of $\beta_s > 0$ indicates the situation in which the deficit is partially financed by inflating debt away and partially by future borrowing.[25]

As argued in Cochrane (2021a), this setup of fiscal policy (5.18)-(5.20) allows for a more (empirically) reasonable response of government debt, which is a keen variable of our interest, in facing deficit shocks: a deficit *raises* instead of *lowers* the value of debt. Furthermore, as we illustrate in the numerical examples in the next section, a deficit shock also yields a positive response in inflation and inflation expectations, and a negative response in the

[22]The proof of $v_t^* = v_t$ in equilibrium can be found in Appendix D.4.

[23]One can refer to Cochrane (2021a,b) for a more thorough discussion for this setup of policy rules.

[24]To see this, notice that if realized inflation differs from the inflation target, v_t will also deviate from v_t^*. But since the fiscal rule does not respond to v_t, this deviation will accumulate and eventually lead to a explosive path of public debt, violating the transversality condition.

[25]The parameter β_s in equilibrium gauges the extent to how much a deficit shock is financed by the effect of inflating away. When $\beta_s = 0$, any deficit is fully repaid by the following surpluses, and thus there is no change in inflation at all. A higher value of β_s indicates that the government is able to inflate away more debt in facing a deficit shock.

convenience yield due to the increase in the path of future public debt.

Lastly, by utilizing the fiscal rule (5.18) and assuming $\theta_{s\pi} = \theta_{sy} = 0$, we can further elaborate the expression of convenience yield in Proposition 2 into the summation of three terms: the current real value of debt-to-GDP, the contemporaneous demand shock, and the exogenous surplus shock. In particular, the coefficient before the surplus shock is positive.[26] Since a negative government surplus shock typically increases inflation and inflation expectations, by controlling the current debt-to-GDP ratio and proxies for the current demand shock as in the empirical exercises in Section 3, the long-run inflation expectation is negatively correlated with the the long-term convenience yield via the surplus channel.

5.5 Equilibrium Characterization with Active Fiscal Policy

Given the policy rules, we are now able to characterize the full equilibrium following Cochrane (2021a) in which the price level is determined by (active) fiscal policy with long-term government debt. The major difference with Cochrane's setup is that the government-issued long-term bonds also provide convenience benefits and so households are willing to hold the bonds with a lower yield. Our purpose is to illustrate the negative association between inflation expectations and the convenience yield via the channel of expected future government surpluses. In particular, we focus on fiscal shocks, since in the empirical analyses, proxies for shocks on fundamentals and the Fed funds rates have already been controlled. We lay out the system of equations characterizing the equilibrium in Appendix D.4.

Since our focus is to illustrate the negative relationship between inflation expectations and the convenience yield with a general parameterization of the model, instead of making a specific quantitative estimation, we therefore follow the calibration of model parameters in Cochrane (2021a) for illustration purpose. We set the model parameters $\beta = 0.99$, $\rho = 0.99$, $\sigma = \psi = 0.5$, $\kappa = 0.5$, $\alpha = 0.2$, $k = 0.9$, $\rho_i = 0.7$, $\rho_s = 0.4$, $\beta_s = 0.14$, and the parameters measuring policy response $\theta_{i\pi} = 0.8$, $\theta_{iy} = 0.5$, $\theta_{s\pi} = 0.25$, and $\theta_{sy} = 1.0$. The steady-state marginal benefit of holding convenience assets relative to consumption is set to be $\gamma = 0.09$.[27] The parameter calibration corresponds to a quarterly model, and one can refer to Appendix D.4 for how these parameters show up in the system of equilibrium conditions. The qualitative results are not sensitive to the calibrated processes of exogenous shocks, or the values of coefficients in those linear policy reaction function, or the value of γ, though they matter quantitatively for the level of impulse responses.

Figure (6) shows the impulse responses (IRFs) of various endogenous variables to a deficit

[26] Specifically, under the assumption of $\theta_{s\pi} = \theta_{sy} = 0$ and noting that $v_t^* = v_t$ in equilibrium, we can rewrite the expression of the convenience yield in (5.14) into $cy_t = \phi_v v_t + \phi_d u_t^d + \sum_{j=1}^{\infty} \phi_{s,j} E_t u_{t+j}^s$, where parameters $\phi_v, \phi_d < 0$ and $\phi_{s,j} > 0$ for any $j \geq 1$. Details can be found in Appendix D.3.

[27] The determinacy condition of the equilibrium requires $\gamma\psi^{-1} + 1 - \alpha < \rho$.

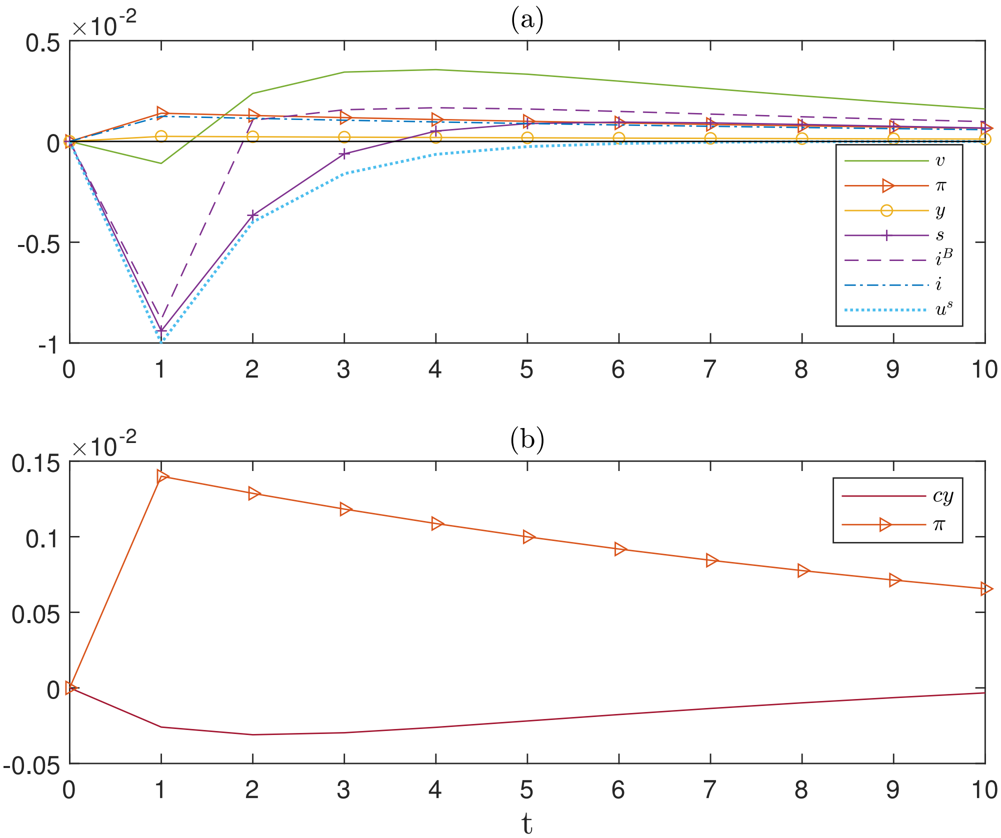

Figure 6: Impulse Responses to a (Fiscal) Deficit Shock

Trajectories describe the impulse responses (IRFs) of various endogenous variables to a deficit shock $\epsilon_1^s = -1$. Panel (a) shows the IRFs of real value of government debt v_t, inflation π_t, output gap y_t, government surplus s_t, short-term nominal return on government-issued perpetuity i_t^B, nominal interest rate i_t, and the exogenous fiscal shock u_t^s; panel (b) shows the IRFs of the long-term convenience yield cy_t and inflation π_t (the inflation path is the same as in panel (a)).

shock $\epsilon_1^s = -1$ happened in period $t = 1$. In particular, panel (a) shows that, following a deficit shock, inflation and output gap have an immediate positive response, indicating a stimulative effect of the deficit shock because of provoked inflation. The response of government surplus features an s-shape – it initially declines, but those deficits raise the real value of government debt, and the debt balance in turn raises the surplus afterwards. Since the IRF of inflation is defined as the changes of expected inflation from the perspective of time $t = 1$ when the deficit shock happens, the trajectory of inflation in Figure (6) effectively indicates the changes in expected inflation. We then see from Panel (a) that higher inflation expectations point to future increased real value of debt balances.

The higher future real value of debt balances in turn implies a lower return on convenience benefits of government bonds and thus a lower convenience yield. Panel (b) confirms this intuition by comparing the IRFs of inflation and the convenience yield on government-issued long-term bonds. In panel (b), a positive response in inflation expectation is associated with a negative movement in the convenience yield.

To sum up, a higher long-run inflation expectation indicates higher future real value of government debt balance and thus is associated with a lower convenience yield for long-term assets. Conditional on shocks on fundamentals and fed funds rates being controlled for, as we do in the empirical exercises, the theoretical model confirms the negative association between long-run inflation expectations and the long-term convenience yield in facing fiscal shocks via the channel of future expected government surpluses and future real value of debt balances.

6 Conclusion

In this paper, we present two new empirical findings and provide a simple model to rationalize both findings. We first show that the convenience yield on long-term treasuries is negatively correlated with long-run inflation expectations, controlling for fundamentals and short-term interest rates. Second, we demonstrate that higher inflation expectations today are associated with higher government debt-to-GDP growth going forward.

To explain these findings, we introduce convenience yields into an otherwise standard general equilibrium model with sticky prices and active fiscal policy rules. There are two insights from the model: first, the convenience yield on the long-term government debt is the discounted sum of all future convenience service provided. Hence the convenience yield is negatively correlated with future debt supply. Second, a fiscal deficit shock is partially funded by higher future debt issuance and partially by inflating debt burden away. As a result, inflation is positively correlated with future debt supply, which in turn is negatively associated with the convenience yield.

Different from existing research, we highlight the dynamic feature embedded in the convenience yield relating to inflation expectations and government fiscal policies. In particular, we show that fiscal policy could also affect government borrowing costs through the channel of convenience yields. It would also be interesting to quantify the size of our channel formally, but doing so will require estimating the policy rules seriously, which will be subject to the issue of observational equivalence as argued in Cochrane (2021a). We thus leave it for future study when evaluating different policies. Our findings also have important implications on the interaction between monetary policies and the government's fiscal space. When the monetary authority manages inflation expectations, the action will have non-trivial consequences on the government's cost of borrowing. One natural question is to think about optimal policy designs to minimize government borrowing costs in this environment. We leave these important questions for future research.

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A Data sources and variable construction

A.1 Daily Data

Below we list the sources and construction of our major variables used in our empirical investigation. Most of the variables are from daily market prices or indicies. Unless otherwise noted, data represent the close value of each day. We aggregate them to the monthly level as follows: For variables in levels, we take the average of daily observations over each month; for first differences, we use the differences in last trading days in each month.

- Inflation swap rates: We obtain inflation swap rates from Bloomberg under the mnemonics “USSWIT[Y] Curncy”, where [Y] is the maturity for inflation swap rates. We use the 10-year inflation swap rate in most of the empirical investigation, unless otherwise specified.

- Treasury yields: We use market yields on U.S. Treasury securities at [X]-year constant maturity from FRED with tickers DGS[X], where [X] represents maturity.

- The AAA yield: We use Moody’s Seasoned Aaa Corporate Bond Yield, retrieved from FRED under ticker DAAA. This index is constructed based on bonds with the AAA rating and maturities 20 years and above. In the baseline, we use the 20-year Treasury yield as the benchmark yield to compute the spread, while in B.1 we also use 10-year and 30-year Treasuries as robustness checks.

- The Refcorp bond yield: To match the horizon of inflation expectations, we use the yield of 10-year Refcorp bond, retrieved from Bloomberg fair value curves under the mnemonics “C09110Y Index”.

- Bond spread net of CDS: we get transaction level bond spread from TRACE Enhanced historical data, bond maturity and rating information from WRDS. Daily credit default swap rates are obtained from Markit.

- The Cleveland Fed inflation expectations: In several specifications we use the inflation expectations estimated by the Cleveland Fed to have longer sequences as early as 1983. Estimates are retrieved from https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx. See Haubrich et al. (2012) for the details of the underlying model and estimation. In their dataset, variables are estimated at the beginning of each month, while our other variables are all at the end of each month. Therefore, we shift 1 month backward for all variable from the Cleveland Fed to be consistent with other variables.

- Public debt supply: We measure the public debt supply using the historical data the website of Federal Reserve of Dallas https://www.dallasfed.org/research/econdata/govdebt#tab3. We use the sequences for privately held debt all the time. We use the market value as the benchmark for forecasting regressions, while use the par value measure for controls in Fact 1 and for robustness checks for Fact 2. The public debt supply is either normalized by nominal GDP or by CPI to reflect the real debt burden.

Below we list the source and construction of control variables used across specifications:

- The inflation risk premium: When the inflation is stochastic, the inflation swap rate also contains the inflation risk premium. Therefore, we also control those risk premia estimated by the Cleveland Fed. The data sequences are obtained from https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx.

- Industrial production (or GDP at the quarter level): We use the Industrial Production Index(INDPRO from FRED) as a proxy for real outputs at the monthly level. The industrial production index is an economic indicator that measures real output for all facilities located in the United States manufacturing, mining, and electric, and gas utilities (excluding those in U.S. territories). At the quarter level, we use the real GDP from FRED (GDPC1).

- VIX: VIX is the CBOE Volatility Index, which measures market expectation of near term volatility conveyed by stock index option prices. It is a common measure in the literature for the financial market sentiment. We retrieve the VIX index at the daily level from FRED under ticker VIXCLS.

- The effective Fed funds rate: Following Nagel (2016), we use the effective Fed funds rate to control for the opportunity cost of money. Data is from FRED under ticker DFF.

- The recession indicator: We construct a recession indicator which equals one during recessions, dated by the NBER Business Cycle Dating Committee, and zero otherwise.

In Section B we check the robustness of our results using other measures of convenience yields. We list the data source here and defer the discussion to their sections.

- The AAA Option-Adjusted Spread (OAS): We use the AAA-OAS as an alternative measure of the convenience yield to address the embedded callable option in corporate

Table 5: Summary Statistics for Control Variables

		Level				First Diff.			
Variable	**Time Period**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
Inflation risk premium	83/01-21/09	0.41	0.07	-0.12	0.59	0.0	0.05	-0.55	0.18
log(Ind. Prod.)	83/01-21/09	4.4	0.23	3.86	4.65	0.0	0.01	-0.15	0.06
VIX	90/01-21/09	19.49	7.69	10.13	62.67	-0.01	4.59	-19.39	21.27

Notes. This table reports summary statistics of control variables at the monthly frequency. Variables are measured in percentage points. Statistics in both levels and first differences are reported separately. For each variable we report its available time period in our sample.

bonds. The sequence is constructed by ICE BofA and retrieved via FRED (ticker BAMLC0A1CAAA). From the descrption on FRED: "This subset includes all securities with a given investment grade rating AAA. The ICE BofA OASs are the calculated spreads between a computed OAS index of all bonds in a given rating category and a spot Treasury curve. An OAS index is constructed using each constituent bond's OAS, weighted by market capitalization."

- The near-10-year AAA yield: As a measure of the 10-year convenience yield, we also construct a spread between a near-10-year AAA yield index and 10-year Treasury yield. The near-10-year AAA yield index is constructed as the average of 7-10 year AAA average yield and 10-15 year AAA average yield. Both indicies are from ICE indicies and retrieved via Bloomberg under mnemonics "C4A1 index" and "C8A1 index".

B Additional Tests for Fact 1

B.1 Term Structure and Embedded Callable Options

A plausible but uninteresting explanation for the negative comovement between AAA-Tbond spread and inflation expectation is that it is due to different payoff structures of AAA corporate bonds and Treasuries. There are indeed several subtle but important differences. First, the AAA yield index from Moody's are based on bonds with maturities 20 years and above, while the benchmark Treasury is chosen at 20-year constant maturity to match the horizon for inflation expectations. One may be concerned that it is different term structures that drive our results. Second, corporate bonds may have an embedded callable option that gives the issuer the right to "call" or buy back its existing bonds prior to maturity when interest rates decline, while most of the U.S. Treasuries in our sample are non-callable.[28] The callable option may also explain the differential exposures to inflation expectations of AAA bonds and Treasuries.

[28] Certain bonds issued before 1985 were also embedded with a callable option. See https://www.treasurydirect.gov/indiv/research/indepth/tbonds/res_tbond_call.htm

To address these concerns, we use the AAA-Treasury option-adjusted spread (OAS) as the dependent variable. The AAA-OASs adjust for the value of the embedded callable option in AAA corporate bonds. In computing the OAS, bonds' cash flows are discounted using the whole Treasury yield curve plus a constant spread, so the concern about term structure is also addressed. The data source and description can be found in 1. We regress AAA-OAS on inflation expectations, and the results are presented in Column (1) of Table 6. If anything, the coefficient for option-adjusted spread is stronger than our baseline estimate (reproduced in Column 3).

Column (2) and Column (4) of Table 6 use 10-year and 30-year Treasuries as the benchmark to compute the AAA-Treasury spread. Compared to the OAS, they serve as model-free robustness checks to address the term structure. The coefficients are comparable to the baseline results in Column (3).

Table 6: Option-Adjusted Spread and Different Maturities

	Δ AAA-OAS	Δ AAA-10yTbond	Δ AAA-Tbond	Δ AAA-30yTbond	Δ 10y AAA-Tbond
	(1)	(2)	(3)	(4)	(5)
$\Delta E\pi$	-0.34	-0.20	-0.23	-0.21	-0.22
	(0.09)	(0.07)	(0.08)	(0.05)	(0.08)
Measure	Swap	Swap	Swap	Swap	Swap
Control	true	true	true	true	true
N	206	206	206	206	205
R^2	0.37	0.29	0.34	0.42	0.24

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first differences at the monthly level. In all columns we use the inflation swap rate as the measure for inflation expectations. In Column (1) the dependent variable is AAA option adjusted spread from ICE BofA. Column (2) reproduces Column (4) in Table 2 by using 10-year Treasury as the benchmark to compute the AAA-Treasury spread. Column (3) and (4) use 20-year and 30-year Treasuries as the benchmark, respectively. Across all columns, the control variables include: (log of) industrial production, CBOE volatility index (VIX), (log of) public debt, real yield, and real and inflation risk premia.

B.2 The Bond-CDS Spread and Inflation Expectations

Table 7: Summary Statistics for Bond-CDS spreads

		Level				First Diff.			
Variable	**Time Period**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**
AA bond-CDS spread	05Q2-19Q2	0.27	0.34	-0.09	1.54	-0.0	0.16	-0.57	0.61
BBB-AA basis-spread	05Q2-19Q2	0.35	0.42	0.04	2.28	0.0	0.26	-0.98	1.39

Notes. This table reports summary statistics for bond-CDS spreads at the quarterly frequency. Variables are measured in percentage points. Statistics in both levels and first differences are reported separately. For each variable, we report its available time period in our sample. See Eq. (B.1) and (B.2) for the definitions.

A credit default swap, or CDS, is a swap contract that allows an investor to "swap" the default risk of some reference assets, such as corporate bonds, with another investor. The buyer of a CDS pays an ongoing premium to the seller until the maturity date of the contract in exchange for the insurance against credit defaulting. By buying a corporate bond and a maturity-matched CDS, an investor can almost replicate the cash flow from a risk-free bond, such as the U.S. Treasury with the same maturity.[29]

Using credit default swaps (CDS), we are able to take the default component out of corporate bond spreads. For each bond i, we construct its *bond-CDS spread*, defined as the bond-Treasury spread net of the CDS spread, i.e.,

$$\text{bond-CDS spread}_{i,t} = (y_{i,t} - y_{Treasury,t}) - CDS_{i,t} \tag{B.1}$$

where the maturity of both the CDS and the Treasury are matched to bond i. Notice our definition of the bond-CDS spread is the opposite of the conventional *CDS-bond basis* commonly used in the literature. By our definition, the sign of the bond-CDS spread is consistent with our other measures of the convenience yield: A positive number indicates Treasuries have higher price (lower spread) relative to the other assets.

In a frictionless world without special demand for convenience services and liquidity concerns, the bond-Treasury spread should be zero by no-arbitrage condition. Nevertheless, positive bond-CDS spreads (or negative CDS basis) are a well-documented empirical fact in the literature, suggesting that Treasuries carry a premium relative to the synthetic risk-free bond.

The source of the premium may be multifaceted. It can be from the illiquidity of corporate bonds (Longstaff et al., 2005; Li and Yu, 2021), or the relative "safety premium" of treasury above that of corporate bonds. For now, we bundle different sources of the premium together as the convenience yield of Treasuries relative to corporate bonds, and defer the discussion of the exact feature of safe assets to Section 5.[30]

High-quality corporate bonds can also carry a safety premium relative to risky bonds. This observation motivates Krishnamurthy and Vissing-Jorgensen (2012) to use the BBB-AAA spread as an alternative measure of the safety premium. Mota (2021) further refines this measure by isolating the non-credit-risk component using CDSs. Following this literature, we construct the BBB-AA basis-spread as the difference between the BBB bond-CDS spread and the AA bond-CDS spread. Notice that the Treasury yield is canceled out in the BBB-

[29] In implementation, there are still subtle differences between the synthetic bond and a Treasury security. For example, the CDS may be subject to the counterparty risks.

[30] Technically the positive spreads can also originate from the the imperfect replication of risk-free bonds due to the frictions on the CDS market, such as counterparty risks, imperfect matching of payoff structure, etc. However, these differences are in general small (e.g., Arora et al., 2012 show the counterparty risk is "vanishingly small"), so we do not take them into account in our analysis.

AA basis-spread (see Equation B.2). Therefore, the BBB-AA basis-spread is a measure of the premium on AA bonds relative to BBB bonds, independent of Treasury yields.

$$\begin{aligned} \text{BBB-AA basis-spread}_t &= \text{bond-CDS spread}_{BBB,t} - \text{bond-CDS spread}_{AA,t} \\ &= (y_{BBB,t} - CDS_{BBB,t}) - (y_{AA,t} - CDS_{AA,t}) \end{aligned} \tag{B.2}$$

If AA bond yields indeed contain a premium relative to BBB bond yields, the BBB-AA basis-spread would be positive. Moreover, if the premium share a similar micro-foundation of U.S. Treasury premium, then the BBB-AA basis-spread should also comove with inflation expectations in the same way does the convenience yield on Tresuries. This is exactly what we find.

Table 7 report summary statistics for bond-CDS spreads. Table 8 presents regressions of bond-CDS spreads on inflation expectations, at the quarterly level. We switch to quarterly frequency to include more matched bond-CDS pairs. The sample is from 2005Q2 to 2019Q2. Inflation expectations are measured by the 10-year inflation swap rates. In Column (1) we regress the AA bond-CDS spread on inflation expectations without covariates. The point estimate is negative, consistent with our baseline results. It shows that a one percentage-point increase in inflation expectations is associated with a 24bp decrease in the AA bond-CDS spread, or 0.72 of its standard deviation. The effect is significant both economically and statistically. In Column (2) we add controls to the regression. The point estimate is slightly smaller, reduced to 17bps, but still highly significant ($p < 0.05$).

Column (3) and (4) investigate the relationship between BBB-AA basis-spread and inflation expectations. As discussed above, it captures the relative premium of AA to BBB. Column (3) shows the result of the univariate regression. The coefficient shows a one percentage point increase in inflation expectations translates into a 33bp (or 0.79 s.d.) decrease in the BBB-AA basis-spread. In other words, with a one percentage point increase in inflation expectations, AA bonds appreciate relative to BBB bonds by 33bps. The coefficient is statistically significant ($p < 0.01$). On Column (4) we add usual controls to the regression, which reduces the point estimate to $-8bp$. The point estimate still points to the correct direction, though no longer statistically significant.

B.3 Hodrick-Prescott Filter

Throughout the main text, we use time-differencing to remove potential spuriously correlated stochastical trends. We show our results are robust to an alternative approach also commonly used in the literature: Hodrick-Prescott (HP) filter.

For each variable, we run the HP filter with the suggested monthly smoothing parame-

Table 8: Bond-CDS Spreads and Inflation Expectations

	Δ AA-Tbond		Δ BBB-AA	
	(1)	(2)	(3)	(4)
$\Delta E\pi$	-0.24	-0.17	-0.33	-0.08
	(0.05)	(0.05)	(0.12)	(0.05)
Measure	Swap	Swap	Swap	Swap
Controls	False	True	False	True
Sample	05Q2-19Q2	05Q2-19Q2	05Q2-19Q2	05Q2-19Q2
N	56	56	56	56
R^2	0.20	0.53	0.15	0.48

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates first difference at the quarterly level. The sample is from 2005Q1-2019Q2. In Column (1)-(2), the dependent variable is the average bond-CDS spread for AA bonds. In Column (3)-(4), the dependent variable is the average BBB-AA basis-spread, defined in Eq. (B.2). Control variables for Column (2) and (4) include quarterly fixed effects, a recession indicator, and first differences in: (log of) real GDP, VIX, debt-to-GDP ratio, the effective Fed funds rate, and the inflation risk premium.

ter 129,600. We then take the deviation of each variable from its low-frequency trends as the regressands and regressors. Table 9 reproduces our baseline results using HP filtered sequences. For notational ease we continue to use Δ (only in this table) to denote the deviation from filtered trends. Across different specifications, the point estimates are negative and comparable to the baseline results in Table 2. The standard errors, however, are much larger, indicating noisier estimation.

C Additional Tests for Fact 2

C.1 Alternative Measures for Debt Growth

As the market value of public debt contains information of discount rates, the growth rate of the market value may reflect the change in discount rates, including changes in inflation expectations and the convenience yield. Therefore, regressing the growth of the market value on inflation expectations may capture the auto-correlation of inflation expectations. Moreover, as GDP is the denominator in our baseline measure, the forecastability can also capture the cyclicality in the GDP. Both concerns are quantitatively insigificant over the long horizon, as we have shown in 4. Here we formally address this concern by using an alternative measure: the real growth rate of the par value of privately held debt, constructed as the growth rate of the nominal par value minus the realized inflation over the same period.

Table 9: The Convenience Yield and Inflation Expectations (HP Filtered)

	Δ AAA-Tbond		Δ Refcorp - TBond	
	(1)	(2)	(3)	(4)
$\Delta E\pi$	-0.60	-0.26	-0.47	-0.21
	(0.10)	(0.06)	(0.17)	(0.08)
Measure	Swap	Swap	Swap	Swap
Controls	False	True	False	True
Sample	04/07-21/09	04/07-21/09	04/07-21/09	04/07-21/09
N	207	207	207	207
R^2	0.51	0.79	0.35	0.65

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. Δ indicates the deviation from the trends filtered by HP filter. The smoothing parameter is set at the suggested monthly level 129,600. In Column (1)-(2), the dependent variables are changes in the AAA-Tbond spread. In Column (3)-(4), the dependent variables are the Refcorp-Tbond spread. Controls variables in Column (2) and (4) are: industrial production, CBOE volatility index (VIX), (log of) public debt, the effective Fed funds rate, the 10-year real interest rate, and real and inflation risk premia. All are filtered using HP filter.

Table 10 presents the same predictive regression as in 4 but with the real par value growth as the predicted variable. The coefficients are highly significant, and comparable with those in Table 4.

Table 10: Inflation Expectations Predict Future Debt/GDP Growth

	1yr	5yr	10yr	1yr	5yr	10yr
	(1)	(2)	(3)	(4)	(5)	(6)
$E\pi$	4.81	31.32	51.34	8.78	48.69	55.10
	(2.08)	(8.22)	(20.83)	(2.31)	(10.85)	(17.20)
Sample	81Q4-20Q3	81Q4-16Q3	81Q4-11Q3	81Q4-20Q3	81Q4-16Q3	81Q4-11Q3
Control	False	False	False	True	True	True
N	156	140	120	156	140	120
R^2	0.06	0.16	0.15	0.22	0.52	0.81

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. The predicted variables are future real debt cumulative growth rates, where debt is measured by the *par value* of privately held public debt, normalized by CPI. Both variables are linearly detrended. Horizons are indicated in each column. The predictor variable is 10-year inflation expectations estimated by the Cleveland Fed (Haubrich et al., 2012). Column (1)-(3) performs the univariate prediction, and Column (4)-(6) control for output growth, the par value of debt to GDP, the inflation risk premium, and the effective Fed funds rate, all at time t.

C.2 Bias-adjusted Estimates

It is known that predictive regressions will have small sample biases when innovations to the predictor and the predicted variables are correlated (Stambaugh, 1999; Boudoukh et al., 2021). Using the formula provided by Boudoukh et al. (2021), we compute the biases in the small sample under the null hypothesis that $\beta_h = 0$, and plot the bias-adjusted coefficients in Figure C.2. The biases are relatively small relative to the point estimates. Our results are still highly significant after adjusting for biases.

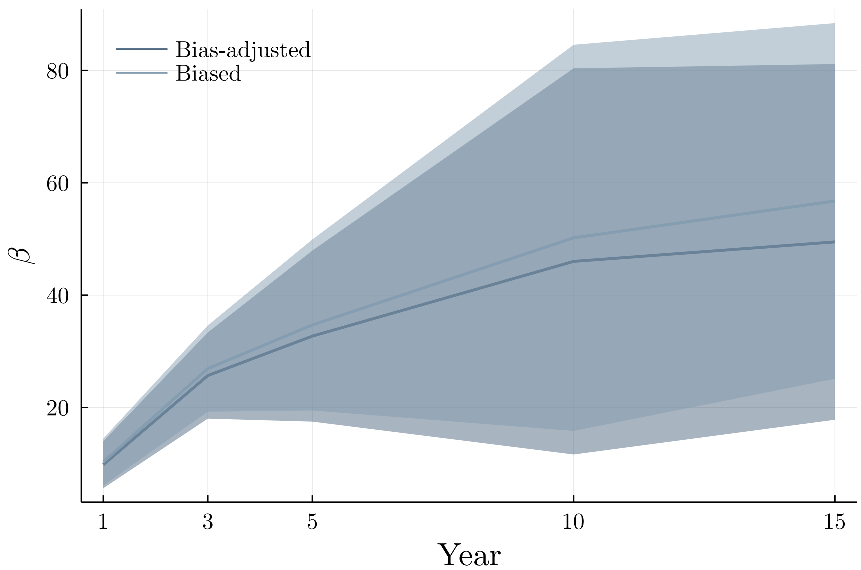

Figure C.7: Bias-adjusted Estimates

This figure plots the estimates of β_h in Eq. 4.1 and the bias-adjusted estimates under the null hypothesis, using the formula from Boudoukh et al. (2021). The shaded area represents the 95% confidence interval with Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)).

C.3 The Predictive Power of the Realized Inflation

As the inflation expectations are highly correlated with the realized inflation (the correlation in our sample is 0.33), we can use the realized inflation as a proxy for inflation expectations to extend our sample further back to 1947. The realized inflation is calculated as the annualized quarterly growth rate of CPI for urban consumers (CPIUCSL). Column (1)-(3) in Table 11 use the realized inflation to predict future debt growth since 1947, controlling for comtemporaneous debt/GDP, GDP growth and a linear trend. The realized inflation also significantly predicts the future debt growth. The coefficients are smaller than inflation expectations in the baseline regression, since the realized inflation is for each quarter while inflation expectataions are over 10 years in the future. This results mirror the findings of Cochrane (2021b), who finds that an innovation to inflation coincides with a rise of public debt value in the future.

Column (4)-(6) performs a horse race between the realized inflation and inflation expectations. After controlling for inflation expectations, the realized inflation lost its predictive power for the 10-year debt/GDP growth. For the shorter horizons the point estimates are still positive but it is no longer statistically significant. These results suggest inflation expectations have better predictive powers than the realized inflation.

Table 11: Predictive Regressions with the Realized Inflation

	1yr	5yr	10yr	1yr	5yr	10yr
	(1)	(2)	(3)	(4)	(5)	(6)
π	0.32	2.21	3.31	0.47	1.58	-0.84
	(0.19)	(0.78)	(1.05)	(0.38)	(0.74)	(0.98)
$E\pi$				10.35	33.08	30.99
				(1.67)	(7.53)	(11.61)
Sample	47Q2-20Q2	47Q2-16Q2	47Q2-11Q2	81Q4-20Q2	81Q4-16Q2	81Q4-11Q2
Control	True	True	True	True	True	True
N	293	277	257	155	139	119
R^2	0.21	0.46	0.60	0.24	0.51	0.82

Notes. Newey-West standard errors (bandwidths optimally chosen following Newey and West (1994)) are shown in parentheses. The predicted variables are future debt/GDP cumulative growth rates, where debt is measured by the market value of privately held public debt. Both variables are linearly detrended. Horizons are indicated in each column. The predictor variable is the realized inflation, calculated as the annualized quarterly growth rate of CPI for urban consumers (CPIUCSL), and the 10-year inflation expectations estimated by the Cleveland Fed (Haubrich et al., 2012). All columns control for output growth and the par value of debt to GDP at time t.

D Proofs for the Theoretical Model

D.1 First-order Conditions of the Household's Problem

The first-order conditions of the household's problem with respect to L_t, B_t, and D_t are given by

$$1 = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{R_t}{\Pi_{t+1}}], \tag{D.1}$$

$$1 - \frac{\mu_v(V_t)}{u_c(C_t)} = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{kQ^B_{t+1}+1}{Q^B_t} \frac{1}{\Pi_{t+1}}], \tag{D.2}$$

$$1 = E_t[\beta \frac{u_c(C_{t+1})}{u_c(C_t)} \frac{kD^D_{t+1}+1}{Q^D_t} \frac{1}{\Pi_{t+1}}]. \tag{D.3}$$

Then to a first-order approximation, the IS curve (D.1) and the demand function of convenience assets (D.2) yield

$$\log R_t = \frac{\mu_v(V_t)}{u_c(C_t)} + E_t \log(\frac{kQ^B_{t+1}+1}{Q^B_t}). \tag{D.4}$$

Similarly, the demand function of non-convenience assets (D.3) with (D.1) yields

$$\log R_t = E_t \log(\frac{kQ^D_{t+1}+1}{Q^D_t}). \tag{D.5}$$

D.2 Proofs for Log-linearizing Households' First-order Conditions

Here we show the steps of deriving equations (5.6), (5.7), and (5.9) in the household's first-order conditions. First, denote i^B_{t+1} as

$$i^B_{t+1} \equiv \log(\frac{kQ^B_{t+1}+1}{Q^B_t} / \frac{k\bar{Q}^B+1}{\bar{Q}^B}),$$

and then by log-linearization, we have

$$i^B_{t+1} = \frac{k\bar{Q}}{k\bar{Q}+1} q^B_{t+1} - q^B_t = kq^B_{t+1} - q^B_t, \tag{D.6}$$

where we have used the steady-state relationship $1 + \bar{i}^B = (k\bar{Q}^B + 1)/\bar{Q}^B$ in the second equality (note that $\bar{i}^B$ is at the scale of first order).

To get equation (5.6), we log-linearize (5.4) and note the market clearing condition $Y_t =$

C_t, i.e.,

$$\begin{aligned}
\log R_t - log\bar{R} &= \frac{\mu_v(V_t)}{u_c(C_t)} - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + E_t \log(\frac{kQ_{t+1}+1}{Q_t}) - \log(\frac{k\bar{Q}+1}{\bar{Q}}),\\
\Rightarrow i_t &= \frac{\mu_v(\bar{V})[1+\frac{\mu_{vv}(\bar{V})\bar{V}}{\mu_v(\bar{V})}\frac{V_t-\bar{V}}{\bar{V}}]}{u_c(\bar{C})[1+\frac{u_{cc}(\bar{C})\bar{C}}{u_c(\bar{C})}\frac{C_t-\bar{C}}{\bar{C}}]} - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + E_t i^B_{t+1}\\
&= \frac{\mu_v(\bar{V})[1-\psi^{-1}(v_t+y_t)]}{u_c(\bar{C})[1-\sigma^{-1}y_t]} - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + kE_t q^B_{t+1} - q^B_t\\
&= \frac{\mu_v(\bar{V})}{u_c(\bar{C})}[1-\psi^{-1}(v_t+y_t)+\sigma^{-1}y_t] - \frac{\mu_v(\bar{V})}{u_c(\bar{C})} + kE_t q^B_{t+1} - q^B_t\\
&= -\gamma\psi^{-1}[v_t+y_t] + \gamma\sigma^{-1}y_t + kE_t q^B_{t+1} - q^B_t,
\end{aligned}$$

where

$$\psi \equiv -\mu_v(\bar{V})/(\mu_{vv}(\bar{V})\bar{V}) > 0, \qquad \sigma \equiv -u_c(\bar{C})/(u_{cc}(\bar{C})\bar{C}) > 0,$$
$$\gamma \equiv \mu_v(\bar{V})/u_c(\bar{C}).$$

Similarly, to get equation (5.7), we log-linearize (5.5), i.e.,

$$\begin{aligned}
i_t &= \frac{k\bar{Q}^D}{k\bar{Q}^D+1}E_t q^D_{t+1} - q^D_t\\
&= kE_t q^D_{t+1} - q^D_t.
\end{aligned}$$

Finally, for equation (5.9), we have the convenience yield $cy_t \equiv r^D_t - r^B_t$ is given by

$$\begin{aligned}
cy_t &= kE_t(cy_{t+1}) + (\bar{Q}^D)^{-1}[-\gamma\psi^{-1}v_t + \gamma(\sigma^{-1}-\psi^{-1})y_t]\\
&= kE_t(cy_{t+1}) + (1+\bar{i}-k)[-\gamma\psi^{-1}v_t + \gamma(\sigma^{-1}-\psi^{-1})y_t]\\
&= kE_t(cy_{t+1}) + (1-k)[-\gamma\psi^{-1}v_t + \gamma(\sigma^{-1}-\psi^{-1})y_t].
\end{aligned}$$

Note that in the first equality, we have assumed $1/\bar{Q}^B \approx 1/\bar{Q}^D$ (to the first-order approximation) in the steady-state; in the second equality, we have used the steady-state relationship $1+\bar{i} = (k\bar{Q}^D+1)/\bar{Q}^D$.

D.3 The Expression of Expected Future Government Real Debt Balances

To nail down the expression of future government real debt balances, we need to log-linearize the government budget constraint (5.2), the Euler equation (5.3), and the trade-off condition between holding convenience assets and short-term riskless assets (D.2). The government budget constraint (5.2) yields

$$\rho v_{t+1} = v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t), \tag{D.7}$$

where s_{t+1} is the scaled real primary surplus to output ratio, and $\rho \equiv e^{-\bar{i}^B} < 1$ is a constant.

Similarly, the IS curve (5.3) yields

$$y_t - u^d_t = E_t(y_{t+1} - u^d_{t+1}) - \sigma(i_t - E_t\pi_{t+1}), \tag{D.8}$$

where $u^d_{t+1} = \rho_d u^d_t + \epsilon^d_{t+1}$ denotes aggregate demand shock (such as preference shock) and ϵ^d_{t+1} is an i.i.d. innovation with mean zero; the trade-off condition of holding convenience assets (D.2) yields

$$i_t = -\gamma\psi^{-1}[v_t + y_t] + \gamma\sigma^{-1}y_t + E_t i^B_{t+1}. \tag{D.9}$$

Combining (D.7)-(D.9) and also noting that $\sigma = \psi = 1$, we have the expected government real debt balance in any time $t + j + 1$ satisfying

$$E_t v_{t+j+1} = \frac{1}{\rho}E_t[(1+\gamma)v_{t+j} - s_{t+j+1} - (u^d_{t+j+1} - u^d_{t+j})] \tag{D.10}$$

for any $j \geq 0$, and then by iterating this equation to time t, it yields

$$E_t v_{t+j+1} = (\frac{1+\gamma}{\rho})^{j+1}v_t - [\sum_{n=1}^{j+1}\frac{1}{\rho}(\frac{1+\gamma}{\rho})^{j+1-n}E_t s_{t+n}] + \frac{1-\rho_d}{\rho\rho_d - (1+\gamma)}[\rho_d^{j+1} - (\frac{1+\gamma}{\rho})^{j+1}]u^d_t. \tag{D.11}$$

One can observe from equation (D.11) that expected future real debt balance of the government equals the summation of three terms: current debt-to-GDP ratio, the future flows of government surpluses, and the contemporaneous demand shock.

Now further consider a fiscal policy rule in which $s_{t+1} = \alpha v_t + u^s_{t+1}$. By substituting s_{t+j+1} into (D.10), we have

$$E_t v_{t+j+1} = \frac{1}{\rho}E_t[(1+\gamma-\alpha)v_{t+j} - u^s_{t+j+1} - (u^d_{t+j+1} - u^d_{t+j})]$$

for any $j \geq 0$, and similarly, by iterating this equation to time t, it yields

$$E_t v_{t+j+1} = (\frac{1+\gamma-\alpha}{\rho})^{j+1}v_t - [\sum_{n=1}^{j+1}\frac{1}{\rho}(\frac{1+\gamma-\alpha}{\rho})^{j+1-n}E_t s_{t+n}]$$
$$+\frac{1-\rho_d}{\rho\rho_d - (1+\gamma-\alpha)}[\rho_d^{j+1} - (\frac{1+\gamma-\alpha}{\rho})^{j+1}]u^d_t.$$

Therefore, in such a case, the convenience yield is given by

$$
\begin{aligned}
cy_t &= -\gamma(1-k)\sum_{j=0}^{\infty} k^j E_t v_{t+j} \\
&= \phi_v v_t + \phi_d u_t^d + \sum_{j=1}^{\infty} \phi_{s,j} E_t u_{t+j}^s,
\end{aligned}
$$

where the constant coefficients are given by

$$
\begin{aligned}
\phi_v &\equiv -\gamma(1-k)\sum_{j=0}^{\infty} k^j (\frac{1+\gamma-\alpha}{\rho})^j < 0, \\
\phi_d &\equiv -\gamma(1-k)\frac{1-\rho_d}{\rho\rho_d-(1+\gamma-\alpha)}\sum_{j=1}^{\infty} k^j [\rho_d^j - (\frac{1+\gamma-\alpha}{\rho})^j] < 0, \\
\phi_{s,j} &\equiv \gamma\frac{1-k}{\rho}\sum_{n=j}^{\infty} k^n (\frac{1+\gamma-\alpha}{\rho})^{n-1} > 0.
\end{aligned}
$$

D.4 Equilibrium Characterization

Denote the log-deviation from steady-sate values by lowercase letters. Market clearing conditions require $y_t = c_t$. Given those first-order conditions from the demand side (D.6)-(D.9), the New Keynesian Phillips curve (5.16) from the supply side, the policy rules (5.17)-(5.18), the government budget constraint (5.12), and the expression of convenience yield (5.9), the equilibrium is fully characterized by the following system of equations:

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{D.12}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \tag{D.13}$$

$$E_t i^B_{t+1} = i_t - \gamma[-\psi^{-1} v_t + (\sigma^{-1} - \psi^{-1}) y_t] \tag{D.14}$$

$$i^B_{t+1} = k q^B_{t+1} - q^B_t \tag{D.15}$$

$$i_t = \theta_{i\pi} \pi_t + \theta_{iy} y_t \tag{D.16}$$

$$s_{t+1} = \theta_{s\pi} \pi_{t+1} + \theta_{sy} y_{t+1} + \alpha v^*_t + u^s_{t+1} \tag{D.17}$$

$$\rho v^*_{t+1} = v^*_t + i^B_{t+1} - \pi^*_{t+1} - s_{t+1} - (y_{t+1} - y_t) \tag{D.18}$$

$$E_t \pi^*_{t+1} = E_t \pi_{t+1} \tag{D.19}$$

$$\Delta E_{t+1} \pi^*_{t+1} = -\beta_s \epsilon^s_{t+1} \tag{D.20}$$

$$\rho v_{t+1} = v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t) \tag{D.21}$$

$$0 = \lim_{T \to \infty} \rho^T E_t v_{t+T} \tag{D.22}$$

$$c y_t = k E_t(c y_{t+1}) + (1-k)\gamma[-\psi^{-1} v_t + (\sigma^{-1} - \psi^{-1}) y_t] \tag{D.23}$$

$$u^s_{t+1} = \rho_s u^s_t + \epsilon^s_{t+1}, \tag{D.24}$$

where v^*_t represents the target level of government real debt balance in period t and π^*_t represents the target level of inflation. The operator $\Delta E_{t+1} \equiv E_{t+1} - E_t$ denotes the unexpected change in time $t+1$. Here for the sake of parsimony, we only include fiscal shocks. One can easily extend the set of types of shocks in the model, such as monetary shocks as in (5.17), aggregate demand or supply shocks as introduced in the IS curve (5.11) and New Keynesian Phillips curve (5.16).

Equation (D.17) generates an s-shaped response of government surpluses to fiscal shocks, which is a much more realistic description of the real data as argued in Cochrane (2021b,a) – a deficit shock increases but not lowers the value of government debt. The purpose of introducing those target (latent) variables and equations (D.18)-(D.20) is to allow the economy to finance a deficit shock partially though inflating debt away and partially through future borrowing (which is captured by the value of β_s). More specifically, equation (D.18) captures the evolution of the target level of government real debt balance v^*_t. Together with the fiscal rule of collecting surpluses (D.17), it generates the desired s-shaped surplus response to a deficit shock. Equation (D.19) and (D.20) describe the latent variable of inflation target, meaning that the expected inflation has to be consistent with the expected inflation target.[31]

[31] One can refer to Cochrane (2021a) for a more detailed discussion on those target variables and the corresponding equations in the equilibrium conditions.

Note that equation (D.18) and (D.21) imply

$$\rho(v_{t+1} - v^*_{t+1}) = (v_t - v^*_t) - (\Delta E_t \pi_{t+1} - \Delta E_{t+1} \pi^*_t).$$

Together with the transversality condition (D.22), we have in equilibrium

$$v_t = v^*_t, \qquad \pi_{t+1} = \pi^*_{t+1}.$$

Thus the equilibrium conditions (D.12)-(D.24) reduces to the following system of equations:[32]

$$\begin{aligned}
y_t &= E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa y_t \\
E_t i^B_{t+1} &= i_t - \gamma[-\psi^{-1} v_t + (\sigma^{-1} - \psi^{-1}) y_t] \\
i^B_{t+1} &= k q^B_{t+1} - q^B_t \\
i_t &= \theta_{i\pi} \pi_t + \theta_{iy} y_t + u^i_t \\
\Delta E_{t+1} \pi_{t+1} &= -\beta_s \epsilon^s_{t+1} - \beta_i \epsilon^i_{t+1} \\
s_{t+1} &= \theta_{s\pi} \pi_{t+1} + \theta_{sy} y_{t+1} + \alpha v_t + u^s_{t+1} \\
\rho v_{t+1} &= v_t + i^B_{t+1} - \pi_{t+1} - s_{t+1} - (y_{t+1} - y_t) \\
c y_t &= k E_t (c y_{t+1}) + (1-k)\gamma[-\psi^{-1} v_t + (\sigma^{-1} - \psi^{-1}) y_t] \\
u^i_{t+1} &= \rho_i u^i_t + \epsilon^i_{t+1} \\
u^s_{t+1} &= \rho_s u^s_t + \epsilon^s_{t+1}.
\end{aligned}$$

D.5 Determinacy Conditions of the Full Equilibrium

Here we characterize the determinacy condition of the full equilibrium. Denote δ as expectational errors in the equations that only tie down expectations. For example, $\delta_{\pi,t+1} = \pi_{t+1} - E_t \pi_{t+1}$.

We can rewrite the system of equations of the full equilibrium conditions in a recursive

[32] The determinacy conditions of the equilibrium and the relevant proofs can be found in Appendix D.5.

form of matrices as

$$
\begin{pmatrix}
1 & \sigma & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -k & 0 & 0 & 0 & 0 \\
-\theta_{sy} & -\theta_{s\pi} & 0 & 0 & 1 & 0 & 0 & -1 \\
1 & 1 & -1 & 0 & 1 & \rho & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ i^B_{t+1} \\ q_{t+1} \\ s_{t+1} \\ v_{t+1} \\ u^i_{t+1} \\ u^s_{t+1} \end{pmatrix}
$$

$$
= \begin{pmatrix}
1+\sigma\theta_{iy} & \sigma\theta_{i\pi} & 0 & 0 & 0 & 0 & \sigma & 0 \\
-\kappa & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta_{iy}+\gamma(\psi^{-1}-\sigma^{-1}) & \theta_{i\pi} & 0 & 0 & 0 & \gamma\psi^{-1} & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_i & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_s
\end{pmatrix}
\begin{pmatrix} y_t \\ \pi_t \\ i^B_t \\ q_t \\ s_t \\ v_t \\ u^i_t \\ u^s_t \end{pmatrix}
+ \begin{pmatrix}
-\sigma\beta_i & -\sigma\beta_s \\
-\beta\beta_i & -\beta\beta_s \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix} \epsilon^i_{t+1} \\ \epsilon^s_{t+1} \end{pmatrix}
$$

$$
+ \begin{pmatrix} \delta_{y,t+1} \\ 0 \\ \delta_{i^B,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{D.25}
$$

where we do not include cy_t into the matrix since it is uniquely pinned down as long as the equilibrium is stationary.

Denote

$$
A \equiv \begin{pmatrix}
1 & \sigma & 0 & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & -k & 0 & 0 \\
-\theta_{sy} & -\theta_{s\pi} & 0 & 0 & 1 & 0 \\
1 & 1 & -1 & 0 & 1 & \rho
\end{pmatrix}. \tag{D.26}
$$

Then the inverse of the matrix on the left-hand side of (D.25) is

$$\begin{pmatrix} A^{-1} & -A^{-1}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ 0 & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}. \tag{D.27}$$

Further denote

$$\begin{aligned} \tilde{A} \equiv & \begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} & 0 & 0 & 0 & 0 \\ -\kappa & 1 & 0 & 0 & 0 & 0 \\ \theta_{iy}+\gamma(\psi^{-1}-\sigma^{-1}) & \theta_{i\pi} & 0 & 0 & 0 & \gamma\psi^{-1} \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha \\ 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\ = & \begin{pmatrix} \begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} \\ -\kappa & 1 \end{pmatrix} & 0 \\ C & \begin{pmatrix} 0 & 0 & 0 & \gamma\psi^{-1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{pmatrix}. \end{aligned} \tag{D.28}$$

Then the eigenvalues of Ω in $X_{t+1} = \Omega X_t$ is ρ_i, ρ_s and eigenvalues of $A^{-1}\tilde{A}$, where

$$X_t \equiv \left(y_t, \pi_t, i_t^B, q_t, s_t, v_t, u_t^i, u_t^s\right)'.$$

We can further rewrite A^{-1} as

$$A^{-1} = \begin{pmatrix} \begin{pmatrix} 1 & -\frac{\sigma}{\beta} \\ 0 & \frac{1}{\beta} \end{pmatrix} & 0 \\ B & \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{k} & -\frac{1}{k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\rho} & 0 & -\frac{1}{\rho} & \frac{1}{\rho} \end{pmatrix} \end{pmatrix}. \tag{D.29}$$

The eigenvalues of $A^{-1}\tilde{A}$ are then the eigenvalues $\{\lambda_1, \lambda_2\}$ of the upper block

$$\begin{pmatrix} 1 & -\frac{\sigma}{\beta} \\ 0 & \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} 1+\sigma\theta_{iy} & \sigma\theta_{i\pi} \\ -\kappa & 1 \end{pmatrix} = \begin{pmatrix} 1+\sigma\theta_{iy}+\frac{\sigma}{\beta}\kappa & \sigma\theta_{i\pi}-\frac{\sigma}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}, \tag{D.30}$$

and the eigenvalues of the lower block

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{k} & -\frac{1}{k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\rho} & 0 & -\frac{1}{\rho} & \frac{1}{\rho} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & \gamma\psi^{-1} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \gamma\psi^{-1} \\ 0 & \frac{1}{k} & 0 & \frac{\gamma\psi^{-1}}{k} \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & \frac{\gamma\psi^{-1}}{\rho}-\frac{\alpha}{\rho}+\frac{1}{\rho} \end{pmatrix}, \tag{D.31}$$

where the eigenvalues of the lower block are $\{0, 0, \frac{1}{k}, \frac{\gamma\psi^{-1}}{\rho}-\frac{\alpha}{\rho}+\frac{1}{\rho}\}$.

To sum up, the eigenvalues of the system are: $\{\rho_i, \rho_s, 0, 0, \frac{1}{k}, \frac{\gamma\psi^{-1}}{\rho}-\frac{\alpha}{\rho}+\frac{1}{\rho}, \lambda_1, \lambda_2\}$, where λ_1 and λ_2 (with $\lambda_1 < \lambda_2$) are solutions to

$$\mathcal{P}(\lambda) \equiv \lambda^2 - (\frac{1}{\beta}+1+\sigma\theta_{iy}+\frac{\sigma}{\beta}\kappa)\lambda + \frac{1+\sigma\theta_{iy}+\sigma\kappa\theta_{i\pi}}{\beta} = 0. \tag{D.32}$$

With two linearly independent expectational errors, the equilibrium is determinant if and only if there are two eigenvalues outside unit circle. Given $1/k > 1$, the equilibrium is determined if and only if $\gamma\psi^{-1} - \alpha + 1 < \rho$ and $|\lambda_1| < 1 < |\lambda_2|$.[33]

Now we focus on the equivalent condition of $|\lambda_1| < 1 < |\lambda_2|$ and show that its necessary and sufficient condition is $\mathcal{P}(1) < 0$.

Note that $\theta_{iy}, \theta_{i\pi} > 0$ and $\sigma, \kappa > 0$ and $0 < \beta < 1$. It is obvious that $\mathcal{P}(1) < 0$ is a sufficient condition, in which the two roots satisfy $0 < \lambda_1 < 1 < \lambda_2$. To show that it is also a necessary condition: conditional on that the two roots satisfy $|\lambda_1| < 1 < |\lambda_2|$, if the two roots form a complex pair, they must have a common modulus, and thus they must be either both outside unit cycle or both within the unit cycle, which contradicts with $|\lambda_1| < 1 < |\lambda_2|$. So the two roots have to be both real roots, which must be the case $0 < \lambda_1 < 1 < \lambda_2$, requiring $\mathcal{P}(1) < 0$.

Therefore, the equilibrium is determined if and only if $\gamma\psi^{-1} - \alpha + 1 < \rho$ and

$$\mathcal{P}(1) = \theta_{iy}(\frac{1}{\beta} - 1) + \frac{\kappa}{\beta}(\theta_{ix} - 1) < 0.$$

[33] Here we ignore the boundary analysis as in Woodford (2003, appx. C).