The Cost of Intermediary Market Power for Distressed Borrowers

Winston Wei Dou Wei Wang Wenyu Wang *

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Abstract

The lending market for distressed loans features an oligopolistic structure, with a few specialist lenders financing a large fraction of loans. It raises the question of how lender market power drives loan pricing in this market. We develop a dynamic game-theoretic model of strategic competition in distress loan markets with endogenous entry. Our model provides several novel implications, including the "entry effect" of collusion capacity on the number of potential specialists and thus the likelihood of ex-post inefficient last-resort lending. We estimate the structural model taking into account collusive lending by specialists and latent heterogeneity. We find that lender market power accounts for up to 90% of default-risk adjusted spreads of distressed loans, with a large fraction attributed to collusive lending and limited participation by specialist lenders. Smaller borrowers are particularly susceptible to lender market power than larger borrowers. Without lender collusion, a large fraction of distressed borrowers would switch from lenders of last resort such as hedge funds to specialized lenders and benefit from a significantly larger loan amount and a lower loan spread. Our policy analysis suggests that there exists an optimal level of spread-cap that can be imposed by regulators.

Keywords: Collusion in Syndication, Blocking Power, Agency Conflicts, Intermediary Asset Pricing, Maximum Likelihood. (**JEL:** G12, G23, G30, L13)

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*Winston Dou is with the Wharton School of University of Pennsylvania, Wei Wang is with Smith School of Business at Queen's University, and Wenyu Wang is with the Kelley School of Business at Indiana University. Emails: wdou@wharton.upenn.edu, wwang@queensu.ca, wenywang@indiana.edu. We are grateful for comments from Fernando Anjos, Alon Brav, Brent Glover, Itay Goldstein, Lars-Alexander Kuehn, Mitchell Petersen, Chester Spatt, Luke Taylor, Toni Whited, seminar participants at BPI and Nova SBE Conference in Corporate Bankruptcy and Restructuring, Carnegie Mellon University (Tepper), and Indiana University (Kelley). We acknowledge the funding support of the Social Sciences and Humanities Research Council of Canada (SSHRC). Wenyu Wang gratefully acknowledges financial support from the Daniel C. Smith Fellowship.

1 Introduction

Financially distressed firms seek urgent financing to support their working capital, investment, and debt repayment. Without such financing, their business operation would be in despair and may end up in a premature liquidation. Despite their desperation to fund operations, financially distressed firms face severe financial constraints and other economic frictions in arranging financing. More importantly, many traditional lending institutions shy away from distressed borrowers, resulting in a limited set of lenders participating in this important segment of the U.S. credit markets.[1] Despite the importance of this market to the survival of distressed firms, academic studies on understanding the economic forces that drive the pricing and quantity of distressed loans are scarce. Our study tries to fill this gap by quantifying the effect of lenders' market power on distressed loan pricing and suggesting implementable policies.

In a perfectly competitive financing market, lenders charge interest and fees that are commensurate with payment default risk and ex post re-contracting efforts with the borrower (Roberts and Sufi, 2009; Berg et al., 2016). However, financially distressed firms face major economic frictions in arranging their financing, which challenge the competitive pricing of these loans.[2] First, the borrowers' bargaining position is weak and their price elasticity of demand for loans can be low. Distressed companies face a dire liquidity situation and are in desperate need to raise capital to fund on-going operations. Importantly, these companies often do not have "unencumbered" assets to pledge for additional secured credit. Moreover, a distressed borrower is reluctant to approach a large number of lenders or even call for an open auction for getting the best priced loan due to concerns of information leakage that could negatively affects its security prices.

Second, lenders are likely to possess significant market power in financing distressed firms. Distressed borrowers' existing lenders' information advantage about borrowers discourages the participation by outsider lenders who are concerned with adverse selection and allows them to hold up borrow-

[1]Financing distressed companies has become an important concern for both policymakers and practitioners as the U.S. enters into a recession during a global pandemic and a record number of large companies filed for bankruptcy in 2020. Albeit not in bankruptcy, many firms suffered large operating losses and became financially distressed.

[2]Recent empirical studies provide supporting evidence. For example, Eckbo et al. (2020) show that bankrupt borrowers pay, on average, 600 basis points spread over the London Inter-Bank Offered Rate (LIBOR) even though the ex post default rates of these loans is minuscule. Schwert (2020) shows that loan lenders earn a large premium relative to the implied spread from prices of bonds issued by the same company, and documents that the premium is especially large for companies with high default risk.

ers for higher interest (Sharpe, 1990; Schenone, 2010; Santos and Winton, 2008). Moreover, lenders who are not familiar with or do not have expertise in restructuring of distressed companies and those with regulatory capital concerns choose not to enter the market.[3] As a result, even without a monopolistic (existing) lender in place, distressed borrowers typically find themselves facing only a clique of specialized lenders, who have incentives to collude in loan pricing (Cai, Eidam, Saunders, and Steffen, 2018; Hatfield, Kroiminer, Lowery, and Barry, 2020).

Distressed loan markets generally consist of two important segments: loans to distressed but not yet bankrupt firms (distressed loans hereafter) and loans to firms already in Chapter 11 bankruptcy, commonly known as the debtor-in-possession (DIP) loans. Despite sharing many similarities such as borrowers' poor financial health and limited debt capacity, strong bargaining power of exiting lenders, and the existence of specialized lenders, the two types of loans are priced quite differently. Specifically, DIP loans carry spreads that are much higher than distressed loans even after adjusting for default risk (Eckbo et al., 2020). Why do loans in the two markets exhibit different levels of spreads given many similarities? Our study provides answers by dissecting the lenders' market power in the two markets separately.

Quantifying the lender market power, however, is empirically challenging. This is because potential lender collusion, explicit or implicit, is unobservable to econometricians, and constructing empirical proxies to lender collusion is very difficult if not completely impossible. In addition, most distressed loans are syndicated by a small group of specialist lenders, and it remains unclear how limited participation by specialist lenders in an oligopolistic market structure drives the loan prices. Moreover, the high loan spreads do not necessarily reflect rent extraction by lenders, because high information asymmetry in this market may impose substantial private costs on lenders. Furthermore, lenders' market power also depends on borrowers' price elasticity of demand for distressed loans, which can be heterogeneous across firms and remain unobservable to outsiders.

In this paper, we overcome these challenges by constructing and estimating a unified model that characterizes both the distressed loan market and the DIP loan market. Our model takes into account the key elements discussed above. Specifically, on the demand side, the model solves the borrowers'

[3] For example, the Basel-III framework requires financial institutions to set aside larger quantity of capital for financing companies with higher default risk because of the higher regulatory risk-weighting of high risk loans.

maximization problem to derive the demand curves (as in Hendel (1999)). On the supply side, the model solves the lenders' participation decisions and optimal lending amount (as well as the equilibrium loan spread that clears the market), taking into account the private costs of lending and the endogenous market power arising from tacit coordination among the participating lenders. In particular, lender market power contains three components. First, lender market features an oligopolistic structure in the sense that a few specialist lenders intermediate a large fraction of loans and these specialists are persistent lenders over decades. Second, existing lenders often possess strong blocking power that prevents the borrowers from reaching out to alternative lenders, which increases the participation costs of specialist lenders and aggravates the limited competition in an oligopolistic market. Third, the clique of specialist lenders may choose to collude in making syndicated loans, reducing the loan amount and charging a high loan spread. Punishment on non-collusive lending acts as an implementable threat to maintain the collusive equilibrium.

The model is highly tractable and it permits semi-analytic solutions that allow us to separately identify demand, costs, and competitive effects. We solve the model and estimate it in the distressed loan market and the DIP loan market separately. We assemble a comprehensive dataset that contains 484 distressed loan facilities and 297 DIP loans to large U.S. public firms that filed for Chapter 11 bankruptcy from 2001–2019. Our sample contains detailed information of these loans, including the loan size, spread, the number of lenders in a syndication, participating lender identities, lender type, as well as characteristics of the borrowers. We estimate the model in the two markets separately, using the Markov Chain Monte Carlo (MCMC) estimator which has seen a quick growth in finance studies (Sorensen, 2007; Johannes and Polson, 2010; Kortweg, 2010). MCMC estimator is a Bayesian approach and it computes the posterior distribution of the model parameters and latent variables conditioning on the observed quantities.

Even though the model is quite generic, parameter estimates of the model can differ substantially across the two markets, which reveal the similarity across the two loan markets as well as their distinct features that help explain the differentials in loan spread. Our estimation delivers a few novel findings. First, we identify intensive lender collusion in both markets. Specifically, non-collusive lending behavior will be punished almost surely and the deviating lender will be pushed to fierce competition by other specialist lenders. Strong enforcement of punishment disciplines specialist lenders and helps maintain a collusive equilibrium. Our estimates suggest that lender collusion contributes to a sizeable component of loan spreads, ranging from 140 to 160 bps, in both markets.

Second, we find that borrowers in both markets exhibit similar and low price elasticity of demand. In other words, the excess spread present in DIP loans cannot be explained by differentials in lender collusion or price elasticity across the two markets.

According to our estimates, the factors that set the two markets apart are the specialist lenders' participation cost and their variable cost of lending. Specifically, the estimated participation cost for lenders in the DIP market is more than two times larger than that in the distressed loan market. Participation cost accounts for 250 bps in DIP loan spread but only 80 bps in distressed loan spread. An important determinant of the specialists' participation cost is the existing lender's blocking power that prevents the borrowers borrowing from alternative lenders. Our results therefore suggest that existing lenders in the DIP market have much stronger blocking power than those in the distressed loan market.

We also find that lenders in DIP market incur significantly higher variable cost of lending. The estimated variable cost is 180 bps in DIP market while only about 10 bps in the distressed loan market. Since the estimator matches the risk-adjusted spread in which both the risk free rate, credit risk premium, and liquidity premium are purged out, the variable cost is largely related to lenders' efforts in monitoring and governing borrowers. Our estimates thus indicate that DIP lenders face higher maintenance costs than distressed loan lenders.

Our decomposition of loan spreads in both market allows us to further analyze which types of borrowers are particularly susceptible to lender market power. We partition the borrowers in our sample into size quintiles and repeat our analyses above on the small (bottome quintile) and large (top quintile) borrowers. Our findings are striking: shutting down lender collusion reduces the loan spread by about 60 bps for large borrowers and by 180-200 bps for small borrowers in both markets. Our analyses thus suggests that small borrowers are more vulnerable to lender market power and policies aiming at helping distressed companies in economic and financial crises should target more small firms given their weak bargaining power and disadvantageous position in the distressed loan market.

Lastly, we use the estimated model as a laboratory to examine the effect of a widely debated regulatory intervention – interest rate cap. We allow the regulator to impose a cap on the specialist lenders' markup in lending and thus restrict the loan spread that can be charged by these lenders. We analyze the specialist lenders' strategic responses to the rate-cap policy and the consequence on borrowers' welfare. Solving the model with rate-cap reveals a few intriguing implications. First, as loan spreads are capped, specialist lenders have less incentive to collude to a small loan size and thus they capture the residual profits by increasing the loan amount. Higher loan amount and lower spread improves the borrowers' welfare when they borrow from specialist lenders (i.e., a positive intensity margin). But meanwhile, rate-cap reduces the expected profits by specialist lenders and thus discourage their participation in the lending market. As fewer specialists are willing to lend in the market, the likelihood for the borrowers to borrow from the lenders of last-resort rises. Since loans made by lenders of last-resort are much more expensive, rate-cap generates an unintended consequence of reducing the depth of specialist lenders' market (i.e., a negative extensive margin). Combining the two effects, we demonstrate that the effect of rate-cap on borrower welfare is hump-shaped and there exists an optimal level of rate-cap. The optimal spread averages about 90 bps for distressed loans and 460 bps for DIP loans, compared with 241 bps and 631 bps observed in the data.

Related Literature. Our paper contributes to the literature on loan pricing and contracting. Prior studies document the effect of information problems, lending relationship, syndicate structure, and lender specific characteristics on the pricing of corporate loans.[4] A few studies suggest that loans to distressed companies are not priced competitively and lenders can capture economic rents (Chatterjee et al., 2004; Li et al., 2019; Hasan et al., 2019; Eckbo et al., 2020). Schwert (2020) shows that loan lenders earn a large premium relative to the bondholders of the same company, and documents that the premium is especially large for companies with high default risk. Cai et al. (2018) and Hatfield et al. (2020) suggest that price collusion by syndicate members may exist in syndicated loan markets.

Our study differs from prior papers on a few important fronts. First, we develop a theoretical model that takes into account the real-world frictions presented in the lending markets, including lenders' private funding costs, borrowers' downward sloping demand curve, and costs of collusion and monitor-

[4]See, Rajan (1992); Petersen and Rajan (1994); Sufi (2007); Ivashina and Scharfstein (2010); Bharath, Dahiya, Saunders, and Srinivasan (2011); Chava and Purnanandam (2011) and Murfin (2012), among others.

ing. Second, fitting this model to our hand-collected novel dataset, we are able to quantify the margins earned by specialized lenders from distressed borrowers as a result of collusion versus private funding costs. More importantly, our study provides policy implications by showing that smaller borrowers are most susceptible to lender collusion and thus adds value to the recent policy debate on whether government should step in to finance bankrupt firms.[5]

2 Game-theoretic model for distressed-loan markets

In this section, we build a novel and flexible game-theoretic model of distressed loan market competition with repeated collaboration relations among specialized lenders and endogenous entries as participants of syndicated lending. We first describe the model's setup, then explain the predictions that form the basis of our estimation. The model features a financially distressed firm which borrows from the distressed loan market dominated in the hands of a few specialized lenders. The repeated oligopoly lending game features a downward-sloping demand system with heterogeneous borrowers with latent characteristics, tacit collusion among specialized lenders, and endogenous market concentration and market power.

2.1 Generic setup

The model starts with M specialized and repeated major lenders in a specific loan market, who are referred to as "specialists" throughout the paper, and M_0 potential lenders, including non-specialized lenders, in the market with $M \ll M_0$. Because of the specialists' dominant position is highly persistent and there are quite a few large deals arrive in the market every year, the M specialists have strong incentives to form tacit collusion and collaborate in terms of syndicating the distressed loans. "Tacit coordination" need not involve any explicit collusion with direct communication and agreement on joint decisions in the legal sense, and an interchangeable term is "tacit collusion" or "noncooperative collusion" or "conscious parallelism" (e.g., Martin, 2006; Ivaldi et al., 2007; Harrington, 2008; Green et al., 2014; Garrod and Olczak, 2018).

[5] In considering the difficulty of smaller distressed borrowers to access financing, the Consolidated Appropriations Act of 2021 ("CAA") that was signed into law on December 27, 2020 allows small debtors in bankruptcy to access the Paycheck Protection Program ("PPP") for loans. However, Small Business Administration ("SBA") holds a different view.

The deals arrive randomly. We assume that the arrivals of deals can be characterized by a Poisson process with intensity η. Each borrower can be one of the K types, and we label a borrower i by k_i for $k_i \in \{1, \cdots, K\}$. The type k_i reflects two fundamental characteristics of borrower i in our model: first, it captures the borrower's size A_i measured by its total value of asset, and second, as to be specified in the next section, our model allows heterogeneous demand curves across different borrowers, and thus k_i captures the demand curve borrower i is attached to, d_i. We denote $k_i = (d_i, A_i)$. Even though size A_i is observable, the demand curve d_i is seen only by the agents inside the model and is latent to the econometricians. We denote the fraction of type k borrowers in the population by $\pi(k)$. Each borrower of type k starts with trying to match with a existing lender with a likelihood, denoted by $\lambda(k)$. If the borrower fails to reach an agreement with the existing lender, then the borrower continues to approach the specialists for funding. It is possible that no specialist is willing to participate in the deal, in which case the borrower has to find the last-resort non-specialized lender. We classify lenders into three types, denoted by $l \in \{1, 2, 3\}$. Particularly, we label the existing lender by $l = 1$, the specialist lender by $l = 2$, and the last-resort non-specialized lender by $l = 3$. Before elaborating the timeline and sequence of lenders' decisions for each deal in Section 2.1.2, we first introduce the basics about borrowers and lenders below in Sections 2.1.1 and 2.1.2.

2.1.1 Demand specification

When the borrower has type k and faces the lenders of type l, the value of the borrower by raising the amount of loan L is

$$W = \underbrace{L^{\beta_l(k)} S_l(k)^{1-\beta_l(k)}}_{\text{PV of running business}} - \underbrace{L\mathbb{E}\left[e^{-r_f} Y\right]}_{\text{PV of borrowing loans}}, \tag{1}$$

where $\beta_l(k) \in (0, 1)$ captures the decreasing return to scale of the amount of distressed loans, r_f is the log riskfree rate, $\mathbb{E}[\cdot]$ is the risk-neutral expectation, Y is the total return of the distressed loan, and $S_l(k)$ is the return shifter, a quantity that captures the effect of firm characteristics and incorporates the interactions between investor and asset characteristics (like in Hendel, 1999). The return shifter $S_l(k)$ is specified as follows:

$$S_l(k) \equiv A e^{\nu_l(k) + \sigma z}, \tag{2}$$

where A is the total asset of the firm, $\nu_l(k)$ captures the effect of the interactions between investor and asset characteristics, and z is a latent firm-specific demand shock.

The total return of the loan Y is e^y with a risk-neutral probability $1-q^*$ and is δe^y with a risk-neutral probability q^*, where the probability q^* and the recovery rate δ are firm specific, and y is log yield on the distressed loan. The risk-neutral default probability q^* and Thus, the firm-level value function can be rewritten as

$$W = L^{\beta_l(k)} S_l(k)^{1-\beta_l(k)} - LR, \tag{3}$$

where $\ln(R) \equiv y - r_f - d$ with $d \equiv -\ln[1-q^*(1-\delta)]$. We refer to R as the risk-adjusted spread of the distressed loan.

The first-order condition for L leads to a demand curve of loan size for a type-k borrower and type-l lenders as follows:

$$\ln\left(\frac{L}{A}\right) = \alpha_l(k) - \varepsilon_l(k)\ln(R) + \sigma z, \tag{4}$$

where

$$\alpha_l(k) \equiv \nu_l(k) + \frac{\ln \beta_l(k)}{1-\beta_l(k)}, \tag{5}$$

$$\varepsilon_l(k) \equiv \frac{1}{1-\beta_l(k)}. \tag{6}$$

The variation in the term $\alpha_l(k) + \sigma z$ captures the demand curve shift across different firms in the population. The coefficient $\alpha_l(k)$ captures the heterogeneous demand level depending on different borrower types k and lender types l. The borrower type k is latent to the econometricians, whereas the lender type l is observable to the econometricians.

The demand function in equation (4) is a standard iso-elastic downward-sloping demand curve. The coefficient $\varepsilon_l(k)$ is effectively the price elasticity of demand. Consistent with the literature (e.g., Atkeson and Burstein, 2008), we assume that $\varepsilon_l(k) > 1$.

We assume that z is i.i.d. distributed according to the standard normal distribution across different cases. The assumption that the demand curve depends on heterogeneous borrowers and differentiated lenders is a natural analog of that in the empirical IO literature emphasizing heterogeneous consumers and differentiated products (e.g., Berry et al., 1995). A higher $\alpha_l(k)$ means that the borrower of type k has higher average demand of loans from the lenders of type l.

Intuitively, charging higher spread R leads to smaller loan amount because $\varepsilon_l(k) > 1$. The slope coefficient $\varepsilon_l(k)$ mainly captures the price elasticity of the type-k borrower's loan demand from the type-l lenders. Similar in spirit to Koijen and Yogo (2019), the slope coefficient is determined by multiple important primitive characteristics of borrowers and lenders. Like the approach adopted by Koijen and Yogo (2015, 2019), among others, in their structural estimation, we start with specifying a flexible demand system on top of a structural agent-optimization model, rather than microfounding the equilibrium relations between the slope coefficient $\varepsilon_l(k)$ and various primitive characteristics of borrowers and lenders. Specifically, $\varepsilon_l(k)$ can reflect the bargaining power of the borrower and the lenders in the loan market, the marginal value of liquidity of the borrower, the intervention style of the lenders, and the non-pecuniary benefits to the firm from borrowing from the relationship lenders. With a larger $\varepsilon_l(k)$, the amount of loans L the firm would like to borrow declines more with an increase in loan spread R required by the lender, meaning that the borrower's demand for loans is more elastic to the loan spread and thus the borrower has more bargaining power. Take an extreme case as an example to illustrate the role of the slope coefficient $\varepsilon_l(k)$. When $\varepsilon_l(k) \to +\infty$, the borrower has full bargaining power over the lenders since it is not willing to pay any spread on top of the risk-adjusted rate. In such an extreme case, a higher $\alpha_l(k)$ obviously reflects a higher intrinsic loan demand from the borrower.

Demand curves can also vary by lender types. Classic banking theories suggest that banks act as traditional financial intermediaries to provide financing and acting as the delegated monitors (e.g. Diamond (1984, 1991)). Banks rarely seek direct control of the borrower through board representation and other mechanisms due to their concerns of legal liabilities (Fischel, 1989). The loan costs to the borrower are mostly reflected in spreads and fees to be paid to the lender. In contrast, alternative investors such as hedge funds and private equity firms use loan instruments as a tool to engage activism and seek control. They often adopt the "loan-to-own" strategies where they lend to firms with an intention to convert debt into ultimate equity ownership (Jiang et al., 2012). In addition to loan interest and fees, these alternative lenders impose strict covenants that may be tied to management changes and governance, which allow them to directly control the borrower's business operation. As a result, to borrowers, loans provided by traditional lenders and alternative lenders may be viewed as different products.

We normalize the loan size L by the total asset A in the modeling of the demand system for distressed loans in equation (4). The main reasons or motivations behind such modeling choice are threefold. First, the additional financial distress risk caused by the newly-added leveraged distressed loan L depends on the total asset of the borrower A. Second, the leveraged distressed loan L is mainly for covering working capital, which in turn is usually proportional to the firm size and thus the total asset level A. Third, in the data, we do find that a strong relationship between the normalized loan size L/A and the spread R within each type of deals.

2.1.2 Supply specification

We first describe the market structure and "technology" of the lenders to make distressed loans. The supply side of distressed loans is characterized by Cournot competition of oligopolistic lenders similar to Berry et al. (1995). The oligopolies can tacit collude usually in the form of syndication, which goes beyond the non-collusive Nash behavior adopted by the BLP framework and captures the highly strategic competition behavior (i.e., tacit coordination) similar to, for example, Chen et al. (2021) and Dou et al. (2021). The literature shows that some credit markets are concentrated in the hands of a few leading financial institutions, and importantly, these institutional lenders compete highly strategically including competition in the form of tacit collusion. For example, Knittel and Stango (2003) documents micro-level evidence suggesting that tacit collusion at non-binding state-level ceilings was prevalent in the credit card lending market during the early 1980's. Moreover, on 5 April 2019, the European Commission published a report – prepared by Europe Economics at the request of the department for competition – on EU loan syndication and its impact on competition in credit markets. The commission had worried that syndicated loan coordination tended to occur tacitly, making anti-competitive collusion on interest rates easier to pull off. The worry is natural because syndicated loans naturally require coordination and communication. The report highlighted the risk that lenders involved in syndicating a loan together would promise future tacit cooperation in exchange for current concessions on limiting loan supply. Nocke and White (2007) and Hatfield et al. (2020) theoretically show that, under certain circumstances, tacit collusion can exist in syndicated markets with repeated interaction of lenders. Further, Carrasco and Manso (2006) theoretically show that syndication is the optimal response of colluding lenders to the communication costs resulting from the negotiations between them for a given loan. Our model builds on the important insights of loan syndication and potential tacit collusion of repeated interacted lenders in the distressed loan markets.

Costs of lending distressed loans. The lender incurs a fixed cost from learning the deal type and participating in lending, and it also incurs a variable cost that depends on the amount of loan made to the borrower L. Specifically, the lender incurs a one-time fixed cost of w if it decides to learn the deal type k and commit to participation. Fixed cost w is random and privately observed by the lender. However, the distribution of w is a common knowledge of all agents and is assumed to be the exponential distribution with mean μ with the following density function:

$$f(w;\mu) = \mu e^{-w/\mu}. \tag{7}$$

Fixed cost w captures both direct costs, such as compensation to talents and other labor costs, as well as indirect costs, such as the loss of other investment opportunities because of limited resources. In addition, the lender of type i incurs variable costs of $e^{\phi_i+\varsigma u}$ for each unit of lending, where u is a standard normal random variable that captures the deal-specific latent cost. Taken together, both latent characteristics z and u are deal specific (or firm specific). We denote $x \equiv (z, u)$. Variable cost $e^{\phi_i+\varsigma u}$ captures both direct costs, such as compensation, as well as indirect costs, such as marginal (shadow) costs of funding for the lender. Consistent with this assumption, we find that the lender explicitly charges proportional fees to cover the variable costs. For any deal, total costs from w and $e^{\phi_i+\varsigma u}$ are denoted by

$$C_i(L) \equiv w\mathbf{1}_{\{L>0\}} + e^{\phi_i+\varsigma u}L. \tag{8}$$

Timing and sequence of lenders' decisions within each deal. Figure 1 illustrates lenders' choices and possible outcomes in each deal, including lending by existing lenders and last-resort lending possibilities. The time span for each deal is divided into two subperiods, "morning" and "afternoon." The shocks, such as whether to punish for deviation or not, the type of the deal k, and the private costs $(w_1, \cdots, w_M)$, are realized in the morning, while the lending decisions L and R are made in the afternoon. Importantly, specialists must make their decisions whether to participate in the syndicated lending by the end of the morning, and no specialist who has already committed can leave without participation after learning the number of participants m in the afternoon.

In the first step of "morning", the firm approaches the existing lender, who can be a specialized or non-specialized lender. We assume that each of the M_0 potential lenders can be the existing lender with equal chances (i.e., $1/M_0$). The lending agreement with this particular existing lender can only be achieved with a small probability $\lambda(k)$ which depends on case type k. If the lending agreement with the existing lender is reached, the monopolistic lender of type $l = 1$ chooses the optimal spread and the loan size according to the demand curve:

$$\Pi_1(k, x) = \max_L \left[\left(e^{\alpha_1(k)+\sigma z} \frac{A}{L} \right)^{1/\varepsilon_1(k)} - e^{\phi_1+\varsigma u} \right] L, \tag{9}$$

where the default probability on the distressed loan does not show up since $R = \left(e^{\alpha_1(k)+\sigma z} A/L\right)^{1/\varepsilon_1(k)}$ is a risk-adjusted loan spread, and $x \equiv (z, u)$. It leads to the optimal monopolistic spread and loan size:

$$R_1(k, x) = \frac{\varepsilon_1(k)}{\varepsilon_1(k) - 1} e^{\phi_1+\varsigma u} \quad \text{and} \quad L_1(k, x) = \left[1 - \frac{1}{\varepsilon_1(k)}\right]^{\varepsilon_1(k)} e^{\alpha_1(k) - \varepsilon_1(k)(\phi_1+\varsigma u)+\sigma z} A, \quad \text{respectively.} \tag{10}$$

Therefore, the optimal profit is

$$\Pi_1(k, x) = \frac{1}{\varepsilon_1(k)} R_1(k, x) L_1(k, x). \tag{11}$$

Loan markup is defined as $\left[R_1(k, x) - e^{\phi_1+\varsigma u}\right] / e^{\phi_1+\varsigma u}$. The loan spread implies that the loan markup is $1/[\varepsilon_1(k) - 1]$, suggesting that the markup ratio decreases with the elasticity coefficient $\varepsilon_1(k)$. The optimal profit $\Pi_1(k, x)$ is $1/\varepsilon_1(k)$ fraction of the revenue $R_1(k, x)L_1(k, x)$. That is, the profit margin is $1/\varepsilon_1(k)$. When the elasticity coefficient $\varepsilon_1(k)$ is lower, the borrower's loan demand is effectively more urgent and pressing, leading to a higher markup and a higher profit margin. The detailed derivations of (10) and (11) are in the appendix.

The expected optimal profit is

$$\begin{aligned}\Pi_1(k) &= \mathbb{E}^x\left[\Pi_1(k,x)\right] \\ &= \frac{1}{\varepsilon_1(k)}\left[1-\frac{1}{\varepsilon_1(k)}\right]^{\varepsilon_1(k)-1} e^{\alpha_1(k)-[\varepsilon_1(k)-1]\phi_1+[\varepsilon_1(k)-1]^2\varsigma^2/2+\sigma^2/2}A.\end{aligned}$$

The game moves to the second stage of "morning" if the firm fails to get a deal from the existing lender. In this case, the competition takes place among the M specialists (type $l=2$). We consider tacit collusion of a few specialized lenders. Let $V^C(k,x,w,m;L^C)$ be the collusive value function of each specialist at the beginning of "afternoon", if the agreed loan plan is $L^C(k,x,m)$, there are m specialists who decided to participate in the deal of syndicated loan, the case is type k, the specialist has private cost w, and the deal-specific characteristics x. There exists an equilibrium threshold w_C^* such that the specialist would participate in the syndicated loan if and only if $w \le w_C^*$.

Note that the privately observed fixed cost w is a sunk cost when the lender chooses the loan amount L. Because $V^C(k,x,w,m;L^C)$ is value function of a specialist at the beginning of the "afternoon" when w, k, and x are already observed, the value function has the following functional form:

$$V^C(k,x,w,m;L^C) \equiv U^C(k,x,m;L^C) - w. \tag{12}$$

The value function $U^C(k,x,m;L^C)$ satisfies the following Bellman equation:

$$U^C(k,x,m;L^C) = \Pi_2(k,x,m;L^C) \quad + \quad \frac{W^C(L^C)}{1-\delta}, \quad \text{where} \tag{13}$$

$$W^C(L^C) = \mathbb{E}^{k'}\left\{\lambda(k')\frac{\Pi_1(k')}{M_0} + \left[1-\lambda(k')\right]\sum_{m'=1}^{M} q(m'|w' \le w_C^*)\left[F(w_C^*)\Pi_2(k',m';L^C) - \int_{w' \le w_C^*} w' dF(w')\right]\right\},$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1,\cdots,K\}$ with probability weight $\pi(k')$ for each k', and $\Pi_2(k,m;L^C) \equiv \mathbb{E}^x\left[\Pi_2(k,x,m;L^C)\right]$ with the profit of the syndicated lending with tacit collusive loan size plan L^C to be

$$\Pi_2(k,x,m;L^C) \equiv \left[\left(e^{\alpha_2(k)+\sigma z}\frac{A}{mL^C(k,x,m)}\right)^{1/\varepsilon_2(k)} - e^{\phi_2+\varsigma u}\right] L^C(k,x,m). \tag{14}$$

and the conditional probability $q(m|w \leq w_C^*)$ is

$$q(m|w \leq w_C^*) = \frac{\mathbb{P}\left\{\text{This specialist and other } m-1 \text{ specialists participate the lending}\right\}}{\mathbb{P}\left\{\text{This specialist participates the lending}\right\}}$$
$$= \binom{M-1}{m-1} F(w_C^*)^{m-1} \left[1 - F(w_C^*)\right]^{M-m}.$$

The derivation of this Bellman equation is in the appendix.

The game moves to the last-resort stage of "afternoon" if no specialists would like to participate the lending. In this case, the last resort (a non-specialized lender, i.e., the lender of type $i = 3$) will lend to the firm as a monopoly with a fixed cost w and variable costs $e^{\phi_3 + \varsigma u}$. The last-resort lender optimally chooses $L_3(k)$ to maximize the profit:

$$\Pi_3(k, x) = \max_L \left[\left(e^{\alpha_3(k) + \sigma z} \frac{A}{L} \right)^{1/\varepsilon_3(k)} - e^{\phi_3 + \varsigma u} \right] L. \tag{15}$$

It leads to the optimal monopolistic spread and loan size:

$$R_3(k, x) = \frac{\varepsilon_3(k)}{\varepsilon_3(k) - 1} e^{\phi_3 + \varsigma u} \quad \text{and} \quad L_3(k, x) = \left[1 - \frac{1}{\varepsilon_3(k)} \right]^{\varepsilon_3(k)} e^{\alpha_3(k) - \varepsilon_3(k)(\phi_3 + \varsigma u) + \sigma z} A, \quad \text{respectively.} \tag{16}$$

Therefore, the optimal profit is $1/\varepsilon_3(k)$ fraction of the revenue $R_3(k, x)L_3(k, x)$ as follows:

$$\Pi_3(k, x) = \frac{1}{\varepsilon_3(k)} R_3(k, x) L_3(k, x). \tag{17}$$

Similar to the case in which the existing lender supplies all the loan, lower elasticity $\varepsilon_3(k)$ leads to a higher markup $1/[\varepsilon_3(k) - 1]$ and a higher profit margin $1/\varepsilon_3(k)$. The detailed derivations of (16) and (17) are in the appendix.

2.1.3 Collusive Nash Equilibrium

The equilibrium outcomes in the lending of existing creditors and that of last-resort lenders can be explicitly characterized in (10) – (11) and (16) – (17), respectively. The equilibrium collusive loan amount is the central piece of model outcome that needs to be pinned down, which we explain in this subsection.

Optimal Lending under Tacit Collusion and Incentive Compatibility Constraint. We first denote the unconstrained optimal loan amount under tacit collusion by $L^C_{max}(k, x, m)$, which solves the following profit maximization problem:

$$L^C_{max}(k, x, m) \equiv \underset{L}{\text{argmax}} \left[\left(e^{\alpha_2(k)+\sigma z} \frac{A}{mL} \right)^{1/\varepsilon_2(k)} - e^{\phi_2+\varsigma u} \right] L. \tag{18}$$

The first-order condition gives the solution for $L^C_{max}(k, x, m)$ as follows:

$$L^C_{max}(k, x, m) \equiv \frac{1}{m} \left[1 - \frac{1}{\varepsilon_2(k)} \right]^{\varepsilon_2(k)} e^{-\varepsilon_2(k)\phi_2 - \varepsilon_2(k)\varsigma u} e^{\alpha_2(k)+\sigma z} A, \tag{19}$$

with the corresponding unconstrained optimal loan spread and lending profit under tacit collusion to be

$$R^C_{max}(k, x, m) = \frac{\varepsilon_2(k)}{\varepsilon_2(k) - 1} e^{\phi_2+\varsigma u} \text{ and } \Pi^C_{max} = \frac{1}{\varepsilon_2(k)} R^C_{max}(k, x, m) L^C_{max}(k, x, m), \text{ respectively.} \tag{20}$$

Comparing (19) – (20) with (10) – (11), the intuition behind the smallest possible collusive lending outcomes immediately follows. That is, the m syndication participants first form the strongest coalition that behaves as if it is a monopoly, and then they split the loan equally.

However, the unconstrained optimal collusive loan size $L^C_{max}(k, x, m)$, derived in (19), is usually unsustainable in the equilibrium because syndication participants would have strong incentives to deviate and reap additional profits by secretly supplying extra loans to the borrower. To sustain the tacit coordination among the specialized lenders as the club members who repeatedly participate in a particular market of distressed syndicated loans, the specialized lenders have an imperfect capacity of monitoring, communication, and ex-post punishment. Specifically, upon a deviation is detected, the specialized lenders will not tacitly collude anymore starting from the next period with a probability ξ as the punishment for deviation. This grim trigger punishment strategy is easy to implement and incentive compatible. There is a probability $1 - \xi$ that the deviation will not be punished. Thus, the parameter ξ captures the tacit collusion capacity in a parsimonious way. A lower ξ reflects a lower tacit collusion capacity, which can be due to more costly monitoring, more costly communications, and higher chances of achieving successful ex-post renegotiation. In our theory and structural estimation, we capture and estimate the tacit collusion capacity by focusing the deep structural parameter ξ without specifying or estimating various possible economic mechanisms that micro-found the imperfect tacit collusion at a more granular level, which is beyond the scope of this paper.

Now we characterize the set of loan sizes that are sustainable under the tacit collusion scheme with collusion capacity captured by parameter ξ, namely, the set of loan sizes that satisfy the incentive-compatibility constraint. Intuitively, the loan size $L^C(k, x, m)$ under the tacit collusion cannot be overly small to ensure that the m participants will have no incentives to deviate. We denote by $\mathcal{L}^C(k, x, m)$ the set of loan sizes for each of the m syndication participants that satisfy the incentive-compatibility constraint, which can be expressed as follows:

$$\mathcal{L}^C(k, x, m) \equiv \left\{ L^C : \mathbb{E}^x \left[U^C(k, x, m; L^C) \right] \geq \mathbb{E}^x \left[U^D(k, x, m; L^C) \right] \right\}, \tag{21}$$

where $U^C(k, x, m; L^C)$ is the value function when all the m syndication participants stick to the tacit coordination scheme of specialized lenders in the market as club members, and $U^D(k, x, m; L^C)$ is the maximum value of the syndication participant that deviates from the given tacit coordination scheme $L^C(\cdot, \cdot, \cdot)$, which is described in detail below.

Intuitively, the tacit coordination scheme on loan size $L^C(\cdot, \cdot, \cdot)$ lies in the set $\mathcal{L}^C(k, x, m)$ if and only if the expected value of not deviating, $\mathbb{E}^x \left[U^C(k, x, m; L^C) \right]$, is not strictly dominated by that of deviating, $\mathbb{E}^x \left[U^D(k, x, m; L^C) \right]$. As explained in Figure 1, the deviation decision is made after the syndication participants learn their private lending cost w, borrower type k, and the number of syndication participants m, but before the latent case-specific characteristics x are learned. As a result, each syndication participant compares the two expected values contingent on not deviating or deviating.

Non-Collusive Nash Equilibrium and Maximum Value of Deviation. To characterize the maximum value of deviation $U^D(k, x, m; L^C)$, we need to first characterize the phase of non-collusion competition among specialized lenders, where the outcome is described by the non-collusive Nash equilibrium. This is because the punishment for deviation is to shift into the phase of non-collusive competition.

The game moves to the second stage of "morning" if the firm fails to get a deal from the prepetition lender. In this case, the competition takes place among the M specialists (i.e. the lenders of type $l = 2$).

We consider the non-collusive Nash equilibrium in which the syndication participants never tacitly coordinate on making the loan. Let $V^N(k,x,w,m)$ be the non-collusive value function of each specialist at the end of "morning," a function of k, x, and m, there are m specialists who decided to participate in the deal of syndicated loan, the case is type k, and the specialist has private cost w. There exists an equilibrium threshold w_N^* such that the specialist would participate in the syndicated loan if and only if $w \leq w_N^*$.

Note that the privately observed fixed cost w is a sunk cost when the lender makes decision on L. Because $V^N(k,x,w,m)$ is value function of a specialist at the end of the morning when w is already privately observed, the value function has the following functional form:

$$V^N(k,x,w,m) \equiv U^N(k,x,m) - w. \tag{22}$$

The value function $U^N(k,x,m)$ prior to paying the fixed cost w and observing the deal-specific characteristics $x = (z,u)$ satisfies the following Bellman equation:

$$U^N(k,x,m) = \Pi_2(k,x,m;L^N) \quad + \quad \frac{W^N}{1-\delta}, \quad \text{with} \tag{23}$$
$$W^N = \mathbb{E}^{k'}\left\{\lambda(k')\frac{\Pi_1(k')}{M_0} + \left[1-\lambda(k')\right]\sum_{m'=1}^{M} q(m'|w' \leq w_N^*)\left[F(w_N^*)\Pi_2(k',m';L^N) - \int_{w' \leq w_N^*} w' dF(w')\right]\right\},$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1,\cdots,K\}$ with probability weight $\pi(k')$ for each k', and $\Pi_2(k,m;L^N) \equiv \mathbb{E}^x\left[\Pi_2(k,x,m;L^N)\right]$ with

$$\Pi_2(k,x,m;L^N) \equiv \max_L \left[\left(e^{\alpha_2(k)+\sigma z}\frac{A}{L+(m-1)L^N(k,m)}\right)^{1/\varepsilon_2(k)} - e^{\phi_2+\varsigma u}\right] L, \tag{24}$$

and the conditional probability $q(m|w \leq w_N^*)$ is

$$\begin{aligned} q(m|w \leq w_N^*) &= \frac{\mathbb{P}\left\{\text{This specialist and other } m-1 \text{ specialists participate the lending}\right\}}{\mathbb{P}\left\{\text{This specialist participates the lending}\right\}} \\ &= \binom{M-1}{m-1} F(w_N^*)^{m-1}\left[1-F(w_N^*)\right]^{M-m}. \end{aligned}$$

The derivation of this Bellman equation is in the appendix.

In the non-collusive equilibrium, the equilibrium loan size $L^N(k,x,m)$ is characterized as follows:

$$L^N(k,x,m) = \underset{L}{\operatorname{argmax}} \left[\left(e^{\alpha_2(k)+\sigma z} \frac{A}{L+(m-1)L^N(k,x,m)} \right)^{1/\varepsilon_2(k)} - e^{\phi_2+\varsigma u} \right] L, \tag{25}$$

which leads to

$$L^N(k,x,m) = \frac{1}{m}\left[1 - \frac{1}{m\varepsilon_2(k)}\right]^{\varepsilon_2(k)} e^{-\varepsilon_2(k)\phi_2 - \varepsilon_2(k)\varsigma u} e^{\alpha_2(k)+\sigma z} A. \tag{26}$$

Therefore, the equilibrium revenue of a syndication participant has the following closed-form expression:

$$\Pi_2(k,x,m;L^N) = \frac{1}{m^2\varepsilon_2(k)}\left[1 - \frac{1}{m\varepsilon_2(k)}\right]^{\varepsilon_2(k)-1} e^{-[\varepsilon_2(k)-1](\phi_2+\varsigma u)} e^{\alpha_2(k)+\sigma z} A. \tag{27}$$

Finally, we now characterize the maximum value of deviating from an agreed tacit coordination scheme $L^C(\cdot,\cdot,\cdot)$. We denote by $V^D(k,x,w,m;L^C)$ the value function for deviation given a fixed tacit coordination scheme $L^C(\cdot,\cdot,\cdot)$ and state variables k, m, and w. We define $V^D(k,x,w,m;L^C) \equiv U^D(k,x,m;L^C) - w$, and thus, the value function $U^D(k,x,m;L^C)$ satisfies the following Bellman equation:

$$U^D(k,x,m;L^C) = \Pi_2^D(k,x,m;L^C) + \frac{(1-\xi)W^C(L^C) + \xi W^N}{1-\delta}, \tag{28}$$

where $W^C(L^C)$ is the continuation value with the tacit coordination scheme $L^C(\cdot,\cdot,\cdot)$, defined in (53), W^N is the continuation value in the phase of non-collusive competition, defined in (23), and the contemporaneous profit gained if the syndication participant deviate from the tacit coordination scheme $L^C(\cdot,\cdot,\cdot)$ is

$$\Pi_2^D(k,x,m;L^C) \equiv \max_{L>0} \left[\left(e^{\alpha_2(k)+\sigma z} \frac{A}{L+(m-1)L^C(k,x,m)} \right)^{1/\varepsilon_2(k)} - e^{\phi_2+\varsigma u} \right] L. \tag{29}$$

Thus, according to (28) and (29), the set of loan size schemes $L^C(\cdot,\cdot,\cdot)$ under tacit collusion competition that satisfy the incentive-compatibility constraint can be rewritten as

$$\mathcal{L}^C(k,x,m) \equiv \left\{ L^C: \ \frac{\xi[W^C(L^C) - W^N]}{1-\delta} \geq \mathbb{E}^x\left[\Pi_2^D(k,x,m;L^C)\right] - \mathbb{E}^x\left[\Pi_2(k,x,m;L^C)\right] \right\}. \tag{30}$$

2.1.4 Endogenous Participation Boundaries

The cutoff points w_N^* and w_C^* are determined as in Li and Zheng (2009). Like solving value functions and optimal policies, we solve w_N^*, then we solve w_C^*. We assume that w is first revealed and specialists choose whether to learn, then k and m are revealed.

We first solve w_N^* according to the condition:

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\pi(k)q(m|w=w_N^*,w^*=w_N^*)\left[U^N(k,m;w_N^*)-w_N^*\right]=\frac{W^N(w_N^*)}{1-\delta}, \tag{31}$$

meaning that the marginal specialist with $w=w_N^*$ is indifferent between participating and not participating the syndicated lending. In other words, the marginal specialist, on average, has zero gain or loss from participating the syndicated lending. Here, $W^N(w_N^*)$ is the continuation value if the cutoff is w_N^*, $\pi(k)$ is the probability of type k, and $q(m|w=w_N^*,w^*=w_N^*)$ is the probability of m participants conditioning on the cost of the specialist is w_N^* and the cutoff w^* is w_N^*. $q(m|w=w_N^*,w^*=w_N^*)$ has the following expression:

$$q(m|w=w_N^*,w^*=w_N^*)=\binom{M-1}{m-1}F(w_N^*)^{m-1}\left[1-F(w_N^*)\right]^{M-m}. \tag{32}$$

The equality (31) can be written as

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\pi(k)q(m|w=w_N^*,w^*=w_N^*)\mathbb{E}^x\left[\Pi_2(k,x,m;L^N)\right]=w_N^*, \tag{33}$$

It can be numerically solved in the following steps:

(i) Solve $\mathbb{E}^x\left[\Pi_2(k,x,m;L^N)\right]$ for each pair of (k,m).

(ii) Guess an initial w_N^*. If $\sum_{m=1}^{M}\sum_{k=1}^{K}\pi(k)q(m|w=w_N^*,w^*=w_N^*)\mathbb{E}^x\left[\Pi_2(k,x,m;L^N)\right]<w_N^*$, we should decrease w_N^*.

(iii) Iterate until the condition is satisfied.

Then, we solve w_C^* according to the condition:

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\pi(k)q(m|w=w_C^*,w^*=w_C^*)\left[U^C(k,m;w_N^*,w_C^*)-w_C^*\right]=\frac{W^C(w_N^*,w_C^*)}{1-\delta}, \tag{34}$$

where w_N^* is solved earlier. This means that the marginal specialist with $w=w_C^*$ is indifferent between participating and not participating the syndicated lending. In other words, the marginal specialist, on average, has zero gain or loss from participating the syndicated lending. Here, $W^C(w_N^*,w_C^*)$ is the continuation value if the cutoff points are w_N^* and w_C^* for non-collusive and collusive Nash equilibria, $\pi(k)$ is the probability of type k, and $q(m|w=w_C^*,w^*=w_C^*)$ is the probability of m participants conditioning on the cost of the specialist is w_C^* and the cutoff w^* is w_C^*. $q(m|w=w_C^*,w^*=w_C^*)$ has the following expression:

$$q(m|w=w_C^*,w^*=w_C^*)=\binom{M-1}{m-1}F(w_C^*)^{m-1}\left[1-F(w_C^*)\right]^{M-m}. \tag{35}$$

The equality (34) can be written as

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\pi(k)q(m|w=w_C^*,w^*=w_C^*)\mathbb{E}^x\left[\Pi_2(k,x,m;L^C)\right]=w_C^*, \tag{36}$$

It can also be numerically solved in the following steps:

(i) Solve $\mathbb{E}^x\left[\Pi_2(k,x,m;L^C)\right]$ for each pair of (k,m).

(ii) Guess an initial w_C^*. If $\sum_{m=1}^{M}\sum_{k=1}^{K}\pi(k)q(m|w=w_C^*,w^*=w_C^*)\mathbb{E}^x\left[\Pi_2(k,x,m;L^C)\right]<w_C^*$, we should decrease w_C^*.

(iii) Iterate until the condition is satisfied.

2.2 Model solution

There is a theoretical property of the model that can ensure the semi-closed-form solution and help immensely simplify the numerical analysis. Specifically, we show that the equilibrium loan sizes under collusive competition, non-collusive competition, and deviation have the following functional form:

$$L^C(k,x,m) \equiv \widehat{L}^C(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A \tag{37}$$

$$L^N(k,x,m) \equiv \widehat{L}^N(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A \tag{38}$$

$$L^D(k,x,m) \equiv \widehat{L}^D(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A. \tag{39}$$

Therefore, all equilibrium outcomes only depend on the discrete state variables k and m in a nonparametric way, while their dependence on the continuous state variables in x has a closed form, which is already known.

We solve the model in three steps. We first solve for U^N and W^N, together with the endogenous cutoff of participation in a non-collusive equilibrium ω_N^*, which is chosen such that a specialist lender with a participation cost of ω_N^* is indifferent between participate or not. Then for any given level of colluded loan amount L^C, we can solve for U^C, W^C, and ω_C^* as functions of L^C. U^D, W^D, and ω_D^* can be solved accordingly, which are also functions of L^C. As the last step, we search for the equilibrium L^C that prevents deviation, as defined in Equation (30).

The model generates solutions for three observable variables of interest, including the number of participating specialist lenders m, the loan size $\frac{L}{A}$, and the DIP rate R. These variables are functions of the DIP deal type observed in the data (i.e., financed by a monopolistic prepetition lender, or by one or multiple specialist lenders, or by a lender of the last resort) as well as the model parameters.

Lastly, to bring the model to the data, we specify the probability of a borrower i belonging to cluster $k_i = (d, A_i)$ as follows:

$$\begin{aligned}\pi\left(k_i\right) &= Prob(d|A_i) \cdot Prob(A_i) \\ &= \frac{e^{\gamma_d+\beta_d(ln(A_i)-E[ln(A)])}}{1+\sum_{j=2}^{D} e^{\gamma_j+\beta_j(ln(A_i)-E[ln(A)])}} \cdot h(A_i)\end{aligned}$$

where $h(A)$ is the PDF of borrower size and $E\left[ln\left(A\right)\right]$ is the average log-size. In this specification, borrower size A_i is observable, and we assume that the likelihood of borrower i belonging to demand curve $d \in \{1,2,...,D\}$ follows the multinomial logistic distribution with parameter γ_d and β_d (and γ_1

and β_1 are normalized to 1). This specification allows the model to capture possible correlation between borrower size and the demand, as it is plausible that large borrowers and small borrowers differ in their overall demand for the loan and the price elasticity.[6]

3 Data Sample

Academics as well as practitioners commonly refer distressed loans as loans provided to firms that are either deeply distressed with the likely prospect of defaulting on its obligations or that are already bankrupt (Altman, Hotchkiss, and Wang, 2019). For our study, we construct a comprehensive data sample that consists of bank loans to both financially distressed but not yet bankrupt U.S. public firms and those public firms already in Chapter 11 bankruptcy from 2001–2019, using two different data filtering methods.

3.1 Distressed loans to firms not yet in bankruptcy

We first assemble a data set of distressed loans to financially distress firms that are not yet in bankruptcy. The academic literature has proposed a number of measures for identifying financially distressed firms. The existing approaches are broadly categorized as accounting ratio based (e.g. Altman Z-score models) and market price based approaches. The advantage of the market price based measure over using accounting ratios is that not only it is forward looking and thus incorporates market view of the future performance of the firm but more importantly, it directly measures how costly it is for a firm to raise financing in the markets. With fast development and high trading liquidity in the credit derivative trading markets, a few of recent studies use prices of credit default swaps (CDS) to take a cue on the financial health of firms (e.g., Hortacsu, Matvos, Syverson, and Venkataraman (2013); Brown and Matsa (2016)). Given that credit ratings are also informative indicators for corporate default likelihood and many institutions rely on ratings for their investment decisions, for our study, we rely on both CDS prices and credit ratings to identify financially distressed firms.

We retrieve monthly five-year CDS prices on senior unsecured bonds of U.S. public issuers from

[6] The model can be solved with a continuous-valued size A and any number of demand curve group g. In estimation, we divide borrowers into deciles based on their size and assume two heterogeneous demand curve $d \in \{1, 2\}$.

IHS Markit database and monthly S&P Long-Term Domestic Issuer Ratings of U.S. public firms from Compustat for the period from 2001–2017. We use two criteria to identify the beginning of a firm's distressed period, namely, whether the five-year CDS price first hits 1000 bps[7] or whether a firm's S&P rating drops to CCC+ or lower[8], whichever occurs first. After identifying the initial starting month of a firm's distress period, we trace the firm's CDS prices and credit ratings until the end of 2017. For firms with CDS prices, the distress period ends when their CDS prices falls below 500 bps or when they file for bankruptcy.[9] For firms with credit ratings, the distress period ends when S&P ratings goes up to B- or higher or the firm defaults according to S&P ratings. Consolidating the distress periods identified using CDS prices and ratings, removing duplicated time periods, and combining two consecutive periods that have an in-between time gap of less than a year, and removing distressed periods that are shorter than 6 months to avoid capturing transitory periods, we have 637 distressed periods of 576 firms from 2001–2017.

We merge the distressed periods with the Dealscan database using the link file by Chava and Roberts to identify loan facilities that have a start date falling into the distress period. We find 765 USD-denominated facilities issued in the U.S. for 243 distressed periods in our sample. After removing facilities that have missing information on lender identities or loan spreads that are measured by the all-in spread drawn (AISD), which is the sum of LIBOR spread and annual fee, and loans that are unsecured or subordinated, we have a final sample of 537 loan facilities (353 packages) in 189 distressed periods.

We first identify whether a lender is a major lender using their titles (e.g. agent bank, lead arrangers, etc.) following Jiang, Li, and Shao (2010). We consolidate all financial institutions to the parent company. For example, JP Morgan Securities would carry the same unique institution ID as JP Morgan & Co. Moreover, we consider institutions' M&As that occurred in our sample period and consolidate the target and the acquirer into one entity after the transaction. Moreover, we remove entities that are special purpose investment vehicles and structured products such as CLOs and CDOs.

[7]Prior studies refer bonds whose yield spread above the government bond is over 1000 bps as distressed bonds (Altman et al., 2019)

[8]Loans that are issued by firms with CCC+ or lower ratings can no longer be widely held by Collateralized Loan Obligations (CLOs), the most important type of investors in the leveraged loan market, because there are limitations on the amount of CCC-rated debt that can be included in the underlying collateral pool of CLOs (typically 7.5% of the collateral pool)

[9]Bankruptcy filings, including both Chapter 11 and Chapter 7 filings, by all U.S. public firms from 2000-2019 from obtained from New Generation Research's Bankruptcydata.com.

Using historical loan issuance information in Dealscan, we are able to determine whether a major lender is an existing lender to a firm—that is, a lender of the distressed loan is also a lender in an earlier loan that has not yet matured. We also determine whether a lender is a private equity fund or hedge fund by searching the lender's website and industry publications (Jiang, Li, and Wang, 2012). Next, we identify the CDS prices at the end of the month immediately preceding the loan issuance date. We also collect Treasury Constant Maturity Rate of different maturities from the Federal Reserve at St. Louis and 3-month LIBOR rate from Bloomberg. Using these information, we are able to determine the fraction of loan spreads that are due to the default risk component (See Appendix). Finally, we retrieve firms' key financial information immediately before loan issuance from Compustat.

We define three types of distressed loans. The first type contains loans provided by only one lender, who is an existing lender and is not a specialist, i.e., existing-lender loans (*Type 1*). The second type contains loans that are provided by at least one specialist, defined as one of the top 10 lenders (measured by the number of distressed loans that it financed in our sample), regardless of the number of lenders in the loan syndicate, specialist loans (*Type 2*). The third type contains loans that have over 50% of the lenders as hedge funds and private equity funds i.e., last-resort loans (*Type 3*). We remove 53 facilities that cannot be classified into any of the above three types. Among the 484 loan facilities in our final sample, the second type of loans it the most dominant type, accounting for 75.2% while Type 1 and Type 3 loans account for 12.2% and 12.6% respectively. Table 1, Panel A, presents the summary statistics of our sample of distressed loans.

3.2 Distressed loans to firms in bankruptcy: Debtor-in-possession (DIP) loans

A U.S. firm filing for Chapter 11 bankruptcy can obtain post-petition financing, commonly known as debtor-in-possession (DIP) financing, to support working capital and pay expenses in bankruptcy under Section 364 of the Bankruptcy Code. This law provision permits the bankrupt firm to arrange a DIP loan with super-priority status over all administrative expenses after notice and a court hearing. With existing lenders' court approval, a DIP loan can be secured by a senior or equal lien on a property that is already subject to a lien (i.e. the "priming lien" provision). These loan contracts are short term and typically contain extensive protective features such as restrictive covenants and milestones that a Chapter 11 firm must need or a default can be called by lenders. (Skeel, 2004; Ayotte and Morrison, 2009; Ayotte and Elias, 2020). With such security and lender protection provision, the default risk of DIP loans are very low, comparable to that of investment-grade loans as shown by Eckbo, Li, and Wang (2020).

The advantages of including DIP loans for our analysis are threefold. First, given their minuscule default rates, we can use the all-in spreads over a riskfree benchmark directly with default-risk adjustment to decompose the effect of lender power on loan pricing, compared to distressed loans for which we have to take out the default risk component of distressed loans using CDS prices. Second, existing lenders' lien on distressed firms' assets and their private information about the borrower allow them to have strong bargaining power and even become monopolistic lenders, which is consistent with our model setup. Third, as Eckbo et al. (2020) suggest, the DIP-lending market is quite concentrated. The top 10 lenders financed more than three quarters of their sample firms. Moreover, the lending syndicate is smaller than that for distressed loans. The highly concentrated lending market and small lending syndicate together create an ideal environment for specialized lenders to collude, where monitoring among lenders can be less costly, making punishment on non-collusive lending a more credible threat.

Our initial study sample, similar to that used in Eckbo et al. (2020), includes all DIP loans to Chapter 11 filings by large U.S public firms (with assets above $100 million in constant 1980 dollars) from 2002–2019, compiled from the UCLA-LoPucki Bankruptcy Research Database, Bankruptcydata.com, the Public Access to Court Electronic Record (PACER), and the Dealscan. We calculate the weighted average LIBOR spread and AISD at the loan package level using facility amount as the weight.[10] We have AISD available for 351 loan packages.

We identify the major lenders of DIP loans using DIP financing motions and master credit agreements and determine whether each lender is an alternative investors such as private equity funds or hedge funds. We also determine whether these lenders hold existing debt. Similar to the approach used to identify specialists for distressed loans, we treat a lender as a specialist if it is one of the top 10 lenders measured by the number of DIP loans in our sample. Also similar to distressed loans, we define three types of DIP loans: existing-lender loans (*Type 1*), specialist loans (*Type 2*), and last-resort loans (*Type 3*). We remove 54 DIP loans that cannot be classified into any of the above three types.[11] Table 1, Panel B,

[10]We use loan packages for DIP loans because it is always the same group of lenders that provide different facilities in a DIP-loan package while for other distressed loans, different facilities could be provided by distinct groups of lenders.

[11]These loans include the case of General Motors as its DIP-loan, the largest size history ($33 billion), was provided by U.S.

presents the summary statistics of our sample firms which obtained DIP-loan packages. In the 297 loan packages in our final sample, the second type of DIP loans it the most dominant type, accounting for 71.7% while Type 1 and Type 3 loans account for 14.5% and 13.8% respectively. Table 1, Panel B, presents the summary statistics of our sample of DIP loans.

4 Estimation

4.1 Likelihood Function

The model solves a few crucial variables for each loan deal based on two deal characteristics. The first characteristics is the cluster k_i that deal i belongs to, defined by borrower size and the latent demand curve the deal attaches to. The second characteristics is the deal type s_i as shown in Figure 1: a loan can be financed by a monopolistic existing lender ($l_i = 1$), by one or multiple specialist lenders ($l_i = 2$), or by a lender of the last resort ($l_i = 3$). Our goal is to estimate the model parameters and identify the latent cluster each deal belongs to based on the observables.

Using the Baysian approach, we estimate the posterior distribution for the variables of interest, $P\left(\{k_i\}_{i=1}^N, \Theta | \{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right)$, in which Θ is the vector of model parameters, m_i is the number of specialist lenders in deal i, $\frac{L_i}{A_i}$ is the loan amount normalized by the borrower's size, and R_i is the loan spread after removing the credit premium and liquidity premium component. The observation index $i = 1, 2, ..., N$ indicates the deals. This posterior distribution describes the estimate of model parameters and the augmented latent cluster variable of each deal based on the observables. Based on Hammersley-Clifford Theorem (Besag, 1974), this posterior distribution is fully characterized by two conditional distributions $P\left(\Theta | \{k_i, l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right)$ and $P\left(\{k_i\}_{i=1}^N | \Theta, \{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right)$, which in turn can be broken down into more lower dimensional conditional distributions.

Conditioning on $\{k_i\}_{i=1}^N$, the distribution $P\left(\Theta | \{k_i, l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right)$ is determined by the distribution $P\left(\Theta | \{k_i\}_{i=1}^N\right)$ and the likelihood function, $P\left(\{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N | \Theta, \{k_i\}_{i=1}^N\right)$, implied

and Canadian governments with a large component of subsidy, and loans that are provided by potential acquirers that use the DIP loan to bridge takeover.

by the model solution. Specifically,

$$
\begin{aligned}
P\left(\Theta|\{k_i, l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right) \propto & P\left(\Theta|\{k_i\}_{i=1}^N\right) \cdot P\left(\{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N|\Theta, \{k_i\}_{i=1}^N\right) \\
= & P\left(\Theta|\{k_i\}_{i=1}^N\right) \cdot \prod_{i=1}^N f_i\left(\Theta, k_i\right)
\end{aligned} \tag{40}
$$

where

$$
\begin{aligned}
f_i\left(\Theta, k_i\right) &= P\left(l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)|\Theta, k_i\right) \\
&= P\left(l_i, m_i|\Theta, k_i\right) \cdot P\left(ln(\frac{L_i}{A_i})|l_i, m_i, \Theta, k_i\right) \cdot P\left(ln(R_i)|l_i, m_i, ln(\frac{L_i}{A_i}), \Theta, k_i\right)
\end{aligned} \tag{41}
$$

is the likelihood function for deal i, conditioning on the model parameters and the cluster it belongs to.

Meanwhile, conditioning on Θ, the distribution $P\left(\{k_i\}_{i=1}^N|\Theta, \{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right)$ is determined by $P\left(\{k_i\}_{i=1}^N|\Theta\right)$ and the likelihood function $P\left(\{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N|\Theta, \{k_i\}_{i=1}^N\right)$:

$$
\begin{aligned}
P\left(\{k_i\}_{i=1}^N|\Theta, \{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N\right) \propto & P\left(\{k_i\}_{i=1}^N|\Theta\right) \cdot P\left(\{l_i, m_i, ln(\frac{L_i}{A_i}), ln(R_i)\}_{i=1}^N|\Theta, \{k_i\}_{i=1}^N\right) \\
= & P\left(\{k_i\}_{i=1}^N|\Theta\right) \cdot \prod_{i=1}^N f_i\left(\Theta, k_i\right)
\end{aligned} \tag{42}
$$

We summarize below the individual component of the likelihood function in Equation (41) for each type of loan.

First, for loans financed by a monopolistic lender,

$$
P\left(l_i = 1, m_i|\Theta, k_i\right) = \lambda(k_i)
$$

where $\lambda(k_i)$ is the probability that deal i is financed by an existing lender. m_i becomes irrelavant here because there is only one existing lender involved. The likelihood of loan size and spread is given as below:

$$P\left(ln(R_i)|l_i, m_i, \Theta, k_i\right) = \Phi\left(ln(\Re\left(k_i, m_i\right)), \varsigma\right) \tag{43}$$

$$P\left(ln(\frac{L_i}{A_i})|l_i, m_i, ln(R_i), \Theta, k_i\right) = \Phi\left(ln(\frac{\mathcal{L}\left(k_i, m_i\right))}{A}, \sigma\right) \tag{44}$$

where Φ is the PDF of the normal distribution, $\mathcal{L}\left(k_i, m_i\right)$ and $\Re\left(k_i, m_i\right)$ are the model-implied loan size and spread when the loan is financed by a monopolistic existing lender, as solved in Equation (10) and the demand curve Equation (4), and ς and σ are the standard deviation of the normal distribution that captures the lender-side and borrower-side shocks.

Second, for loans financed by a lender of the last resort, it must be the case that no specialist lenders participate and thus $m_i = 0$

$$P\left(l_i = 3, m_i = 0|\Theta, k_i\right) = \left(1 - \lambda(k_i)\right)\left(1 - F(\omega^*_{C,i}|k_i)\right)^M$$

where $1 - \lambda(k_i)$ is the probability that deal i is not financed by an existing lender and $\left(1 - F(\omega^*_{C,i}|k_i)\right)^M$ is the probability that none of the M specialist lenders participate. The likelihood of loan size and spread is given as in Equation (43) and (44).

Third, for loans financed by one or multiple specialist lenders, there must be at least one specialist lender ($m_i \geq 1$), and we have

$$P\left(l_i = 2, m_i|\Theta, k_i\right) = \left(1 - \lambda(k_i)\right)\begin{pmatrix} M \\ m_i \end{pmatrix} F(\omega^*_{C,i}|k_i)^{m_i}\left(1 - F(\omega^*_{C,i}|k_i)\right)^{M-m_i}$$

where $1 - \lambda(k_i)$ is the probability that deal i is not intermediated by an existing lender and $\begin{pmatrix} M \\ m_i \end{pmatrix} F(\omega^*_{C,i}|k_i)^{m_i}\left(1 - F(\omega^*_{C,i}|k_i)\right)^{M-m_i}$ is the probability that m_i out of M specialist lenders participate in this deal.

4.2 MCMC Estimator

To implement MCMC estimator, for each deal i in the data, we define a vector z_i of dimension K (i.e., the number of possible cluster) so that the jth element in the vector describes the probability of this deal belonging to cluster j (i.e., $\pi(k_i = j)$) so that $z_{ij} \geq 0$ and $\sum_{j=1}^{K} z_{ij} = 1$. Then we apply the Metropolis-Hastings algorithm with the following procedure:

Step 1: for each deal i, update $z_i^{(g)}$ using the EM method based on outputs from the last iteration round $g-1$:

$$z_{ij}^{(g)} = \frac{\pi^{(g-1)}(k_i = j) f_i(\Theta, k_i = j)}{\sum_{\kappa=1}^{K} \pi^{(g-1)}(k_i = \kappa) f_i(\Theta, k_i = \kappa)} \tag{45}$$

where $\pi^{(g-1)}(k_i = j)$ is the probability of $k_i = j$ (determined by the model parameters obtained from the last iteration), and $f_i(\Theta, k_i)$ is the likelihood function of observables evaluated at the parameter values from the last iteration. Then we follow the SEM algorith (Celeux, 1985; Celeux and Diebolt, 1992) and draw the realization of $k_i^{(g)}$ from the multinomial distributions with weights in Equation (45). We throw out the draw and repeat this step if for any cluster j, the total number of deals assigned to this cluster is less than a cutoff (e.g., 3%).

Step 2: update other parameters in Θ using the random walk Metropolis-Hastings algorithm based on the updated draws of $\{k_i^{(g)}\}_{i=1}^{N}$ from step 1. We compute the acceptance/rejection threshold:

$$\alpha\left(\Theta^{(g)}, \Theta^{(g-1)}\right) = min\left(\frac{P\left(\Theta^{(g)} | \{k_i^{(g)}\}_{i=1}^{N}\right) \prod_{i=1}^{N} f_i\left(\Theta^{(g)}, k_i^{(g)}\right)}{P\left(\Theta^{(g-1)} | \{k_i^{(g)}\}_{i=1}^{N}\right) \prod_{i=1}^{N} f_i\left(\Theta^{(g-1)}, k_i^{(g)}\right)}, 1\right)$$

where $\Theta^{(g-1)}$ is the parameter vector from the last iteration, and $\Theta^{(g)} = \Theta^{(g-1)} + \Sigma\epsilon$ is the vector of proposed parameters.

Step 3: repeat the procedure in step1 and step 2 and generate a MCMC chain for the classification of cluster and the estimated parameters.

4.3 Estimation Results

In this section, we first discuss what empirical patterns in the data help the MCMC estimator to identify model parameters. We then present the estimation results and our empirical findings for the two distressed loan markets.

A key parameter in our model, ξ, controls the tacit coordination among specialized lenders. In case of perfect collusion, lenders in a syndicate group coordinate as a cartel, and thus the total loan amount lent by the cartel equals the loan amount lent by a monopolistic lender. However, as collusion intensity falls, syndicated lenders lend more aggressively, leading to larger loan amount by the syndicate group than that by a monopolistic lender. As a result, the ratio of the loan size made by a syndicate lender group (with two or more specialist lenders) to the loan size made by a single monopolistic specialist, $\frac{L/A_{m>1}}{L/A_{m=1}}$, helps pin down ξ. Clearly, as ξ increases, this ratio declines. Two important things are worthnoting here. First, when the profits from deviation are large enough, even the toughest punishment (i.e., $\xi = 1$) cannot fully restore perfect collusion, and therefore it is possible to observe the loan size ratio to remain above one even if $\xi = 1$. Second, collusion is harder to achieve in face of large borrowers, because profits from deviation increase as the demand grows. As a result, the model suggests that the loan size ratio is higher for large borrowers than for small borrowers given the level of ξ.

Figure 3 illustrates how the loan size ratio varies with ξ for small borrowers (left) and large borrowers (right). Panel (a) shows the results for the distressed loan sample while panel (b) shows the results for the DIP sample. In these figures, the solid line depicts the model-implied loan size ratio as a function of ξ, the dash line indicates the average loan size ratio measured in the corresponding samples, and the red circle marks the model prediction with ξ set to its estimated value (Table 2). In both markets, a low level of ξ is sufficient to support a high level of tacit coordination among the specialist lenders in face of small borrowers. Specifically, with a ξ being around 0.3, the loan size ratio is already quite close to one, both in the model and in the data. We, however, observe that the loan size ratio is monotonically decreasing as ξ increases for loans made to large borrowers, and even with $\xi = 1$, the loan size ratio is still above one. In all panels of the figure, the model-predicted loan size ratio (red circle) is very close to their empirical counterparts (dash line), suggesting that the parameter ξ is well identified in our estimation. Since we estimate ξ in MCMC, its estimated value is also affected by other observable variables such as loan size, loan spread, and the number of specialized lenders, and thus the loan size ratio is not the only factor that pins down ξ. However, the fact that the model is able to match the loan size ratio so closely even if the loan size ratio is not used explicitly in forming the aggregate likelihood lends strong support to the model's underlying mechanism and the identification.

The fraction of loan deals financed by the existsing lender helps identify the parameter $\lambda(k_i)$. This linkage is established exogenously in the model by assuming that there is a probability $\lambda(k_i)$ that the existing lender has the exclusive right to provide the financing and thus block the firm from borrowing from outside lenders.

Different type of lenders (i.e., exissting lenders, specialist lenders, and lenders of the last resort) may incur different variable costs (e.g., monitoring costs). Part of the spread R is used to compensate for these variable costs. By comparing R across deals financed by different types of lenders (after controlling for the number of lenders involved), the estimator is able to back out the parameter ϕ_l for existing lenders ($l = 1$), specialist lenders ($l = 2$), and lenders of the last resort ($l = 3$).

Parameters related to the demand curves are estimated based on the classification of clusters, which in turn is achieved as augmented latent variables in MCMC. The relation between R and $\frac{L}{A}$ *within* each cluster helps identify the intercept and elasticity of the corresponding demand curve, $\alpha(k)$ and $\varepsilon(k)$. It is worthnoting that this approach is different from estimating the demand curve coefficients by simply regressing the quantity ($ln\,(\frac{L}{A})$) on price ($ln\,(R)$), which is known to produce biased estimates. The key identification challenge in estimating the demand curve is that, both the demand curve and supply curve can shift simultaneously due to unobservable, fundamental factors, and thus the observed quantity and price are produced by the intersections of multiple demand and supply curves (i.e., in multiple equilibriums). Many empirical studies overcome this challenge by utilizing exogenous shocks or instruments on the supply side and thus fixing the demand curve and shifting the supply, while our approach tackles this issue by characterizing the full joint distribution of both demand and supply in equilibrium. Specifically, our estimator first classifies the observations (loans) to different demand curves based on the model-implied distribution of demand and supply in equilibrium. This classification addresses the identification challenge that both the quantity and price are endogenous by exploring the fundamental factors in the model that simultaneously drive the observed quantity and price. Conditioning on the classification, estimating the coefficients of each demand curve becomes feasible, and it produces unbiased estimates.

MCMC estimator generates Markov chains for each model parameter and the augmented variables.

We first present the estimation of cluster classification. For each loan deal i in our sample, our estimator assigns it a vector $z_i^{(g)}$ that describes the likelihood of the deal belonging to each cluster in the gth iteration on the MCMC chain, as specified in Equation (45). We take the average of $z_i^{(g)}$ over the chain and classify loan i to the cluster with the highest likelihood. Based on the classification results, Figures 4 depicts the demand curves for the distressed loan sample and DIP loan sample, respectively. Each dot in the figure represents an observation of loan, and the size of the dots indicts the borrower's size. The two dash lines, in black and gray, depict the two estimated demand curves. Black dots are loans classified to the first demand curve and the white dots are loans classified to the second demand curve. As we discussed above, the classification is performed based on the full likelihood of demand and supply in the model equilibrium and thus it differs from a simple classification based on the observed loan size $ln\left(\frac{L}{A}\right)$ and spread $ln\left(R\right)$. This explains why some dots located closer to one demand curve according to the observed loan size and spread are instead classified to the other demand curve, highlighting the importance of controlling for the endogeneity problem in estimating the demand curve coefficients. In both samples, borrowers on the first demand curve (black dots) have a lower intercept of loan size but a higher price elasticity of loan demand compared with those classified as the second demand curve (white dots). It indicates that borrowers on the first demand curve have a lower level of overall demand for loan (relative to their firm size) and they are more sensitive to the loan price. Empirically, we find that borrowers on the first demand curve are also larger in size and have higher total revenues, which may explain partly the difference we identify in their demand curves.

Besides the classification of clusters and their the demand curves, the MCMC estimator also delivers the posterior distribution of each model parameter. We report in Tables 2 the point estimate and the standard errors of each parameter for two samples respectively. The point estimate is taken as the parameter set that produces the highest likelihood along the chain, and the standard errors is computed as the standard deviation of the parameter draws along the chain.

The likelihood of punishment on deviation, ξ, is estimated to be 0.991 for distressed loans. It suggests that if a specialist lender deviates from the perceived cartel equilibrium, it will be pushed into a non-collusive equilibrium almost surely. This parameter controls the collusion intensity in the model and a value of 0.991 implies a very high level of tacit coordination among specialists. Our estimates show that the punishment on deviation is also large for DIP loans with ξ estimated to be 0.951. ξ is estimated with relatively small standard errors in both samples, suggesting that the data pattern strongly rejects a model without tacit coordination among specialist lenders. As we discussed above, the ratio of loan size made by a syndicate group of lenders to the loan size made by a monopolistic lender disciplines the identification of tacit coordination among specialist lenders. Stronger coordination leads to a low loan size ratio, consistent with the data.

Our estimates of ς and σ show that unobservable heterogeneity, possibly driven by the lender-side and borrower-side shocks, is important in explaining the cross-sectional variation of the observed loan size and spread. A simple variance decomposition suggests that with the estimated unobservable heterogeneity, our model is able to explain 51% (14%) of variation in $ln(R)$ and 16% (18%) of variation in $ln(\frac{L}{A})$ for the DIP sample (distressed loan sample).

It is also interesting to note that the lenders' participation cost, μ, is estimated to differ significantly across the two samples. Specifically, participation cost by DIP lenders is more than two times larger than that by distressed loan lenders. A crucial determinant of the participation cost is the blocking power by existing lenders (Eckbo et al., 2020): if the existing lender has a strong power in blocking the borrower from reaching out to other lenders even when it does not make the distressed loan to the borrower, then such blocking power can manifest as a large estimated participation cost born by specialist lenders (i.e., it is more difficult for specialist lenders to participate).

The mid panel of Tables 2 reports the lender-specific variable costs. It is worth noting that since we have purged out the risk-free rate and the risk premium in loan spread before estimation, the variable costs here should not be interpreted as funding costs, instead, they reflect other variable costs associated with lending to distressed firms. Monitoring cost and costs of lender participating in the restructuring process can be such variable costs. For example, as a distressed borrower approaches bankruptcy or is already in bankruptcy, lenders need to pay close attention to the borrower's business operations and legal challenges in the court process. That the estimated variable costs of DIP loans are much higher than those for distressed loans are consistent with more uncertainties faced by lenders in bankruptcy court.

An interesting evidence that stands out in the DIP-loan sample is that the estimated variable costs are about 170 bps for existing lenders and specialist lenders but 240 bps for lenders of last-resort (mostly hedge funds). Unlike the existing lenders who possess information advantage or the specialist lenders who have expertise in this market, hedge funds are rarely frequent players in this market. Most of them appear only once or twice in our sample as the lenders of last-resort, and therefore it is plausible that they face higher variable costs. More importantly, hedge funds often play an active role in the governance of bankrupt firms such as seeking board representation and appointing managers in pursuing a "loan-to-own" strategy (Jiang et al., 2012; Li and Wang, 2016; Ayotte and Elias, 2020), which requires hedge funds' significant efforts and resources.

We report the parameters that govern the demand curves in the bottom panel of Tables 2. We estimate the level of demand, $\alpha(\kappa_i)$, and the price elasticity of demand, $\varepsilon(\kappa_i)$. Within each market, the estimated intercept and elasticity just reiterate the two demand curves shown in Figure 4. Across the two market, we find that DIP borrowers exhibit higher elasticity of demand than distressed loan borrowers. Our estimates, therefore, suggest that the striking difference in loan spread between the two markets cannot be explained by the price elasticity of demand.

5 Decomposing Loan Spread

Using the estimated model as a laboratory, we quantify the lenders' market power in the two separate distressed loan markets and decompose their market power into three components: (i) the potential collusion among specialists, (ii) limited participation by these lenders, and (iii) the oligopolistic structure in this market.

Both markets exhibit a few unique and interesting features. First, there is a club of specialist lenders that intermediate a large fraction of loan deals. Indeed, 83% of distressed loans and 78% of DIP deals in our sample are financed by the top 10 specialist lenders. This lending market, therefore, resembles an oligopolistic structure. To illustrate the dominant position of top lenders in the two loan markets, Figure 2 shows the loan market network for distressed loans and DIP loans, respectively. It is clear that the top lenders in each market form a strong network with other top lenders in syndicating the loans. Second, most deals are syndicated by a small group of specialist lenders as shown in Table 1. It implies limited participation in lending by specialists, which further reduces lender competition. Third, the lender clique may give rise to possible collusion among specialists that is hard to be detected by outsiders. In this section, we employ the estimated model to perform a few counterfactual benchmarks and decompose the market power of specialist lenders.

Starting from the baseline model with the estimated parameters, we first quantify the effect of lender collusion. To do so, we set the key parameter ξ to zero. In this counterfactual model, all specialist lenders deviate from collusion because no punishment is imposed, and as a result, a non-collusive equilibrium emerges. We keep other model parameters at their estimated values so that the non-collusive equilibrium still features limited participation by specialist lenders and an oligopolistic market structure. We examine how lender collusion affects the average loan spread, loan size, and the number of participating lenders by comparing the outputs from the two models.

Tables 3 shows the results. Columns (1) and (5) report the baseline model predictions for the distressed loan sample and DIP sample respectively, and columns (2) and (6) report the non-collusive model outputs for the two samples. Notably, the average loan spread for specialist lenders declines by 141 bps (from 235 bps in the baseline model to 94 bps in the non-collusive model) for distressed loans, representing a 60% reduction in spread if specialist lenders compete in a non-collusive equilibrium. DIP loan spread declines by 159 bps (from 610 bps to 451 bps) as lender collusion is eliminated, representing a 26% decline. Without collusion, specialists compete more aggressively and lend a larger amount in aggregation. The average loan amount, relative to the borrowers' size, rises from 0.495 to 3.364 for distressed loans and from 0.216 to 0.424 for DIP loans. Intense competition has a moderate deterrence effect on lender participation, evident by a slight drop in the average number of syndicated lenders.

Next, we examine the effect of limited participation by specialist lenders. In our model, each lender incurs a stochastic participation cost that is assumed to follow an exponential distribution with the mean parameter μ. We construct a counterfactual benchmark of full participation by setting μ to zero. In this counterfactual model, we still keep $\xi = 0$ to measure the incremental effect of removing limited participation from the non-collusive model. The results for this counterfactual model are reported in columns (3) and (7) of Tables 3 for the two samples. The direct change is that all specialist lenders now participate in loan syndication and thus the number of lenders reaches the full capacity of 10. Full participation increases competition among specialist lenders, and the loan spread decreases by another 80 bps for distressed loans and a striking 246 bps for DIP loans. This result suggests that limited participation by specialist lenders has a much more pronounced effect in DIP market than in distressed loan market. Because participation costs in our model incorporate both the costs of learning about a loan deal and the existing lenders' power on blocking other lenders from participating, our estimates indicate that such costs are higher for lenders in the DIP-loan markets. This is consistent with earlier studies that document that new DIP lenders cannot prime "liens" of existing lenders without their consent and thus "prepetition" lenders possess a strong bargaining position in deciding who can participate in a DIP loan syndicate (Skeel, 2004; Ayotte and Morrison, 2009). Moreover, because borrowers tend to be selective in which lenders to approach before bankruptcy filing, it can be costly for some lenders to learn about the deal type (Eckbo et al., 2020). This is generally less of a case for distressed loans.

As the last step, we quantify the effect of an oligopolisitc market structure. To turn an oligopolistic market into a competitive market, we increases the total number of specialist lenders from 10 to positive infinity. We build this counterfactual upon the non-collusive, full participation model to capture the incremental effect. In columns (4) and (8) of Tables 3, we find that the average loan spread drops by just 1 bp and 8 bps for distressed loans and DIP loans financed by specialist lenders, respectively. Interestingly, this finding shows that the oligopolistic structure is not the main cause of high loan spread in either market, and the market would be competitive enough if all the 10 specialist lenders can participate.

Moving from the baseline model to the non-collusive, full participation, competitive model, we find that the lenders' market power accounts for about 94% of distressed loan spread (i.e., $\frac{235-13}{235}$) and 68% of the DIP spread (i.e., $\frac{610-197}{610}$). A further decomposition of the market power suggests that lender collusion contributes to 64% (i.e., $\frac{235-94}{235-13}$) of the market power for distressed loans and 38% of the market power for DIP loans (i.e., $\frac{610-451}{610-197}$), while limited participation contributes to 36% of the market power (i.e., $\frac{94-14}{235-13}$) for distressed loans and 60% of the market power (i.e., $\frac{451-205}{610-197}$) for DIP loans, Finally, the oligopolistic market structure contributes to the remaining negligible 0.4% (i.e., $\frac{14-13}{235-13}$) and 2% (i.e., $\frac{205-197}{610-197}$) for the two markets respectively.

We summarize the decomposition of loan spread for the two markets in Figure 5. Specifically, we compare each component of the loan spread for distressed loans and DIP loans. This figure reveals two main conclusions that are central to our paper. First, tacit coordination among specialist lenders is prevalent in both markets, which allows the lender cartel to extract sizeable rent from borrowers (around 150 bps). Second, the excess loan spread in DIP market, compared with that in the distressed loan market, is mainly explained by the existing lenders' strong power in blocking the borrowers from reaching out to alternative lenders (i.e., high participation costs) and the high monitoring costs in DIP market (i.e., variable costs).

Among the different components that contribute to lender market power, lender tacit coordination often attracts most attention by regulators, and anti-trust policies often target at breaking down such coordination. We then examine which borrowers are more susceptible to lender market power by measuring the effect of lender tacit coordination on borrowers of different size.

In the final part of the analysis, we partition the borrowers into size quintiles and repeat our analyses of the baseline model and the non-collusive model performed above. We report the changes in the average loan spread, loan size, and the number of participating specialist lenders as we shut down lender coordination in Tables 4. We compare small borrowers (bottom size quintile) with large borrowers (top size quintile).

When lender coordination is eliminated, we find that small borrowers seem to benefit more. Specifically, the distressed loan spread drops by 108 bps (78 bps) for small (large) borrowers. DIP loan spread drops 197 bps (64 bps) for small (large) borrowers. The increase in loan size due to the elimination of collusion is also more pronounced for small borrowers than large borrowers in both markets. Overall, our analyses here suggest that small borrowers in both markets are more susceptible to lender market power and the effect of collusion on smaller borrowers more than doubles that for large borrowers. Our findings therefore imply that anti-trust policies that deter lender collusion may benefit small lenders more, and in economic or financial crises, financial aids towards distressed companies should target at small firms. The implication is consistent with a recent policy proposal by legal scholars to encourage the U.S. government to provide direct funding to small firms that filed for bankruptcy during the COVID-19 pandemic.[12]

[12] See "Use of Chapter 11 and Federal Lending to Help Small Businesses," by Kathryn Judge and Jared A. Ellias, July 27, 2020, Letter to the Office of Senator Sherrod Brown, as Ranking Member of the Committee on Banking, Housing, and Urban Affairs.

6 Policy Analysis

Our estimation and quantification results so far suggest that specialized lenders in the distressed loan market and DIP loan market extract rents from borrowers by charging excess risk-adjusted loan spread above their marginal costs; moreover, a major part of these rents is made possible through the market power enjoyed by the few specialized lenders, especially through the tacit collusion in the form of syndication. For such implicit coordination is often hard to detect, let alone make prosecution and enforcement, a simple regulatory intervention is to impose a cap on interest rates. Our findings support the proposals on government intervention by disciplining both markets of distressed loans, reducing the borrowing costs and facilitating borrowers' credit accessibility, to mitigate the damage of inefficient bankruptcy waves owing to an economy-wide cash-flow pause (e.g., DeMarzo et al., 2020; Conti-Brown and Skeel, 2020). In this section, we focus on interest rate cap regulation (i.e., usury regulation), and use the estimated model to examine the effect of such policies.

Regulations and laws pertaining to interest rate caps have been one of the few most ubiquitous economic legislations historically and geographically (Blitz and Long, 1965). As of today, they still play a vital role in economic activities across different economies. Specifically, our setting is about commercial loans, defined as loans made primarily for business, commercial, investment, agricultural, or similar purposes, in contrast to consumer loans. For instance, in New York, corporations and limited liability companies (LLCs) cannot be charged more than 16% interest per annum, and specifically, loans to businesses under $2,500,000 are generally exempt from the 16% civil usury cap for consumer loans, but are subject to the 25% cap in 2021.

6.1 Formulation of Polices in the Model

The interest rate cap regulation can be implemented after conditioning on the characteristics of a borrower and adjusting the risk premium to better account for borrower risk heterogeneity and risk pricing. This can substantially improve the effectiveness of the interest rate cap policy because unsophisticated constant interest rate caps have severe limitations on balancing the tradeoff between borrower production and credit access (e.g., Cuesta and Sepúlveda, 2021). Assuming that the regulator can observe or estimate the marginal cost of providing distressed loans $e^{\phi+\varsigma u}$ and the risk premium of the loan, we can directly consider the interest rate cap that is imposed on the risk-adjusted spread R in the following form:

$$R_{max}(x) \equiv \mathcal{R}_{max} e^{\phi + \varsigma u}, \tag{46}$$

where $\mathcal{R}_{max}$ is a positive constant.

According to the demand system of the borrowers, the loan amount per specialized lender corresponding to the ceiling on the risk-adjusted spread is

$$L_{min}(k, x, m) = \frac{1}{m} \mathcal{R}_{max}^{-\varepsilon(k)} e^{[\alpha(k) - \varepsilon(k)\phi] - \varepsilon(k)\varsigma u + \sigma z} A,$$

where m is the number of specialized lenders in the syndication, k indicates the type of the borrower, and x contains the characteristics of the deal. In fact, under the interest rate cap specified in (46), the loan amount $L_{min}(k, x, m)$ is the minimum loan size each specialized lender will offer in equilibrium for the syndication characterized by (k, x, m).

Given that we focus on the risk-adjusted interest rate cap imposed on the spread, the optimal loan sizes for each specialized lender under non-collusive syndication, collusive syndication, and deviation have the following respective functional forms:

$$L^i(k, x, m; \mathcal{R}_{max}) \equiv \widehat{L}^i(k, m; \mathcal{R}_{max}) e^{[\alpha(k) - \varepsilon(k)\phi] - \varepsilon(k)\varsigma u + \sigma z} A, \quad \text{with } i \in \{N, C, D\}. \tag{47}$$

Non-collusive equilibrium with interest rate cap $\mathcal{R}_{max}$. Under the interest rate cap regulation, the value function prior to paying the fixed cost w and observing the deal-specific characteristics $x = (z, u)$, denoted by $U^N(k, x, m; \mathcal{R}_{max})$, satisfies the following Bellman equation:

$$\begin{aligned} U^N(k, x, m; \mathcal{R}_{max}) &= \Pi_2(k, x, m; L^N, \mathcal{R}_{max}) \quad + \quad \frac{W^N(\mathcal{R}_{max})}{1 - \delta}, \quad \text{where} \\ W^N(\mathcal{R}_{max}) &= \mathbb{E}^{k'} \left\{ \lambda(k') \frac{\Pi_1(k'; \mathcal{R}_{max})}{M_0} \right\} \\ &+ \mathbb{E}^{k'} \left\{ [1 - \lambda(k')] \sum_{m'=1}^{M} q(m' | w' \leq w^*_{N,\mathcal{R}_{max}}) \left[F(w^*_{N,\mathcal{R}_{max}}) \Pi_2(k', m'; L^N, \mathcal{R}_{max}) - \int_{w' \leq w^*_{N,\mathcal{R}_{max}}} w' dF(w') \right] \right\}, \end{aligned} \tag{48}$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1, \cdots, K\}$ with probability weight $\pi(k')$ for each k', and the cutoff $w^*_{N,\mathcal{R}_{max}}$ is determined in the same way as w^*_N but they can be different in the equilibrium.

The symmetric non-collusive Nash equilibrium can be characterized by the following condition:

$$L^N(k, x, m; \mathcal{R}_{max}) = \underset{L \geq L_{min}(k,x,m)}{\operatorname{argmax}} \left[\left(e^{\alpha(k)+\sigma z} \frac{A}{L + (m-1)L^N(k, x, m; \mathcal{R}_{max})} \right)^{1/\varepsilon(k)} - e^{\phi+\varsigma u} \right] L. \tag{49}$$

Plugging (47) into (49) results in the following relation:

$$\widehat{L}^N(k, m; \mathcal{R}_{max}) = \underset{\widehat{L} \geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}}{\operatorname{argmax}} \left\{ \left[\widehat{L} + (m-1)\widehat{L}^N(k, m; \mathcal{R}_{max}) \right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}, \tag{50}$$

which leads to

$$\widehat{L}^N(k, m; \mathcal{R}_{max}) = \max \left\{ \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}, \ \frac{1}{m} \left[\frac{m\varepsilon(k)}{m\varepsilon(k) - 1} \right]^{-\varepsilon(k)} \right\}. \tag{51}$$

Collusive equilibrium with interest rate cap $\mathcal{R}_{max}$. Under the interest rate cap regulation, the value function of a specialist at the beginning of the "afternoon" when w, k, and x are already observed, denoted by $V^C(k, x, w, m; L^C, \mathcal{R}_{max})$, has the following functional form:

$$V^C(k, x, w, m; L^C, \mathcal{R}_{max}) \equiv U^C(k, x, m; L^C, \mathcal{R}_{max}) - w. \tag{52}$$

The value function $U^C(k, x, m; L^C, \mathcal{R}_{max})$ satisfies the following Bellman equation:

$$U^C(k, x, m; L^C, \mathcal{R}_{max}) = \Pi_2(k, x, m; L^C, \mathcal{R}_{max}) \ + \ \frac{W^C(L^C, \mathcal{R}_{max})}{1-\delta}, \quad \text{where} \tag{53}$$

$$W^C(L^C, \mathcal{R}_{max}) = \mathbb{E}^{k'} \left\{ \lambda(k') \frac{\Pi_1(k', \mathcal{R}_{max})}{M_0} \right\}$$
$$+ \mathbb{E}^{k'} \left\{ [1 - \lambda(k')] \sum_{m'=1}^{M} q(m'|w' \leq w^*_{C,\mathcal{R}_{max}}) \left[F(w^*_{C,\mathcal{R}_{max}})\Pi_2(k', m'; L^C, \mathcal{R}_{max}) - \int_{w' \leq w^*_{C,\mathcal{R}_{max}}} w' dF(w') \right] \right\},$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1, \cdots, K\}$ with probability weight $\pi(k')$ for each k', and the cutoff $w^*_{C,\mathcal{R}_{max}}$ is determined in the same way as w^*_C but they can be different in the equilibrium.

For a given scheme of collusive loan size captured by $\widehat{L}^C(k, m)$, the optimal deviation in terms of loan size is the one that maximizes the expected deviation profit, characterized as follows:

$$\widehat{L}^D(k,m;\mathcal{R}_{max}) = \underset{\widehat{L}\geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}}{\operatorname{argmax}} \left\{ \left[\widehat{L} + (m-1)\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}. \tag{54}$$

The benefit of deviation is the difference between the maximal expected deviation profit and the expected collusive profit without deviation, denoted by $\Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max}) \equiv \mathbb{E}^x\left[\Pi_2^D(k,x,m;\widehat{L}^C,\mathcal{R}_{max})\right]$ and $\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max}) \equiv \mathbb{E}^x\left[\Pi_2(k,x,m;\widehat{L}^C,\mathcal{R}_{max})\right]$, respectively. Given the collusive scheme $\widehat{L}^C(\cdot,\cdot;\mathcal{R}_{max})$ and the interest rate cap regulation captured by $\mathcal{R}_{max}$, the maximal expected deviation profit $\Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max})$ is achieved at the optimal deviation $\widehat{L}^D(k,m;\mathcal{R}_{max})$, that is,

$$\begin{aligned}
&\Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max})\\
&\quad = \left\{ \left[\widehat{L}^D(k,m;\mathcal{R}_{max}) + (m-1)\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}^D(k,m;\mathcal{R}_{max})\\
&\qquad \times \exp\left\{\alpha(k) + \frac{1}{2}\sigma^2 + [1-\varepsilon(k)]\phi + \frac{1}{2}[1-\varepsilon(k)]^2\zeta^2\right\} A.
\end{aligned} \tag{55}$$

Given the collusive scheme $\widehat{L}^C(\cdot,\cdot\,;\mathcal{R}_{max})$ and the interest rate cap regulation captured by $\mathcal{R}_{max}$, the expected collusive profit $\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max})$ is

$$\begin{aligned}
&\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max})\\
&\quad = \left\{ \left[m\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1 \right\} \widehat{L}^C(k,m;\mathcal{R}_{max})\\
&\qquad \times \exp\left\{\alpha(k) + \frac{1}{2}\sigma^2 + [1-\varepsilon(k)]\phi + \frac{1}{2}[1-\varepsilon(k)]^2\zeta^2\right\} A.
\end{aligned}$$

We define the IC compatible set of functionals $\widehat{L}^C(\cdot,\ ;\mathcal{R}_{max})$ as follows:

$$\begin{aligned}
\widehat{\mathcal{L}}^C(\mathcal{R}_{max}) \equiv \Bigg\{ \widehat{L}^C :\ & \frac{\xi[W^C(\widehat{L}^C,\mathcal{R}_{max}) - W^N(\mathcal{R}_{max})]}{1-\delta} \geq \Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max}) - \Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max}),\\
& \widehat{L}^C(k,m) \geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)},\ \ \forall\, k,m \Bigg\}.
\end{aligned}$$

In the collusive Nash equilibrium we focus on, the loan size is

$$\widehat{L}^C(\cdot,\cdot) = \underset{\widehat{L}\in\widehat{\mathcal{L}}^C(\mathcal{R}_{max})}{\operatorname{argmax}} \mathbb{E}\left[U^C(k,x,m;\widehat{L},\mathcal{R}_{max})\right]. \tag{56}$$

6.2 The Effects of Interest Rate Cap

After we solve the model with interest rate cap, we investigate its effects on borrowers' welfare. In particular, we are interested in how this interest rate cap influences the loan spread that borrowers pay and the loan amount that they obtain. Since the effects of interest rate cap are similar qualitatively in both markets, we present the results for the DIP loan market as an example.

We start with demonstrating the intended consequences of the interest rate cap policy. In panels (c) and (d) of Figure 6, we plot the loan spread (left) and loan amount (right) as a function of the tightness of interest rate cap controlled by the parameter $\mathcal{R}_{max}$. As $\mathcal{R}_{max}$ decreases, the lenders' markup declines and the equilibrium loan spread drops, indicating a tight interest rate cap control.

As expected, a tighter cap on interest rate reduces the loan spread almost mechanically. Meanwhile, since loan spread is capped, coordination among specialist lenders that aim at restricting the total loan amount for lifting loan spread becomes ineffective, and thus specialist lenders lend more aggressively in order to capture the profits from a larger loan size. Accordingly, we observe a sharp increase in the loan amount as interest rate cap tightens. Both the decline in loan spread and increase in loan size by specialist lenders benefit the borrowers.

Even though interest rate cap improves borrower welfare through borrowing from the specialist lenders (i.e., the intensive margin), an unintended consequence of this policy is to further discourage the participation of specialist lenders and thus hinders the depth of this market (i.e., the extensive margin). Intuitively, specialist lenders decide to participate only when their expected profits are higher than the participation costs. Interest rate cap reduces the expected profits earned by specialist lenders and therefore excludes more specialist lenders from participating. If no specialist lenders are willing to participate in a specific deal, then the borrower is forced to borrow from the lenders of last-resort who are private investors in the market and may not be restricted by the interest rate cap. Lack of competition among the last-resort lenders and the high variable costs they bear make the loan very expensive. Panel

(e) and (f) confirm this model prediction. As $\mathcal{R}_{max}$ declines and the spread-cap tightens, the likelihood for the borrowers to borrow from lenders of the last-resort climbs sharply from 10-20% to above 60%. Meanwhile, even if some borrowers can still borrow from specialist lenders, the average number of participating lenders becomes significantly smaller.

Combining the positive effect of interest rate cap on the intensive margin and its negative effect on the extensive margin, Panel (a) and (b) illustrate the net effect. The net effect captures the likelihood of the borrowers borrowing from different types of lenders and the loan spread charged by these lenders. We observe a U-shaped relation between the average loan spread paid by borrowers and the tightness of rate cap and a hump-shaped relation between the average loan amount obtained by these borrowers and the tightness of rate cap. These findings suggest that there exists an optimal level of rate cap that maximizes the borrowers' welfare. Specifically, the optimal interest rate cap maps to an average loan spread of about 420 bps across all types of lenders in in the DIP loan market.

7 Conclusions

The lending market for distressed loans features an oligopolistic structure, with a few specialist lenders financing a large fraction of loans. It raises the question of how lender market power drives loan pricing in this market. We develop a dynamic game-theoretic model of strategic competition in distress loan markets with endogenous entry. Our entry and competing model provide several novel implications, including the "entry effect" of collusion capacity on the number of potential specialists and thus the likelihood of ex-post inefficient last-resort lending. Taking into account collusive lending by specialists and latent heterogeneity, we then use a comprehensive data sample that contains both distressed loans to bankrupt firms and those not yet in bankruptcy to estimate the structural model. We find that lender market power accounts for more than 90% of default-risk adjusted loan spread of distressed loans and for up to two-thirds of the DIP-loan spreads. More than half of the lender market power is attributed to collusive lending in distressed loans and limited participation, likely due to lenders' blocking power, in DIP loans. Smaller borrowers are particularly susceptible to lender market power than larger borrowers, calling for more attention from policy makers towards small borrowers in difficult times. Without lender collusion, a large fraction of distressed borrowers would switch from lenders of last resort such as hedge funds to specialized lenders and benefit from a significantly larger loan amount and a much lower loan spread. Our policy analysis on interest rate cap suggests that such policy has a hump-shaped effect on the borrowers' welfare due to a positive intensive margin counteracting a negative extensive margin. Specifically, there exists an optimal level of rate-cap that can be imposed by regulators.

Appendix

A Solution method:

We conjecture

$$L^C(k,x,m) \equiv \widehat{L}^C(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A \tag{57}$$

$$L^D(k,x,m) \equiv \widehat{L}^D(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A. \tag{58}$$

For each $\widehat{L}^C(k,m)$, we numerically solve

$$\widehat{L}^D(k,m) = \underset{\widehat{L}}{\text{argmax}} \left[\left(\frac{1}{\widehat{L} + (m-1)\widehat{L}^C(k,m)} \right)^{1/\varepsilon_2(k)} - 1 \right] \widehat{L}. \tag{59}$$

The first-order condition is the maximization problem above is

$$1 - \frac{1}{\varepsilon_2(k)} + \frac{1}{\varepsilon_2(k)} \frac{(m-1)\widehat{L}^C(k,m)}{\widehat{L}^D + (m-1)\widehat{L}^C(k,m)} = \left[\widehat{L}^D + (m-1)\widehat{L}^C(k,m) \right]^{1/\varepsilon_2(k)} \tag{60}$$

Thus, with the optimal deviation $\widehat{L}^D(k,m)$ pinned down in (60), the optimal deviation profit is

$$\mathbb{E}^x \left[\Pi_2^D(k,x,m;L^C) \right] = \left[\left(\frac{1}{\widehat{L}^D + (m-1)\widehat{L}^C} \right)^{1/\varepsilon_2(k)} - 1 \right] \widehat{L}^D e^{\alpha_2(k)+\frac{1}{2}\sigma^2+(1-\varepsilon_2(k))\phi_2+\frac{1}{2}(1-\varepsilon_2(k))^2\zeta^2} A \tag{61}$$

and

$$\mathbb{E}^x \left[\Pi_2(k,x,m;L^C) \right] = \left[\left(\frac{1}{m\widehat{L}^C} \right)^{1/\varepsilon_2(k)} - 1 \right] \widehat{L}^C e^{\alpha_2(k)+\frac{1}{2}\sigma^2+(1-\varepsilon_2(k))\phi_2+\frac{1}{2}(1-\varepsilon_2(k))^2\zeta^2} A. \tag{62}$$

We now try to get $\widehat{L}^C(k,m)$ for $L^C(k,x,m) \equiv \widehat{L}^C(k,m)e^{\alpha_2(k)-\varepsilon_2(k)\phi_2-\varepsilon_2(k)\varsigma u+\sigma z}A$. We define the IC compatible set of functionals $\widehat{L}^C(\cdot,\cdot)$ as follows:

$$\widehat{\mathcal{L}}^C \equiv \left\{ \widehat{L}^C : \frac{\xi(W^C(\widehat{L}^C) - W^N)}{1-\delta} \geq \mathbb{E}^x \left[\Pi_2^D(k,x,m;\widehat{L}^C) \right] - \mathbb{E}^x \left[\Pi_2(k,x,m;\widehat{L}^C) \right], \ \forall\, k,m \right\}. \tag{63}$$

In the collusive Nash equilibrium we focus on, the loan size is

$$\widehat{L}^C(\cdot,\cdot) = \underset{\widehat{L}\in\widehat{\mathcal{L}}^C}{\operatorname{argmax}}\, \mathbb{E}\left[U^C(k,x,m;\widehat{L})\right]. \tag{64}$$

Let $\widehat{L}^C_{max}(k,m) \equiv \frac{1}{m}\left[1-\frac{1}{\varepsilon_2(k)}\right]^{\varepsilon_2(k)}$. According to the definitions, it follows that $\widehat{L}^C_{max}(\cdot,\cdot)$ is the optimal loan size if $\widehat{L}^C_{max}(\cdot,\cdot) \in \widehat{\mathcal{L}}^C$.

B Estimation of Credit Spreads Based on CDS Prices

We assume that both credit spreads and CDS premia only mainly reflect the same credit risk. We observe T-year CDS premium, which is paid every period of Δ. The frequency $\Delta = 0.5$ is semiannual. We also have reliable estimation on the expected recovery rates δ and δ_L for the corporate bond and the distressed loan, respectively. Moreover, we have the information on zero-coupon risk-free bond prices, denoted by $Z(0, i\Delta)$ with $i = 1, 2, \cdots, T/\Delta$. Let $n = T/\Delta$ and $t_i = i\Delta$ for $i = 1, \cdots, n$.

Back out constant hazard rate p^*. If CDS only reflects credit risk, the non-arbitrage CDS premium formula is

$$s = \frac{1}{\Delta}\frac{(1-\delta)\sum_{i=1}^{n}\left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] Z(0,t_i)}{\sum_{i=1}^{n} P^*(0,t_i)Z(0,t_i)}, \tag{65}$$

where $P^*(0,t)$ is the risk-neutral probability of survival up to time t, modeled as

$$P^*(0,t) \equiv \exp(-p^* \times t). \tag{66}$$

We can estimate p^* using the equations (65) and (66) based on the data of s, δ, and $Z(0,t_i)$.

Estimate loan credit spreads. If a loan has maturity $T_L = t_m$, it holds that

$$1 = P^*(0,t_m)Z(0,t_m) + \sum_{i=1}^{m}\left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] Z(0,t_i)\delta_L + y\Delta\sum_{i=1}^{m} P^*(0,t_i)Z(0,t_i),$$

where y is the annualized loan yield. The equality above recognizes that convention that loans are sold at par, and that the loan yield is exactly equal to the coupon rate.

Then, the annualized yield of the loan is

$$y = \frac{1}{\Delta} \frac{1 - P^*(0, t_m) Z(0, t_m) - \sum_{i=1}^{m} \left[P^*(0, t_{i-1}) - P^*(0, t_i) \right] Z(0, t_i) \delta_L}{\sum_{i=1}^{m} P^*(0, t_i) Z(0, t_i)}.$$

and the credit spread is $y - r_f$ with $r_f = -\ln Z(0, 1)$.

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Figure 1: Model Timeline

This figure describes the timeline of the model. Each lending period is divided into two subperiods – morning and afternoon. The existing lender first decides whether to lend to the borrower with a probability $\lambda(\kappa)$, if the existing lender does not lend to the borrower, the borrower approaches a group of specialist lenders who can decide whether to participate in a syndicate loan. The specialist lenders need to pay a fixed cost if they participate. If no specialist lenders are willing to participate, the borrower turns to the lenders of last-resort.

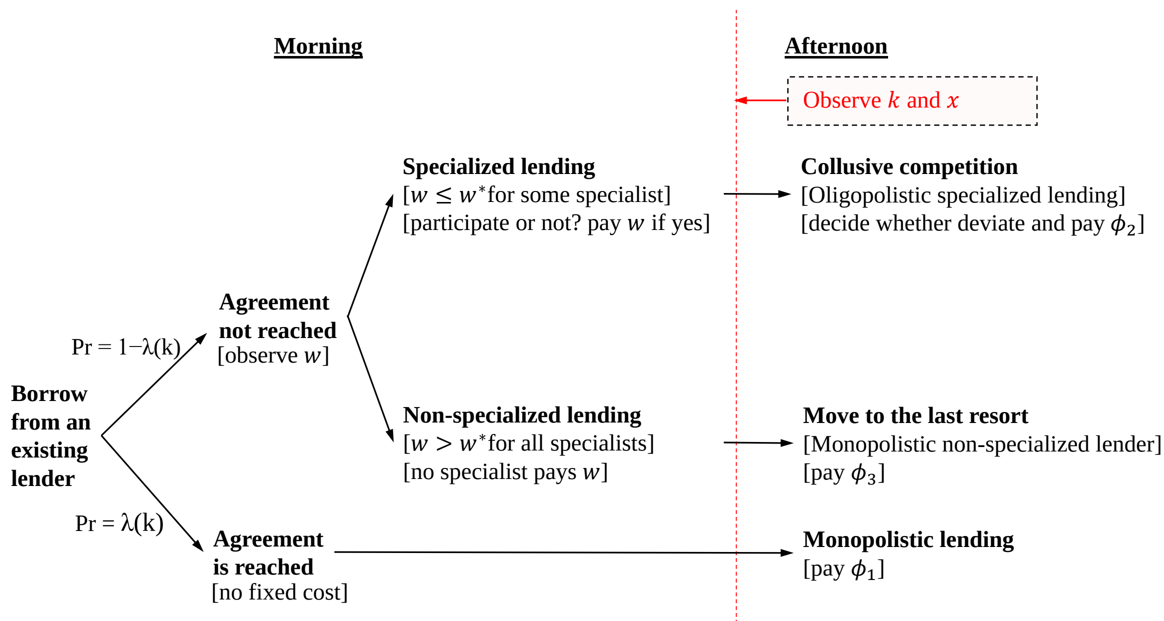


Figure 2: Loan Market Network

This figure describes the network of the distressed loan markets and the DIP loan market. Each circle represents one unique lender, with the size of the circle depending on the market share of the lender in the corresponding loan market. The tiny circles on the outer layer are for lenders whose market share is negligible. Each line between the lenders represents one coordination in a syndication.

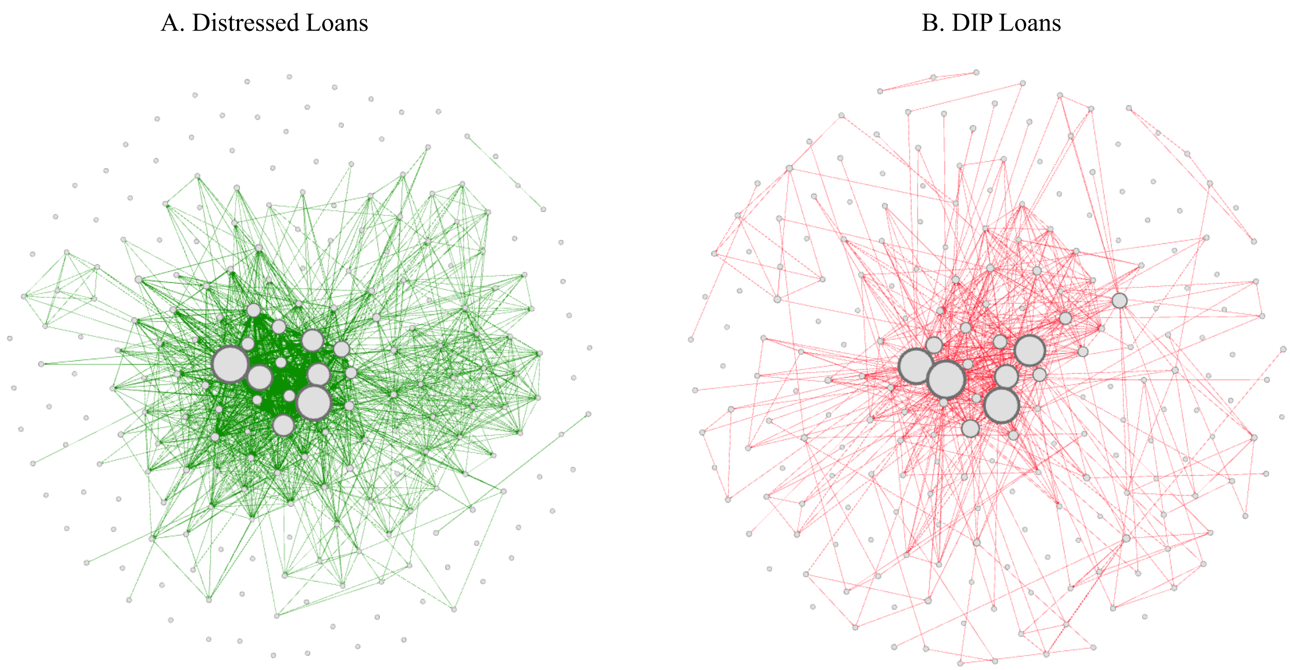
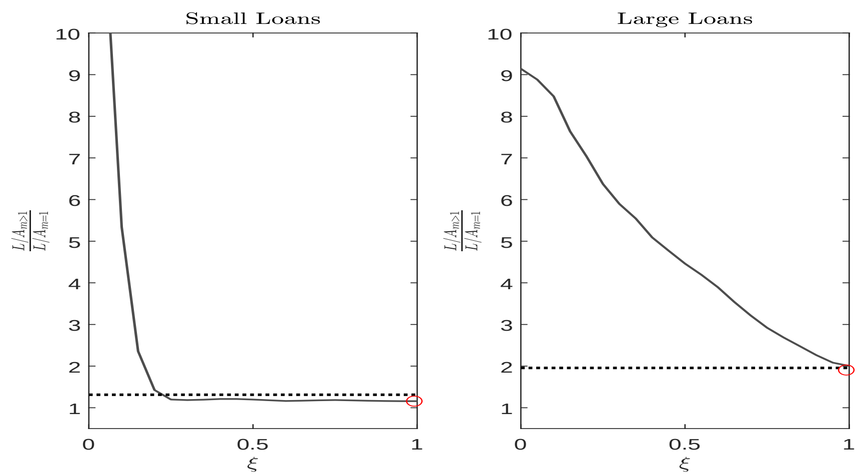
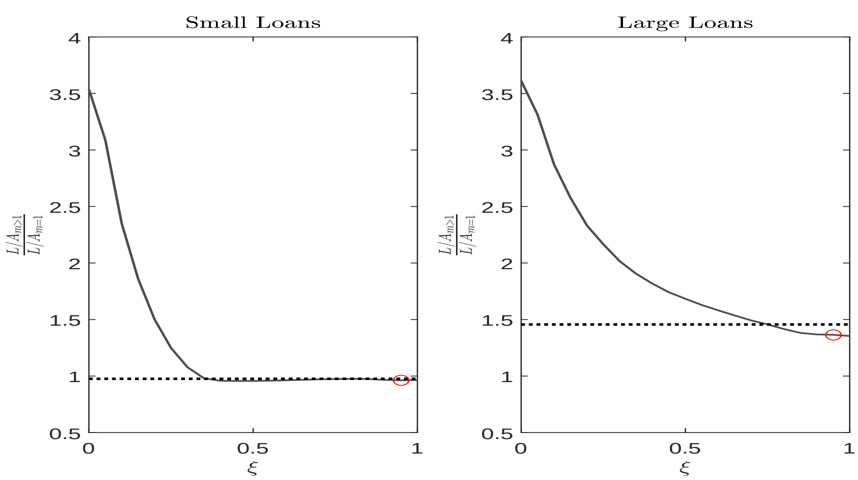


Figure 3: Identification of ξ

This figure plots the ratio of loan amount by a syndicated lender group ($m > 1$) to loan amount by a single lender ($m = 1$) for small borrowers (left) and large borrowers (right). Small (large) borrowers are the borrowers in the bottom (top) tercile of firm size in our sample. The solid line depicts how this ratio varies as the collusion intensity, ξ, changes. The dash line shows the empirical value of this ratio observed in the data for small and large borrowers. The red circle marks the estimated value of ξ and the model-implied loan size ratio. Panel (a) illustrates the results for the distressed loan sample, and panel (b) illustrates the results for DIP sample.



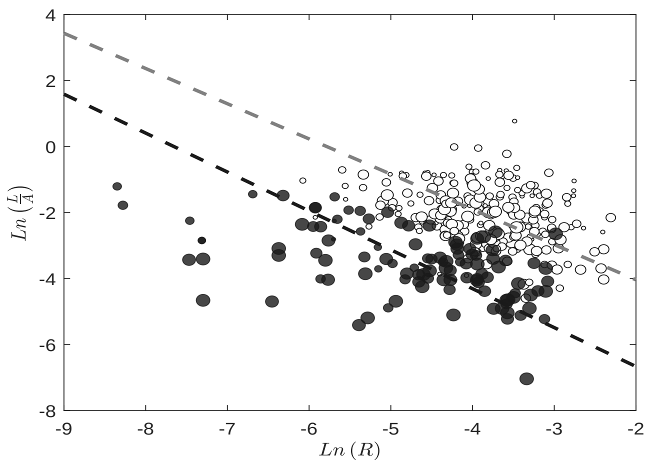
(a) Distressed Loans



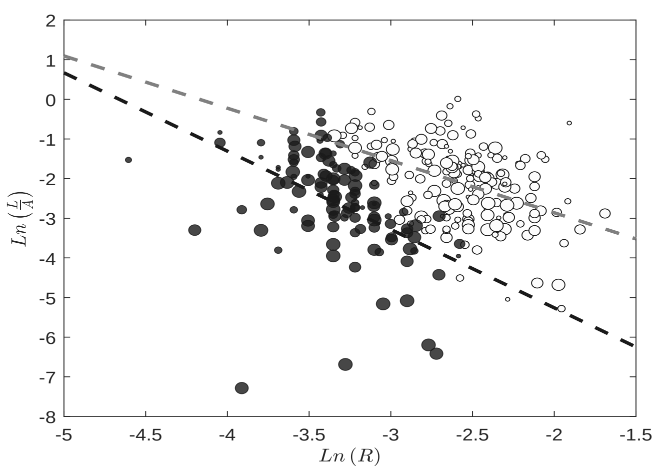
(b) DIP Loans

Figure 4: Demand Curves by Clusters

This figure shows the classification of borrowers to different demand curves. Each dot in the figure represents an observation in our sample, and the dash lines are the estimated demand curves. The dots in black represent the borrowers belonging to the demand curve marked in black dash line and the dots in white represent the borrowers belonging to the demand curve marked in gray dash line. The intercept and slope of the demand curves represent the constant term and the elasticity of the demand curve function, specified in Equation 4. Panel (a) depicts the classification in the distressed loan market and panel (b) depicts the classification in the DIP market.



(a) Distressed Loans



(b) DIP Loans

Figure 5: Spread Decomposition

This figure compares different components of loan spread in the distressed loan market and DIP loan market. Collusion represents the component of loan spread that arises from the specialist lenders' tacit coordination in syndicated lending; Limited participation represents the component of loan spread that arises from the limited competition in small syndicated lending groups, which in turn is a consequence of high participation costs in the estimated model; Oligopoly represents the component of loan spread due to the oligopolistic market structure with a few concentrated lenders; and variable costs represent the component of loan spread used to compensate the lenders for their lending costs (not include funding costs) such as monitoring costs.

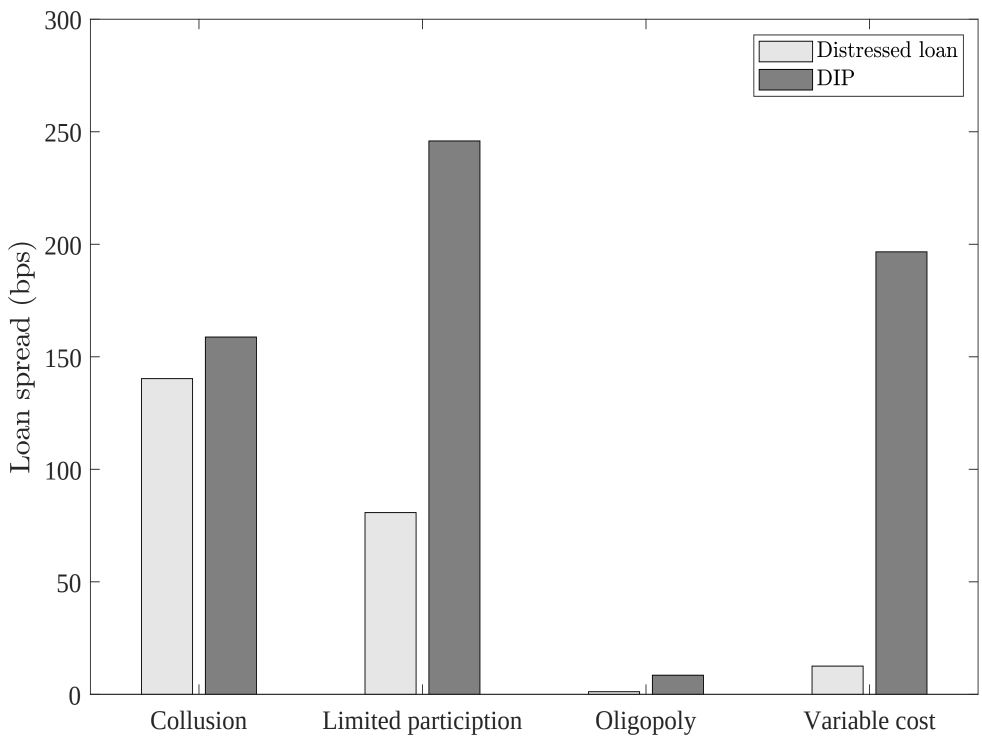


Figure 6: The Effects of Interest Rate Cap Policy

This figure shows the effects of the spread-cap policy. The policy details are described in section 6. Panels (a) and (b) illustrate the overall effect of rate-cap on the average loan spread and loan amount; panel (c) and (d) depict the intensive margin of rate-cap on reducing the loan spread charged by specialist lenders and increasing the corresponding loan amount in distressed loan market and DIP-loan market; panel (e) and (f) show the extensive margin of rate-cap on reducing the number of participating specialist lenders and increasing the chance of the borrowers turning to lenders of last-resort.

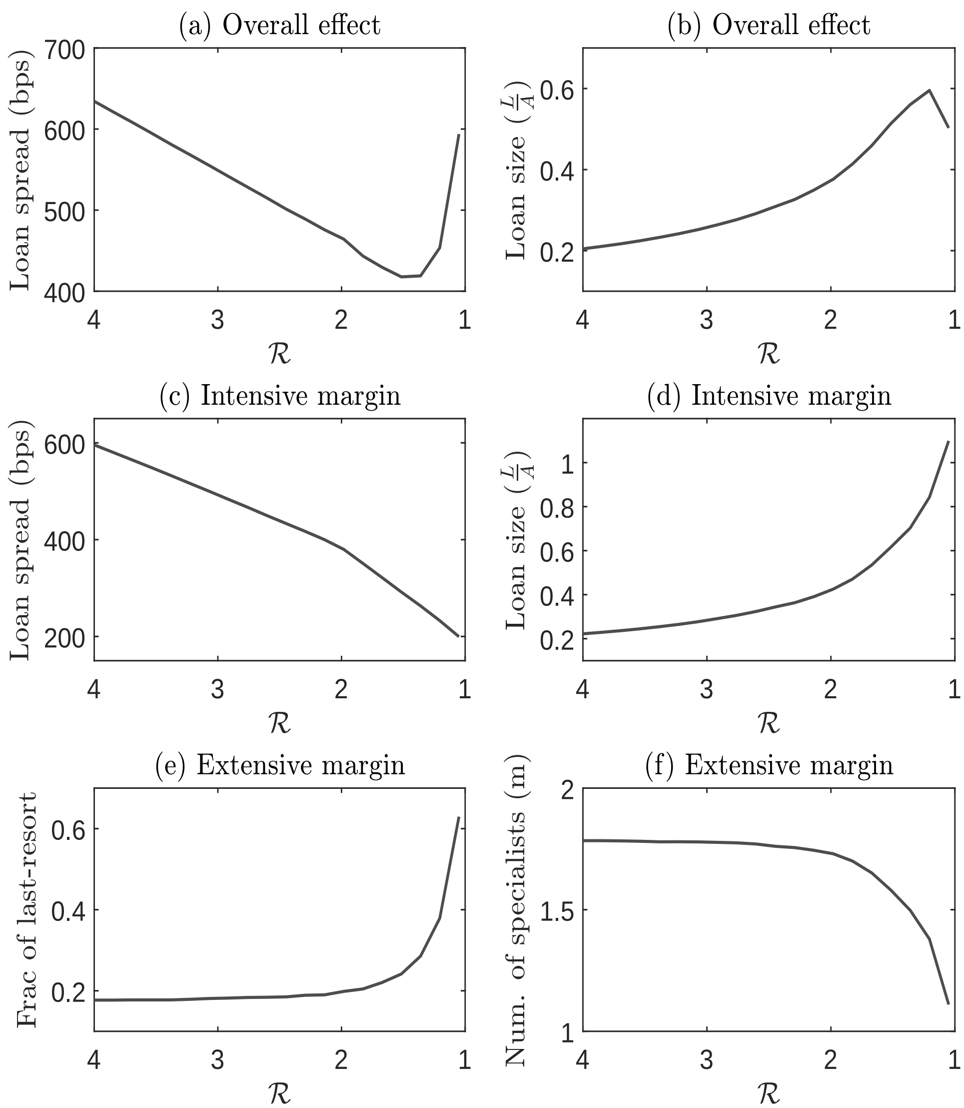


Table 1: Summary Statistics

This table presents the summary statistics of our sample firms. Our sample consists of 484 distressed loans (in Panel A) and 297 DIP loans (in Panel B) to U.S. public firms between 2001 and 2019. All financial variables are taken from the last fiscal year reported immediately prior to loan initiation, retrieved from Compustat. Assets, Liabilities and Sales are book assets, book liabilities, and revenue measured in millions of dollars, respectively. Leverage is the ratio of book liabilities to book assets. ROA is EBITDA scaled by book assets. PP&E/assets is the ratio of net property, plant and equipment to book assets. Cash/assets is the ratio of cash and short-term securities to book assets. Loan amount (L) is in millions of dollars. L/A is the ratio of DIP amount to book assets. AISD (R) is all-in-spread drawn in basis points. Number of lenders is the number of unique institutions in a syndicate. Loan type 1, Loan type 2 and Loan type 3 are indicator variables for loans provided by an existing lender, specialist lenders, and lenders of last resort, respectively.

	N	Mean	Std	25%	Median	75%
Panel A: Distressed Loans						
Assets	484	7,980	12,786	616	2,492	10,292
Liabilities	484	7,877	13,697	562	2,357	9,864
Sales	484	5,527	10,129	436	2,257	5,979
Leverage	484	1.037	0.434	0.793	0.949	1.140
ROA	478	0.066	0.129	0.037	0.072	0.114
PP&E/assets	484	0.385	0.242	0.169	0.356	0.545
Cash/assets	484	0.060	0.065	0.013	0.034	0.088
Loan amount (L)	484	430.945	644.333	69.000	200.000	500.000
L/A	484	0.134	0.167	0.033	0.081	0.181
AISD (R)	484	432.168	211.336	275.000	400.000	525.000
Number of lenders	484	3.744	2.787	2	3	5
Loan type 1	484	0.122	0.328	0	0	0
Loan type 2	484	0.752	0.432	1	1	1
Loan type 3	484	0.126	0.332	0	0	0
Loan type 1 (# of lenders)	59	1	0	1	1	1
Loan type 2 (# of lenders)	364	4.264	2.637	2	4	5
Loan type 3 (# of lenders)	61	3.295	3.348	2	2	2
Number of specialized lenders	484	2.285	1.644	1	2	3
Panel B: DIP Loans						
Assets	297	3,383	10,333	412	694	1,916
Liabilities	297	3,220	8,402	438	778	1,883
Sales	282	2,287	4,700	404	791	1,733
Leverage	281	1.084	0.521	0.832	0.972	1.228
ROA	280	0.031	0.207	-0.003	0.056	0.104
PP&E/assets	281	0.389	0.266	0.168	0.354	0.604
Cash/assets	282	0.052	0.065	0.012	0.028	0.068
Loan amount (L)	297	298.361	709.853	41.250	100.000	275.000
L/A	297	0.162	0.155	0.052	0.119	0.217
AISD (R)	297	636.439	296.998	400.000	600.000	809.091
Number of lenders	297	2.128	1.739	1	1	3
Loan type 1	297	0.145	0.352	0	0	0
Loan type 2	297	0.717	0.451	0	1	1
Loan type 3	297	0.138	0.346	0	0	0
Loan type 1 (# of lenders)	43	1	0	1	1	1
Loan type 2 (# of lenders)	213	2.394	1.912	1	2	3
Loan type 3 (# of lenders)	41	1.927	1.104	1	2	3
Number of specialized lenders	297	1.202	1.007	1	1	2

Table 2: Parameter Estimates

This table reports the estimated model parameters together with the standard errors obtained from MCMC. The top panel shows the general model parameters including the likelihood of punishment on deviation ξ, the demand and supply shock σ and ς, the parameters that control the correlation between borrower size and the demand curve, γ and β, as well as the average participation cost μ. The mid panel shows the lender-specific model parameter ϕ that captures the lending costs for each type of lenders (i.e., existing lenders, the specialist lenders, and lenders of the last resort). The bottom panel shows the borrower-specific parameters including the demand curve coefficient α and ε and the likelihood for the existing lender to finance the loan λ.

	DIP Loans				Distressed loans		
			General Parameters				
	ξ	σ	ς		ξ	σ	ς
	0.951	1.008	0.339		0.991	1.036	0.906
	(0.103)	(0.040)	(0.025)		(0.032)	(0.050)	(0.030)
	γ	β	μ		γ	β	μ
	0.435	-0.330	75.42		1.516	-0.672	29.32
	(0.823)	(0.237)	(12.10)		(0.421)	(0.310)	(3.34)
			Lender-specific Parameters				
	Existing lender	Specialists	Last-resort		Existing lender	Specialists	Last-resort
e^{ϕ}	0.017	0.018	0.024	e^{ϕ}	0.0009	0.0008	0.001
	(0.003)	(0.002)	(0.003)		(0.0002)	(0.0001)	(0.0002)
			Borrower-specific Parameters				
		Demand curve 1	Demand curve 2			Demand curve 1	Demand curve 2
α		-9.213	-5.498	α		-8.86	-6.085
		(1.496)	(0.232)			(0.331)	(0.126)
ε		1.976	1.319	ε		1.105	1.043
		(0.329)	(0.061)			(0.031)	(0.011)
λ		0.134	0.178	λ		0.117	0.169
		(0.022)	(0.031)			(0.019)	(0.027)

Table 3: Counterfactual Models and The Decomposition of Lender Market Power

This table reports the model implications for the estimated baseline model and a few counterfactual models. The Estimated model is the baseline model. In the non-collusive model, we shut down collusion by setting $\xi = 0$. In non-collusive, full participation model, we further zero out the participation cost for all specialist lenders and therefore all of them participate in a loan deal. In non-collusive, full participation, unlimited lenders model, we further increase the total number of specialist lenders to positive infinity so that it turns an oligopoly market to a competitive market.

	Distressed Loans				DIP Loans			
	Estimated Model	Non-collusive Model ($\xi = 0$)	Non-collusive, Full Participation ($\xi = 0, \mu = 0$)	Non-collusive, Full Participation, Unlimited Lenders ($\xi = 0, \mu = 0,$) ($M- >$ inf)	Estimated Model	Non-collusive Model ($\xi = 0$)	Non-collusive, Full Participation ($\xi = 0, \mu = 0$)	Non-collusive, Full Participation, Unlimited Lenders ($\xi = 0, \mu = 0,$) ($M- >$ inf)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Avg. Total Spread (bps)	246	139	44	43	646	551	268	261
Frac. Existing	0.098	0.098	0.098	0.098	0.157	0.157	0.157	0.157
Avg. Spread (bps)	325	325	325	325	607	607	607	607
Frac. Specialist	0.844	0.798	0.902	0.902	0.666	0.639	0.843	0.843
Avg. Spread (bps)	235	94	14	13	610	451	205	197
Frac. Last-Resort	0.059	0.104	0	0	0.177	0.204	0	0
Avg. Spread (bps)	289	305	NaN	NaN	817	822	NaN	NaN
Avg. m	2.500	2.135	10	Inf	1.784	1.707	10	Inf
Avg. L/A	0.495	3.364	7.725	8.297	0.216	0.424	1.006	1.073

Table 4: Heterogeneous Effects of Lender Collusion

This table reports the effect of shutting down lender collusion on borrowers of different size. We divide borrowers into size quintiles by their book assets and compare counterfactuals between small borrowers (bottom quintile) and the large borrowers (top quintile). The estimated model is the baseline model, and in the non-collusive model, we shut down lender collusion by setting ξ to zero. The effect of shutting down collusion is calculated as the difference between the results from the non-collusive model and those from the baseline model.

		Distressed loans			DIP loans		
	Borrower size	Baseline	Non-collusive	change	Baseline	Non-collusive	change
R	Small	289	108	-181	679	482	-197
	Large	136	78	-58	488	424	-64
L/A	Small	0.348	4.381	4.032	0.199	0.427	0.228
	Large	1.008	2.199	1.191	0.258	0.368	0.110
m	Small	2.510	2.110	-0.400	1.777	1.715	-0.062
	Large	2.484	2.117	-0.366	1.753	1.633	-0.120