Market Integration, Risk-Taking, and Income Inequality*

Lin William Cong[†] Ron Kaniel[‡] Yizhou Xiao[§]

Abstract

A pandemic or nationalism can dial back global integration as much as advancements in IT and transportation spur it. We study a parsimonious general equilibrium model of occupational choice, risk-taking, and income inequality against backdrop of market (dis)integration and certain services in inelastic supplies. In a decentralized, segmented environment, entrepreneurship and risk-taking are inefficiently low; in an integrated market, they can be socially excessive and entrepreneurship is non-monotone in the service supply. As transportation and information technologies improve, occupational risk-taking and total production increase, with ambiguous welfare consequences. In a dynamic setting with inter-generational inheritance, wealth inequality is exacerbated by income inequality, but faces a long-term reversal when service supply is affected by total production.

JEL Classification: E24, G12, G18, H23, J24, J31, J38

Keywords: Entrepreneurship, Globalization, Inequality, Relative Wealth.

*We thank Luofeng Zhou for excellent research assistance. This research was funded in part by the Ewing Marion Kauffman Foundation and the contents of this article are solely the responsibility of the authors.

[†]Cornell University SC Johnson College of Business and NBER. Email: will.cong@cornell.edu

[‡]University of Rochester. Email: ron.kaniel@simon.rochester.edu

[§]The Chinese University of Hong kong. Email: yizhou@cuhk.edu.hk

1 Introduction

The past decades have witnessed a rapid pace of globalization and digitization, promoting a more integrated market, followed by a recent revival of nationalism and deglobalization. Meanwhile, a large number of studies document rising income inequalities around the globe.[1] Many researches (e.g., Gozgor and Ranjan, 2017; Dorn, Fuest, and Potrafke, 2018) argue that globalization and market integration amplify the capital allocation efficiency, resulting in a larger gap between capital return and labour income, contributing to the significant and persistent losses for low-wage earners and increased income inequality. The effect of market integration on income inequality receives a lot of attention as events such as the Brexit, trade wars, and the ongoing pandemic all seem to disrupt the trend of market integration and exude anti-globalization sentiments.

We propose risk-taking as a new channel that connects the market integration (induced by globalization and information technology progresses) and income inequality. To this end, we build a tractable model to study how innovations in information technology, transportation, globalization, etc., make markets more integrated and affect agents' career choices and income inequality, with potential welfare consequences. Here, technological innovation is generically interpreted as new matching and allocation mechanisms on products or services or talents (e.g., in health care, marriage market, restaurant industry, etc.). As matching technologies improve, the products or services can be allocated among a larger group of people, improving allocative efficiency but altering agents' occupational choices and risk-taking, which in turn affect income inequality and welfare.

In our baseline specification, agents choose their occupations and compete for goods or services in inelastic supply. Specifically, each agent takes on a safe job ("iron rice bowl" or "job for life") with a risk-free income, or a risky job that may succeed or fail with different levels of skewness. Agents differ in the probability of success if one chooses to take risky occupations. After the realization of his career outcome, each agent learns about his desired service or product and competes for one unit of it under different allocation mechanisms.

[1] Piketty and Saez (2003) demonstrate that the percentage of all pre-tax income (excluding capital gains) in the United States that was received by the top 0.1 percent of income earners rose strikingly from 2.2 percent to 8.0 percent between 1981 and 2006. Tables and figures updated through 2010 at http://elsa.berkeley.edu/ saez/TabFig2010.xls, March 2012. Middle-class incomes have grown at a slower rate than upper-tier incomes over the past five decades. From 1970 to 2018, the median middle-class income increased from $58,100 to $86,600, a gain of 49%. By comparison, the median income for upper-tier households grew 64% over that time, from $126,100 to $207,400.

The inelastic supply of unit-demand service is important in our model in that it constitutes a device to make income inequality matter in an endogenous way through a market price mechanism.[2]

Intuitively, any limited resource would be more likely allocated to high income earners with strong demand because they can bid higher prices. Therefore, agents with mediocre income who have a decent chance to enjoy the limited resource in segmented market find "gambling" to become rich more attractive in an integrated market. Taking that into account, more agents opt to take a more skewed risk profile in their occupational choices (occasionally referred to as entrepreneurs), resulting in excessive risk-taking, unequal income distribution, and potential over-investment in high-risk sectors.

To see this, consider a segmented environment (in which pairs of agents represent segmented markets) in which all agents are randomly assigned into groups of two. Each group maybe endowed with zero, one, or two units of service, representing no supply, short supply, or full supply of inelastic goods. Agents compete through a second price auction if there is only one unit of service available. One can interpret each group as a segmented market and the lack of information technology makes it too costly to search and match across markets. In the integrated economy, the advance of information technology reduces the search cost and enables an integrated market for products and services of inelastic supply (which we generically refer to "service").

In the segmented environment, the formation of groups is independent of occupational choices. While workers still have a decent chance to win the service when they are matched with other workers or failed entrepreneurs, entrepreneurs face substantial risk because they may fail and thus in a disadvantageous position in the service competition. We show that in the unique equilibrium, the concern about limited supply of service discourages occupational risk-taking and agents tend to be conservative. They are less likely to become entrepreneurs (extensive margin) and all entrepreneurs take low risk (intensive margin). As a result, the economy experiences a small income inequality. Overall, relative to what would be socially efficient, we may have sub-optimal risk-taking. Surprisingly, an increment in service supply does not mitigate the friction. More service supply implies a higher chance to miss service

[2] It can be interpreted as natural resources, clear air, etc. of which the supply cannot be quickly adjusted. It can be equally interpreted as goods or services in a winner-takes-all environment. For example, high income households have been shifting consumer demand in favor of goods whose value stems from the talents of the top few (Frank, 2014).

goods upon failure, making agents even more conservative.

For an integrated economy, however, the efficient allocation of service goods implies that most potential competitors in the service market are successful entrepreneurs who can afford a high price. In order to have a decent chance to win the competition, one needs to become a successful entrepreneur as well. We show that in the unique equilibrium, the concern about the inelastic supply of service encourages risk-taking in both extensive and intensive margin. Agents tend to be aggressive in their risk-taking profile and the economy in aggregate experiences a significant income inequality. Overall, risk-taking is socially excessive in aggregate.

Under both environments, successful entrepreneurs are richer and are more willing to pay higher price to win the service. But if the entrepreneur fails, then he is less likely to afford the price and enjoy the service. The occupation choice affects not only the agent's income but also his service consumption. We emphasize the aggregate increase in income inequality, not how individuals are persistently high-income earner over time or whether inequality is self-limiting in that "rags to rags in three generations." In other words, we focus on distributional changes over time, not on social mobility. That said, we numerically illustrate how technology-driven market integration affects occupational choice and increases risk-taking as well as total production outputs. We also discuss how in a dynamic environment, the mechanism we highlight may exacerbate wealth inequality under inheritance and how wealth inequality faces a reversal in the long run when service supply is affected by total production and economic growth.

It is also interesting to notice that in the integrated economy, an increment in the supply of services have different effects on the extensive and intensive margin of risk-taking. More supply of services lower the equilibrium service goods price. As a result, entrepreneurs do not need to become super rich to find the service goods affordable and the intensive margin of risk-taking decreases. However, the effect on extensive margin (decision to become an entrepreneur) is non-monotonic. When the total supply is low, the price of service is relatively high, and workers and failed entrepreneurs are very unlikely to be able to purchase the service. An increment in supply lowers the price, but only successful entrepreneurs can enjoy the benefit, making the choice to become an entrepreneur more attractive. When the total supply is high, the price is low, and workers have reasonable access to service. Because workers are relatively more sensitive to service price, an increment in supply lowers the price and makes the service consumption relatively more attractive to the workers, reducing the incentive distortion for entrepreneurial career.

Technological progresses lead to more and more service goods allocated via integrated market. Consequently, we find through numerical simulation that a higher ratio of integrated market increases intensive margin. this is due to the fact that a higher ratio of integrated market implies agents are more likely to compete with other successful entrepreneurs, encouraging entrepreneurship and that a higher ratio implies more competitions among agents in service markets, resulting a lower expected utility gain, discouraging entrepreneurship. Interestingly, we observe non-monotonicity in extensive margin, and ranking flip (high service supply suggests a more sensitive relationship)

We then simulates the dynamics of an integrated market: to-start, for fixed service supply, dynamic further increase the inequality because rich people are more risk tolerant and are willing to take risks. We find that long term reversal. short term high risk-taking increase the social output, and hence service supply in later stage. The service supply discourages risk-taking and the inequality decreases.

We do not claim that the particular channel of occupational risk-taking is the only one or the most prominent in explaining and understanding income inequality. However, empirical patterns and anecdotes provide good motivations to examine it closely. For example, Feng and Tang (2019) documents that during 1992-2009, labor market factors collectively contributed more than three quarters of the total increase in income inequality in urban China. DeMarzo, Kaniel, and Kremer (2004) also quote from Wall Street Journal a money manager: "I've been saving like crazy. I'm expecting that when I'm 80 and need part-time nursing care, I'm going to be bidding against a lot of people for that."

Moreover, the interaction of occupational risk-taking and market integration have not been examined before. There is a large body of evidence indicating that globalization shocks can lead to important labor market disruptions (e.g., Pierce and Schott, 2016). For instance, David, Dorn, and Hanson (2013) show that American workers in regions specialized in goods facing steeper competition from China are less likely to work in manufacturing, more likely to be unemployed, and more likely to rely on disability insurance. Similarly, Dix-Carneiro and Kovak (2017, 2019) show that Brazilian regions that were more exposed to foreign competition as a result of the 1990s trade liberalization experienced prolonged periods of lower formal employment and wages. The market integration we model in abstraction includes globalization and can provide new insights.

Our paper adds to the literature on income inequality. The literature has documented how inequality is associated with crime (Choe, 2008), gambling (Freund and Morris, 2006), and greater consumer debt (Frank, 2013). Inequality is also associated with social and health problems, including higher rates of violence, drug use, and shorter life expectancies (e.g., Pickett and Wilkinson, 2015). Similarly, prior studies have shown that inequality affects with risk-taking at societal and individual levels (e.g., Mishra, Hing, and Lalumiere, 2015; Payne, Brown-Iannuzzi, and Hannay, 2017). Meanwhile, many studies discusses the causes of wealth or income inequality. Some are information-based (Azarmsa, 2019), some are related to risk aversion (Gomez et al., 2016) or financial ownership frictions (Peter, 2019), some are behavioral (e.g., Frank, Levine, and Dijk, 2014).

However, little is known about how occupational choices and risk-taking affect inequality. Moreover, technological innovation and economic integration have pushed up demand for services in highly inelastic supply (including skilled knowledge workers). Few theoretical studies relate income inequality to technological advancement and globalization; they also do not jointly analyze the occupational choice/labor market risk-taking with income inequality. We believe and demonstrate that they contribute to the rising income inequality in an intuitive way.

In that regard, we contribute to the literature relating income inequality to technological progress and IT innovations: Aghion, Akcigit, Bergeaud, Blundell, and Hémous (2018) show that innovation-led growth and creative destruction by entrants are is a source of top income inequality and important determinants of its dynamics. Bell, Chetty, Jaravel, Petkova, and Van Reenen (2018) also document that the most successful innovators cause a sharp rise in income. Jones and Kim (2018) build a Schumpeterian model to explain the Pareto distribution of the top income bracket. Our findings are also consistent with Murphy and Topel (2016) that market fundamentals favoring more skilled workers are driving the rising inequality. However, they do not consider endogenous occupational choices being affected by inequality.

Information technology in the workplace has been contributing to growing inequality because it complements the skills of the educated labor force (Acemoglu, 1998; Bresnahan, Brynjolfsson, and Hitt, 2002; Bartel, Ichniowski, and Shaw, 2007). Meanwhile, information technology seem to have leveled the playing field for consumers (Morton, Zettelmeyer, and

Silva-Risso, 2003; Tucker and Yu, 2019). Studies on superstars such as Cook and Frank (2010) also argues that top salaries have been growing sharply in virtually every labor market because technological forces greatly amplified small increments in performance and increase competition for the services of top performers. New online and international markets for talented managers has affected executive salaries in the same way that free agency affected the salaries of professional athletes in recent decades.

Technologies that allow scaling and greater information delivery certainly play important roles. Instead of focusing on top talents or superstars as the literature does, we examine the impact of market integration on income inequality and the general populace's occupational choice and risk-taking. We do not contradict Acemoglu and Autor (2011) who posit wage dispersion in a perfect labor market as a consequence of skill differentials, but given that the Skill-biased-technological-change hypothesis falls short as a unicausal explanation for the evolution of the U.S. wage structure (Card and DiNardo, 2002), we complement by showing how endogenous occupational risk-taking can create and amplify the dispersion. We also capture the consensus view that technological progress has made managerial skills more general, increasing competition for agents from segregated markets to a single economy-wide market (e.g., Piketty and Saez, 2006). We demonstrate how progresses in IT and transportation have led to greater market integration, which is beneficial only in moderation. A fully integrated market with inelastic supplies of goods and services may lead to socially inefficient and excessive occupational risk-taking.

Our emphasis on relative consumption power over scarce goods is related to relative compensation (Abel, 1990; DeMarzo, Kaniel, and Kremer, 2004, 2008), behavioral reference points (Frank, Levine, and Dijk, 2014), Tournaments (Cook and Frank, 2010), and status (Becker, Murphy, and Werning, 2005; Ray and Robson, 2012). In particular, (Becker, Murphy, and Werning, 2005) predict that there is more risk-taking behaviour in more equal societies and that the middle class should be the most risk-taking. Ray and Robson (2012) show in a dynamic setting that a concern for status implies that persistent and inefficient risk-taking hinders the attainment of full equality. We differ in that relative wealth matters through the consumption of services of inelastic supply and that we incorporate a new dimension of heterogeneity in skill. In addition, we are among the first to demonstrate how market (dis)integration interacts with risk-taking and income inequality.

We organize the remainder of the article as follows. Section 2 sets up the model; Sections 3 and 4 analyze equilibria under decentralized (segmented) markets and integrated markets respectively; Section 5 provides numerical analysis of the model in dynamic settings; Section 6 concludes; Appendix A contains all the proofs.

2 Model Setup

A simple economy with a discount rate normalized to 1 features two periods, 0 and 1, and a unit measure of agents. In $t = 0$, each agent i chooses his occupation and then the corresponding production technology λ_i. If an agent $i \in [0, 1]$ chooses to be a worker, we set $\lambda_i = 0$; otherwise, Agent i becomes an entrepreneur with an entrepreneurial production technology $\lambda_i \geq 1$. With $Prob(i, \lambda_i)$, he succeeds and generates a total income of $w_i = V_{\lambda_i}$; with probability $1 - Prob(i, \lambda_i)$, he fails and the output drops to $w_i = V_f$. We refer to the occupation choices (worker versus entrepreneur) as the extensive margin and the entrepreneurial production technology choices ($\lambda_i \geq 1$) as the intensive margin.

In $t = 1$, with probability α Agent i desires to consume a service in scarce supply, which delivers him a utility of $\log A > 0$. One can interpret the scarce service goods to be medical treatment for health issues, quality education for children, and so on. We let $s_i \in \{0, 1\}$ be the indicator of Agent i's service preference which is learned only at the start of $t = 1$. To capture the fact that many vital services and scarce goods, such as medical treatment, and education degrees, are indivisible in nature, we assume that each agent either consumes one unit service or not. We also assume a service production cost $\nu > 0$. Service providers choose not to provide service goods if the price is below ν. The service goods are scarce in the sense that service providers can produce at maximum a total supply of $\pi \in [0, 1)$.

Agent i's utility can be described as:

$$U_i = \log(w_i - p_i \mathbb{1}_i) + \mathbb{1}_i s_i \log A, \quad s.t. \ w_i - p_i \mathbb{1}_i \geq 1 \tag{1}$$

where w_i is agent i's income, $p_i > 0$ is the service price charged to Agent i and $\mathbb{1}_i$ is the indicator function for service purchase. The budget constraint reflects the minimum consumption level for survive, which is normalized to 1. Agent i who demands the scarce service ($s_i = 1$), when given a wealth w_i, purchases the service if $\log(w_i - p_i) + \log A \geq \log w_i$, i.e., $p \leq \frac{A-1}{A} w_i$. Notice that the price threshold satisfies the budget constraint. Intuitively, agents' production choices affect not only their consumption, but also the competition for the service.

To highlight the effect of limited services, we set the entrepreneurial production technology such that, absent concerns about limited services, entrepreneurs should be indifferent among production technology choices λ_i. Specifically, the probability of success is $Prob(i, \lambda_i) = \frac{2i}{1+\lambda_i}$, and the output is $V_0 = m > 1$, $V_{\lambda_i} = m^{1+\lambda_i}$, and $V_f = 1$. By construction,

$$\frac{2i}{1+\lambda_i}\log(m^{1+\lambda_i}) + (1 - \frac{2i}{1+\lambda_i})\log(1) = 2i\log m. \tag{2}$$

Then without the concern about the service market, the marginal entrepreneur i_e should be $i_e = \frac{1}{2}$. In this setup, failed entrepreneurs are unable to purchase the service.

In this model, we consider two types of markets for allocation/matching. The first is a traditional technology allocating each service within a group of two randomly drawn agents (two-agent islands). The second allows services to be distributed among all agents.

2.1 Segmented Market

We start with the two-agent island case. In this economy, agents are randomly assigned to groups of two. Services are allocated with equal probability to each group. Let $P_n(T)$ be the probability that a group is endowed with $n \in \{0, 1, 2\}$ units of services when there are T units of service available. We have:

$$\frac{1}{2}P_1(T) + P_2(T) = T. \tag{3}$$

We allow for general allocation rules but assume that they admit a monotonic increasing allocation ratio $\frac{P_2(T)}{P_1(T)}$. The monotonic increasing ratio assumption is general and is mechanic when there are enough service ($P_1(T) + P_2(T) = 1$). It implies that as more and more service becomes available, agents are more likely to enjoy the service goods.

In a group of one unit of service, agents compete through a second price auction. In a group of two units of service, both agents enjoy the service (if they are not failed entrepreneurs).[3] One example is that people might face uncertain medical conditions and the local medical resources could be random. If one happens to be born in a populated village with limited number of doctors, she has to compete with others for the service locally. Let

[3]One can motivate this by assuming that each unit of service charges some arbitrary small cost.

$b_i(w_i, A_i)$ be agent i's bidding strategy in second price auction. It is straightforward to see that the optimal bidding strategy is to bid the maximum price he can take. Now we can define the equilibrium for the two-agent case:

Definition 1. *[Two-agent Equilibrium] A two-agent equilibrium consists of agents' strategies* $\{\lambda_i, b_i\}_i$ *such that agent i's strategy* $\{\lambda_i, b_i\}$ *maximizes his expected utility, given other agents' strategies* $\{\lambda_j, b_j\}_{j\in[0,1]/i}$*, i.e.,* $\{\lambda_i, b_i\} \in \text{argmax}_{\{\lambda,b\}} \mathbb{E}[U_i]$.

2.2 Integrated Market

With information technology innovations, services can now be allocated among all agents. Everyone faces the same service price p. The equilibrium for the integrated market is:

Definition 2. *[Integrated Market Equilibrium] An integrated market equilibrium consists of strategies* $\{\lambda_i, \mathbb{1}_i\}_i$ *and service price p such that: (i)Individual optimality: Given p, Agent i's strategy* $\{\lambda_i, \mathbb{1}_i\}$ *maximizes the expected utility, i.e.,* $\{\lambda_i, \mathbb{1}_i\} \in \text{argmax}_{\{\lambda,\mathbb{1}\}} \; E[U_i]$*, and (ii) the service market clears, i.e.,* $\int_0^1 \mathbb{1}_i di = T$.

3 Equilibrium Characterization in Segmented Markets

We analyze the agent's expected utility gain from the service auction based on his career and entrepreneurial production technology choice. We then analyze the occupation choice and production technology choice in equilibrium. Finally, we prove the existence and uniqueness of equilibrium.

In a two-agent island, when there are two units of goods, or the other agent is not interested in service goods, as long as the agent is not a failed entrepreneur, the utility gain from the service is simply $\log A$. With one unit of the service goods available, however, the two agents have to compete. Consider worker i. If he is matched with another worker j, then both of them bid $b_i = b_j = \frac{A-1}{A}m$, and they are indifferent between winning or losing the auction.

When worker i is matched with a successful entrepreneur j, then

$$b_j = \frac{A-1}{A} m^{1+\lambda_j} > \frac{A-1}{A} m = b_i, \tag{4}$$

and agent i loses the auction. When he is matched with a failed entrepreneur j, agent i's gain from the auction is $\log A$.

As a worker, agent i's unconditional expected utility gain from the auction is

$$E_w(i) = \alpha^2 P_1(T)\pi_f \log A + (\alpha(1-\alpha)P_1(T) + \alpha P_2(T)) \log A, \tag{5}$$

where π_f is the probability that agent i is matched with a failed entrepreneur. Notice that the matching probability is independent of agent i's own career choice. The first term characterized the expected gain when both agents desire service goods and compete in the auction, while the second term characterize scenarios that there is no competition and agent i just pay the reserved price if he wants.

We now considers entrepreneurs. If agent i is a failed entrepreneur, he does not participate the auction. When agent i is a successful entrepreneur with production technology λ_i, if he is matched with a worker, then the other agent bids $\frac{A-1}{A}m$. Agent i's gain from the auction is

$$\begin{aligned} \log(m^{1+\lambda_i} - b_j) + \log A - \log(m^{1+\lambda_i}) &= \log\left(Am^{1+\lambda_i} - (A-1)m\right) - \log(m^{1+\lambda_i}) \\ &= \log\left(A - \frac{A-1}{m^{\lambda_i}}\right). \end{aligned} \tag{6}$$

If he is matched with another successful entrepreneur j with production technology λ_j, then the other agent j bids $\frac{A-1}{A}m^{\lambda_j}$. Agent i's gain from the auction is

$$\begin{cases} 0 & \text{if} \quad \lambda_i \leq \lambda_j \\ \log(A - \frac{A-1}{m^{\lambda_i - \lambda_j}}) & \text{if} \quad \lambda_i > \lambda_j. \end{cases} \tag{7}$$

If agent i is matched with a failed entrepreneur, then he pays the reserved price and his gain from the auction is $\log A$.

The equilibrium analysis is challenging for two reasons. First, each entrepreneur may choose different production riskiness λ_i; Second, the matching probabilities are endogenous and co-move with both extensive and intensive margins. The following lemma shows that, when the probability of service preference is sufficiently low, in the two-agent island case entrepreneurs choose conservative production technology $\lambda = 1$.

Lemma 1. *There exists $\overline{\alpha} \in (0,1)$ such that if $\alpha \leq \overline{\alpha}$, then in any equilibrium, all en-*

trepreneurs choose $\lambda = 1$.

Lemma 1 states that if the probability of service preference is low, then in any equilibrium entrepreneurs choose the most conservative entrepreneurial production technology $\lambda = 1$. If entrepreneur i chooses a more risky entrepreneurial production technology $\lambda > 1$, then conditional on being successful, he has more income and can afford a higher bidding, resulting a higher utility gain from the service auction. On the other hand, with a more risky entrepreneurial production technology $\lambda > 1$, agent i is likely to fail and thus loses the service competition. In the two-agent island case, agent i needs to compete with another random assigned agent, who may be in low demand of service, or is a failed entrepreneur. In either case, income advantage is marginal in terms of increasing potential utility gain from the service competition. Taking that into account, entrepreneurs finds it suboptimal to take additional risk and choose the most conservative entrepreneurial production technology.

Given Lemma 1, entrepreneur i's expected utility gain from auction is

$$\begin{aligned} E_e(i) =& i\alpha^2 P_1(T)\left[\pi_w \log(A - \frac{A-1}{m^{\lambda_i}}) + \pi_f \log A + \pi_s \times 0\right] \\ &+ i\left(\alpha(1-\alpha)P_1(T) + \alpha P_2(T)\right)\log A, \end{aligned} \tag{8}$$

where π_w, π_f and π_s are the probabilities that the agent is matched with a worker, a failed entrepreneur, and a successful entrepreneur, respectively. The term $\pi_s \times 0$ comes from the fact that all entrepreneur choose $\lambda = 1$. The next lemma characterizes the extensive margin in equilibrium.

Lemma 2. *In any equilibrium, there exists a marginal agent $\theta \in [0, 1]$ such that all agents with $i > \theta$ become entrepreneurs, and all agents with $i < \theta$ become workers.*

Lemma 2 states that in any equilibrium, there exists a threshold θ for occupational choice. Without loss of generality, one can then rewrite those matching probabilities as functions of the threshold θ. Given the marginal agent θ, the equilibrium matching probabilities can be computed as $\pi_w(\theta) = \theta$, $\pi_f(\theta) = (1-\theta)\frac{1-\theta}{2}$, and $\pi_s(\theta) = (1-\theta)\frac{1+\theta}{2}$.

Our first main result shows that, when the probability of service preference is sufficiently low, there exists an unique equilibrium and $\theta > i_e$. That is, in the two-agent case, the concern about services discourages entrepreneurship (more broadly, risky occupational choices).

Proposition 1. *If $\alpha \leq \overline{\alpha}$, there exists a unique equilibrium. In the equilibrium, all entrepreneurs choose $\lambda = 1$. Moreover, $\theta > i_e$ if $\alpha \leq \min\{\frac{8}{11}, \overline{\alpha}\}$.*

For the two-agent case, the formation of groups is independent of career choices. While workers still have a decent chance to win the service when they are matched with failed entrepreneurs and agents with no service demand, entrepreneurs face substantial risk because they may fail and thus in a disadvantageous position in service competition. We show that in the unique equilibrium, the concern about limited supply of service discourages entrepreneurial activities and agents tend to be conservative and the economy experiences a small income inequality. Moreover, relative to what would be socially efficient, we could have under-supply of entrepreneurial activities.

One might expect an increment in service supply T can mitigate the distortion in service market. However, the following corollary shows the opposite.

Corollary 2. *Let $\theta(T)$ be the marginal agent when the total supply of limited service is T. Then the equilibrium $\theta(T)$ is strictly increasing in T.*

In the two-agent case, each group consists of two agents, and agents are conservative because the concern about potential failure dominates the benefit of income advantage after success. When the total supply of service increases, the agent is more likely to be assigned in groups with full supply of services, and agents are more concerned about entrepreneurial risk, resulting in a more severe under-supply of entrepreneurial activities.

4 Equilibrium Characterization in Integrated Markets

With new IT or transportation innovations, limited services can now be allocated among all agents. The price p is the same for every agent and those who are rich and in high demand would purchase the service. Because entrepreneurs are indifferent on production technology choice absent the service market, and a higher wealth enables the entrepreneurs to consume the service, entrepreneurs find it optimal to always taking enough risk to make them rich enough to consume the service upon success. Agent i's expected utility when he becomes an entrepreneur is:

$$\begin{aligned}&\frac{2i}{1+\lambda_i}[\alpha \log \left((m^{1+\lambda_i}-p)A\right)+(1-\alpha)\log m^{1+\lambda_i}]\\ =&\frac{2i\alpha}{1+\lambda_i}\log(A-\frac{Ap}{m^{1+\lambda_i}})+2i\log m.\end{aligned} \tag{9}$$

Similarly, if he becomes a worker, his expected utility is:

$$\alpha[\log((m-p)A)-\log m]\mathbb{1}_{p\leq\frac{A-1}{A}m}+\log m. \tag{10}$$

Again, the analysis is potentially difficult because the supply function and thus the equilibrium price co-move with both extensive and intensive margins. We start our analysis by showing that in any equilibrium, there is a threshold type for career choice.

Lemma 3. *In any equilibrium, there exists a marginal agent $\theta \in [0,1]$ such that all agents $i > \theta$ choose to be entrepreneurs, and all agents $i < \theta$ choose to be workers.*

Lemma 3 states that in any equilibrium, there exists a threshold θ for occupational choice. Let θ be the marginal agent, then in any equilibrium:

$$\frac{2\theta\alpha}{1+\lambda_\theta}\log\left(Am-\frac{Ap}{m^{1+\lambda_\theta}}\right)+2\theta\log m=\alpha\left[\log((m-p)A)-\log m\right]\mathbb{1}_{p\leq\frac{A-1}{A}m}+\log m \tag{11}$$

The next lemma characterizes the equilibrium entrepreneurial technology choice.

Lemma 4. *In any equilibrium, all entrepreneurs choose the same entrepreneurial production technology λ. There exists a price $\underline{p}$ such that if $p > \underline{p}$, then $\lambda > 1$.*

In integrated markets, all entrepreneurs face the same service price, and the same relative risk among production technologies. As a result, in equilibrium they pick the same optimal choice of production technology. Lemma 4 implies that when the price is high, in equilibrium entrepreneurs may choose risky entrepreneurial production technology $\lambda > 1$. A more risky entrepreneurial production technology makes the price more affordable to the successful entrepreneur, but is more likely to fail. In an integrated market, a high equilibrium price means that rich agents are more likely to involve in service competition, and agent i needs to compete with other successful entrepreneurs. Taking that into account, entrepreneurs find it optimal to take more risk.

We next show that there exists a unique equilibrium may involve risky entrepreneurial production technologies ($\lambda > 1$) and over-entrepreneurship ($\theta < i_e$). That is to say, in the equilibrium the economy faces oversupply of entrepreneurial activities (and hence more income inequality), and entrepreneurs choose risky technologies (a higher degree of income skewness). These results hold regardless of parameter values ($\forall m > 1,\ 1 < A < m,\ \textit{and}\ \alpha \in (0,1)$).

Proposition 3. *There exists a unique equilibrium. In equilibrium, $p'(T) \leq 0$, and there exists a price $p^{i_e} \equiv m - \frac{\sqrt{4A(m-1)m+1}-1}{2A} \in \left(0, \frac{A-1}{A}m\right)$ such that $\theta < i_e$ if and only if $p > p^{i_e}$.*

For the integrated market case, the efficient searching and matching imply that most potential competitors in the service market are successful entrepreneurs. In order to have a decent chance to win the competition, one needs to become a successful entrepreneur as well. We show that in the unique equilibrium, the concern about limited supply of service encourages entrepreneurial activities and agents tend to be aggressive in their risk-taking and the economy in aggregate consequently faces a large income inequality.

Corollary 4. *Entrepreneurs' production technology choice λ is decreasing in T. Let $\theta(T)$ be the marginal agent when the total supply of limited service is T. For T_w satisfying $p(T_w) = \frac{A-1}{A}m$, $\theta'(T)$ is strictly increasing when $T > T_w$, and strictly decreasing when $T < T_w$.*

Corollary 4 states that the effect of service supply is non-monotonic in extensive margin, and may affect extensive and intensive margins differently. In the integrated market case, when the total supply is low, the price of service is relatively high, entrepreneurs thus have a strong incentive to take additional risk. However, in the equilibrium every entrepreneur takes excessive risk and the expected return for entrepreneurial activities is low. An increment in supply lowers the price, and the successful entrepreneurs are taking less risk in equilibrium, while they find the service consumption more attractive. When the total supply is high, the price is low, and workers have reasonable access to service. Because workers are relatively more sensitive to service price, an increment in supply lowers the price and makes the service consumption relatively more attractive to the workers, reducing the incentive distortion for entrepreneurial career. Figure 1 illustrates how marginal agent θ and the production technology choice λ_i responses to a change in the limited service supply.

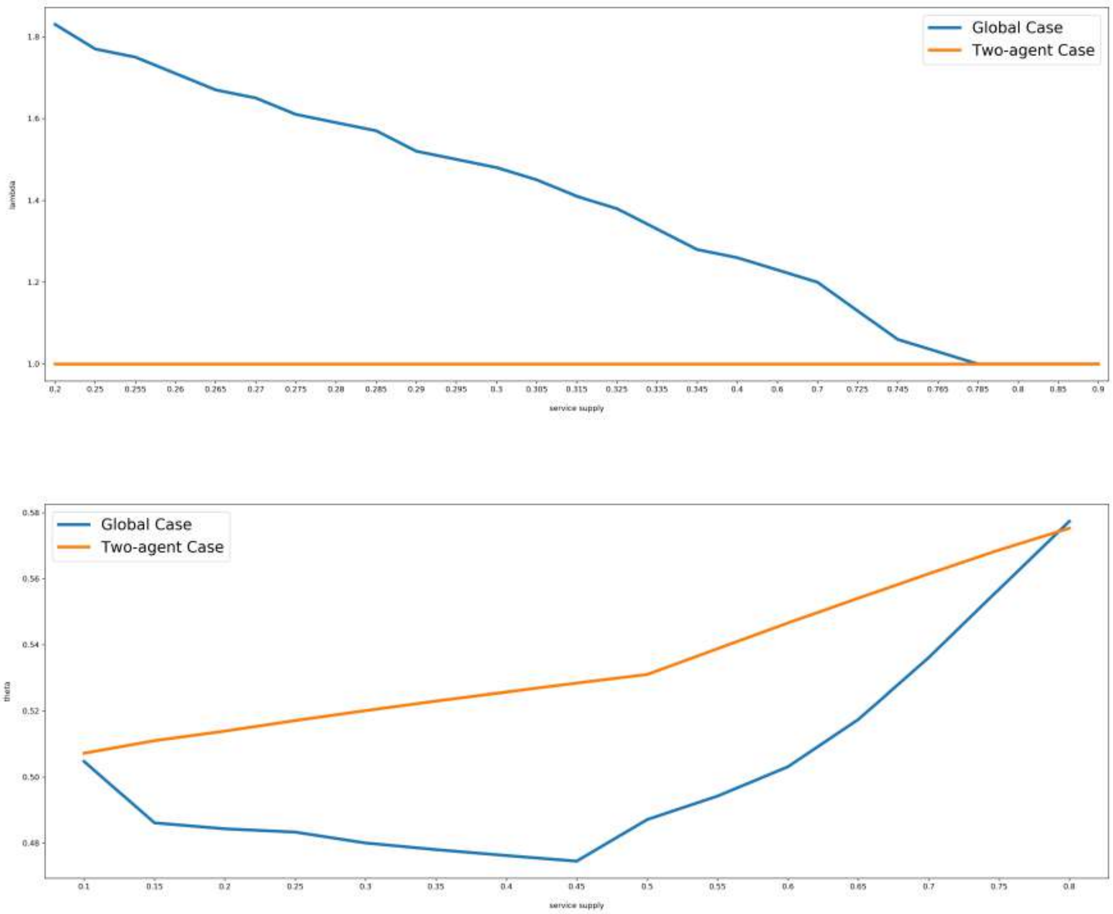

Figure 1: θ (entrepreneurship threshold) and λ (production technology choice) vs service supply: two-agent and global cases. $\alpha = 0.8$, m=1.4, A=1.3.

5 Occupational Risk-Taking and Inequality Dynamics

In this section we numerically analyze how the evolution of technology affects occupation choice and income inequality.

5.1 Service Supply and Ratio

We start our analysis by studying how the evolution of information technology changes the equilibrium occupation choices and risk-taking, and hence the income inequality. To characterize advances in information technology, we introduce a new "integrated market ratio," $\omega \in [0, 1]$, the proportion of limited services that are matched through advanced information technology and hence allocated effectively in an integrated market. Agents learn ω at $t = 0$ but each of them enters the integrated service market with a probability ω.

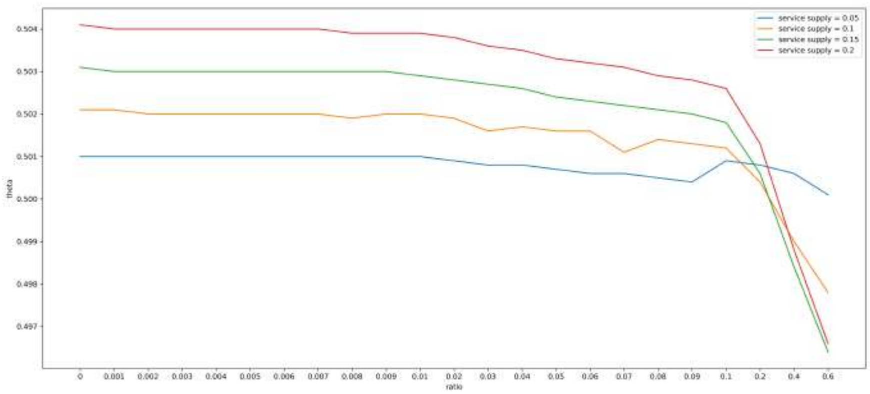

Figure 2: Occupation Choice and Technology Progress. $\alpha = 0.65$, m=2.3, A=1.25.

Figure 2 plots the occupation choice θ as information technology improves. There are two economic forces. On the one hand, a higher ratio of integrated market implies that agents are more likely to compete with other successful entrepreneurs in limited service market, motivating more entrepreneurial activities. On the other hand, the competition in integrated market suggests a lower expected utility gain from limited services, discouraging entrepreneurship. As Figure 2 shows, information technology progress in general encourages entrepreneurial activities. The effect can be non-monotonic, especially when the total supply of limited service is low and the corresponding competition is intensive in integrated market.

It is also interesting to see that the rankings of occupation choices with different service supplies may flips as the technology progresses. When there is little technology progress, agent's occupation choices are largely driven by the two-agent scenario, and a high level of service supply implies more concerns about service market distortion and agents are less likely to become an entrepreneur. As the technology progresses, a high level of service supply introduces a big effect of potential competition in integrated markets scenario, and agents change their risk-taking behavior at a faster speed.

We next studies the intensive margin. Figure 3 illustrates that as information technology improves, entrepreneurs are more concerned about competition with other successful entrepreneurs in the integrated market, and they have strong incentive to take more risky

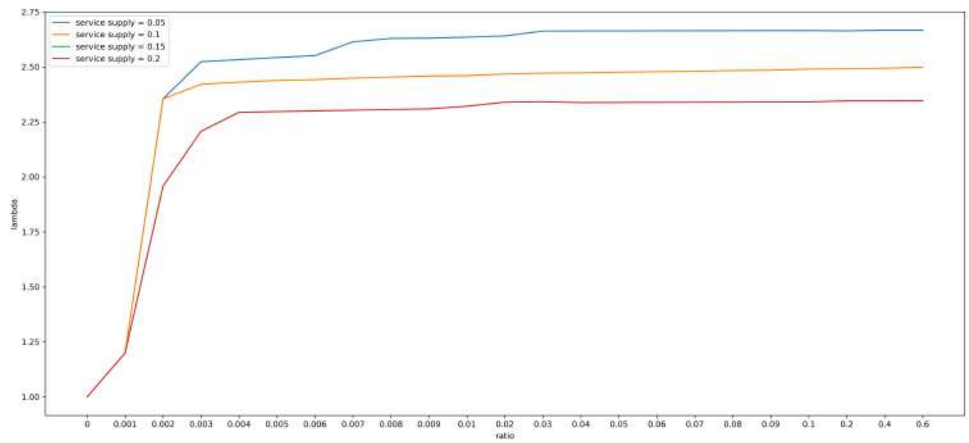

Figure 3: Risk-taking and Technology Progress. $\alpha = 0.65$, m=2.3, A=1.25.

projects. The smaller the total supply of limited service is, the more concerns about the service market competition and more risk-taking.

Closely related is the total output of agents. When entrepreneurs become more risk tolerant, projects are more innovative and the expected output increases. Figure 4 confirms this intuition by showing that as information technology advances, the total production output increases.

However, increasing in total production does not necessarily mean agents are better off. Figure 5 shows that agents may be worse off as information technology advances, even when the total output increases and the limited service market allocation becomes more efficient. This comes from the fact that the improvement in allocation efficiency largely benefits service providers, and intensive competition enables providers to extract more surplus. In a sense, globalization and market integration benefits those with scarce service goods.

5.2 Wealth Dynamics Under Unequal Income and Inheritance

We extend the baseline model to allow overlapping generations in order to numerically characterize the wealth dynamics in the population. For an agent i, if the corresponding older generation agent has w in the end (production minus expense), the productivity of agent i changes from m to $m + \gamma w$. Intuitively, if agent i chooses to be a worker, its income can be viewed as the income earned by its work (m) plus the money left to him γw; if agent

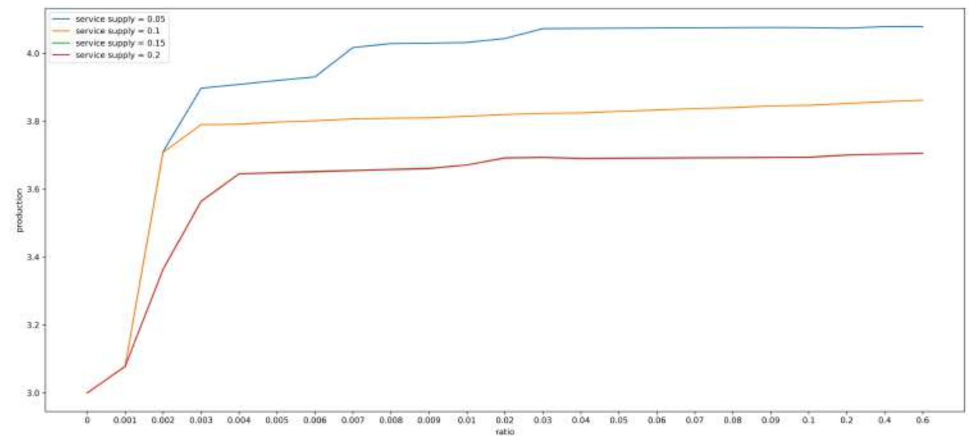

Figure 4: Production Output and Information Technology Progress. $\alpha = 0.65$, m=2.3, A=1.25.

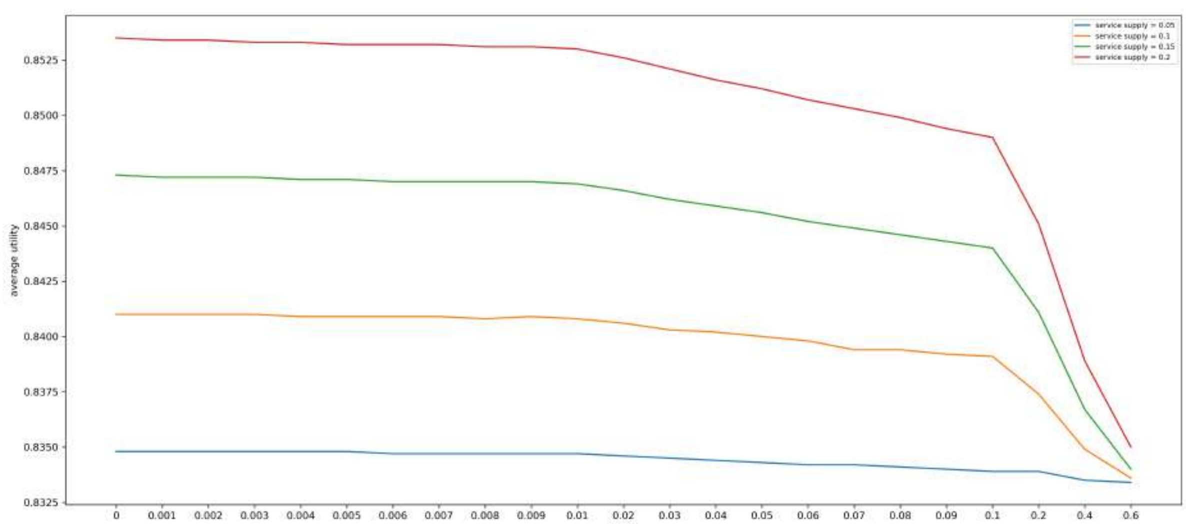

Figure 5: Agents Welfare and Information Technology Progress. $\alpha = 0.65$, m=2.3, A=1.25.

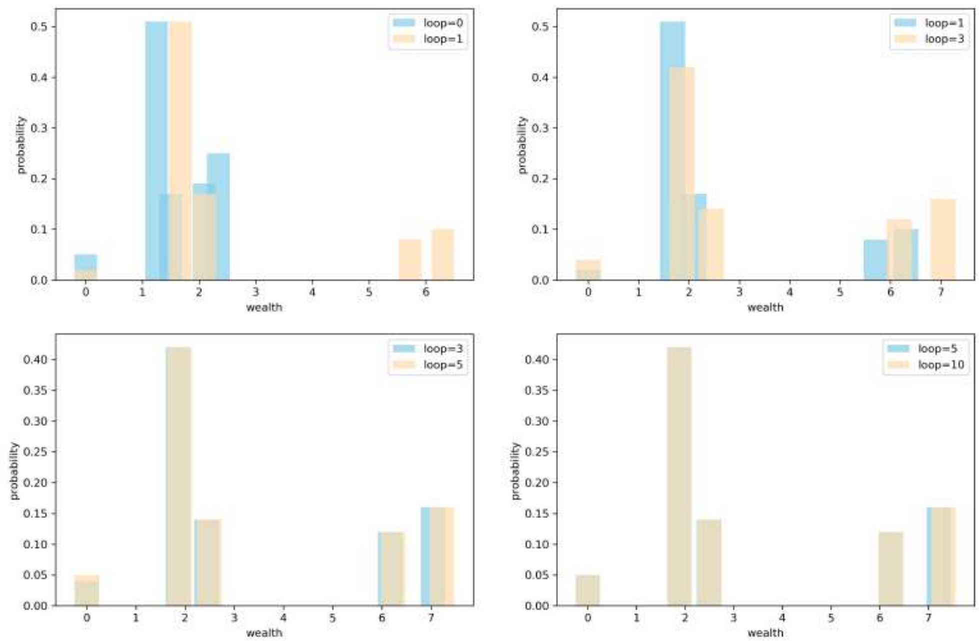

Figure 6: Wealth Dynamics with Fixed Service Supply. $\alpha = 0.75$, m=1.5, A=1.2.Each Loop is one generation in the overlapping generation model.

i chooses to be an entrepreneur with λ, it invests its human capital (opportunity cost as a worker, m) plus the money left to him γw to entrepreneurial activities and its income structure is $(m + \gamma w)^{1+\lambda}$ if he succeeds and 0 otherwise. We assume $\gamma = 0.34$ and $T = 0.35$ and start with uniformly 0 initial wealth endowment.

Figure 6 shows the evolution of wealth distribution. As the economy evolves, wealthy agents are less risk-averse and are more likely to involve entrepreneurial activities and risky choice of production technology. As a result, the wealthy agent becomes relatively more wealthy and wealth inequality becomes more skewed as it converges to the steady state.

In the analysis above we assume a fixed supply of limited services. In the long run, the long term supply of limited services may evolve and is determined by the total production. For example, it takes a long time and resources to build up education and healthcare sectors. To be more specific:

$$T_{t+1} = \rho T_t + \beta Z_t, \tag{12}$$

where Z_t is the total production (total income of workers and successful entrepreneurs) at

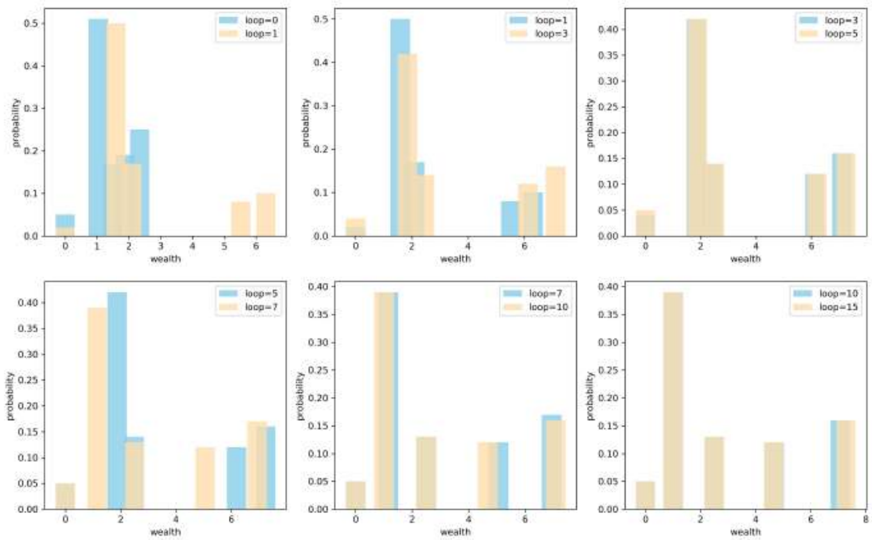

Figure 7: Heritage with Evolving Service Supply. $\alpha = 0.75$, m=1.5, A=1.2.

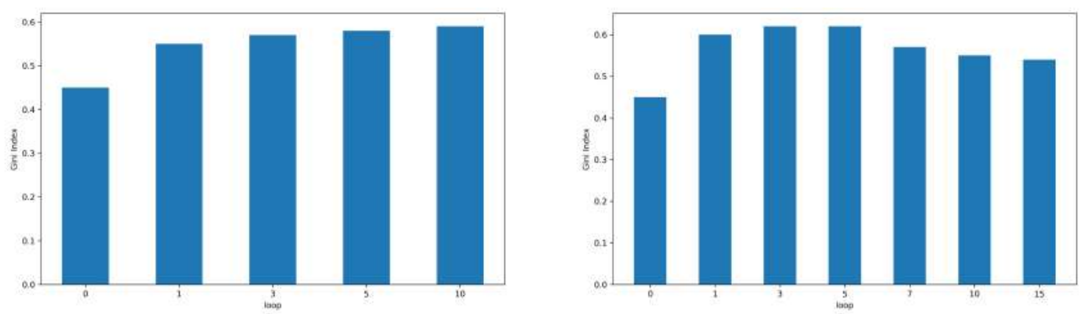

Figure 8: Gini Coefficients with Fixed and Evolving Service Supply

time t. It is obvious that the steady state solves:

$$T^* = \frac{\beta}{1-\rho} Z^*. \tag{13}$$

We assume that $\rho = 0.75$, $\beta = 0.01$, $, T_0 = 0.25$.

Figure 7 shows the evolution of wealth distribution, and figure 8 shows the change of Gini coefficients in both fixed and evolving service supply scenarios. Similar to the fixed service supply case, as the economy evolves, wealthy agents are less risk-averse and are more likely to involve entrepreneurial activities and risky choice of production technology. However, the evolving service supply introduces another economic force. When economy has more entrepreneurs and risky production technologies, the total production output is high, suggesting a larger supply of limited services next period. A large supply of service weakens the market competition, and agents become less aggressive resulting in a lower income inequality. As a result, after the initial jump in wealth, wealthy agents become relatively less wealthy as the economy and total supply of limited services converge.

6 Conclusion

A pandemic or nationalism can dial back global integration as much as advancements in IT and transportation spur it. We study a parsimonious general equilibrium model of occupational choice, risk-taking, and income inequality against backdrop of market (dis)integration with inelastic supplies of products and services. In a decentralized and segmented environment, entrepreneurship and risk-taking are inefficiently low; in an integrated market, they can be socially excessive and entrepreneurship is non-monotone in the inelastic supply of service. As transportation and information technologies improve, occupational risk-taking and total production increase, with ambiguous welfare consequences. In a dynamic setting with inter-generational inheritance, wealth inequality is exacerbated by income inequality, but faces a long-term reversal when service supply is affected by total production.

References

Abel, Andrew B, 1990, Asset prices under habit formation and catching up with the joneses, Discussion paper, National Bureau of Economic Research.

Acemoglu, Daron, 1998, Why do new technologies complement skills? directed technical change and wage inequality, *The Quarterly Journal of Economics* 113, 1055–1089.

——— , and David Autor, 2011, Skills, tasks and technologies: Implications for employment and earnings, in *Handbook of labor economics*vol. 4 . pp. 1043–1171 (Elsevier).

Aghion, Philippe, Ufuk Akcigit, Antonin Bergeaud, Richard Blundell, and David Hémous, 2018, Innovation and top income inequality, *The Review of Economic Studies* 86, 1–45.

Azarmsa, Ehsan, 2019, Investment sophistication and wealth inequality, .

Bartel, Ann, Casey Ichniowski, and Kathryn Shaw, 2007, How does information technology affect productivity? plant-level comparisons of product innovation, process improvement, and worker skills, *The quarterly journal of Economics* 122, 1721–1758.

Becker, Gary S, Kevin M Murphy, and Ivan Werning, 2005, The equilibrium distribution of income and the market for status, *Journal of Political Economy* 113, 282–310.

Bell, Alex, Raj Chetty, Xavier Jaravel, Neviana Petkova, and John Van Reenen, 2018, Who becomes an inventor in america? the importance of exposure to innovation, *The Quarterly Journal of Economics* 134, 647–713.

Bresnahan, Timothy F, Erik Brynjolfsson, and Lorin M Hitt, 2002, Information technology, workplace organization, and the demand for skilled labor: Firm-level evidence, *The quarterly journal of economics* 117, 339–376.

Card, David, and John E DiNardo, 2002, Skill-biased technological change and rising wage inequality: Some problems and puzzles, *Journal of labor economics* 20, 733–783.

Choe, Jongmook, 2008, Income inequality and crime in the united states, *Economics Letters* 101, 31–33.

Cook, Philip J, and Robert H Frank, 2010, *The winner-take-all society: Why the few at the top get so much more than the rest of us* (Random House).

David, H, David Dorn, and Gordon H Hanson, 2013, The china syndrome: Local labor market effects of import competition in the united states, *American Economic Review* 103, 2121–68.

DeMarzo, Peter M, Ron Kaniel, and Ilan Kremer, 2004, Diversification as a public good: Community effects in portfolio choice, *The Journal of Finance* 59, 1677–1716.

——— , 2008, Relative wealth concerns and financial bubbles, *The Review of Financial Studies* 21, 19–50.

Dix-Carneiro, Rafael, and Brian K Kovak, 2017, Trade liberalization and regional dynamics, *American Economic Review* 107, 2908–46.

——— , 2019, Margins of labor market adjustment to trade, *Journal of International Economics* 117, 125–142.

Dorn, Florian, Clemens Fuest, and Niklas Potrafke, 2018, Globalization and income inequality revisited, .

Feng, Shuaizhang, and Gaojie Tang, 2019, Accounting for urban china's rising income inequality: The roles of labor market, human capital, and marriage market factors, *Economic Inquiry* 57, 997–1015.

Frank, Robert, 2013, *Falling behind: How rising inequality harms the middle class* . vol. 4 (Univ of California Press).

Frank, R, A Levine, and O Dijk, 2014, Expenditure cascades, *Review of Behavioral Economics* 1, 55–73.

Frank, ROBERT H, 2014, The vicious circle of income inequality, *New York Times* 11.

Freund, Elizabeth A, and Irwin L Morris, 2006, Gambling and income inequality in the states, *Policy Studies Journal* 34, 265–276.

Gomez, Matthieu, et al., 2016, Asset prices and wealth inequality, *Unpublished paper: Princeton.*

Gozgor, Giray, and Priya Ranjan, 2017, Globalisation, inequality and redistribution: Theory and evidence, *The World Economy* 40, 2704–2751.

Jones, Charles I, and Jihee Kim, 2018, A schumpeterian model of top income inequality, *Journal of Political Economy* 126, 1785–1826.

Mishra, Sandeep, Leanne S Son Hing, and Martin L Lalumiere, 2015, Inequality and risk-taking, *Evolutionary Psychology* 13, 1474704915596295.

Morton, Fiona Scott, Florian Zettelmeyer, and Jorge Silva-Risso, 2003, Consumer information and discrimination: Does the internet affect the pricing of new cars to women and minorities?, *Quantitative marketing and Economics* 1, 65–92.

Murphy, Kevin M, and Robert H Topel, 2016, Human capital investment, inequality, and economic growth, *Journal of Labor Economics* 34, S99–S127.

Payne, B Keith, Jazmin L Brown-Iannuzzi, and Jason W Hannay, 2017, Economic inequality increases risk taking, *Proceedings of the National Academy of Sciences* 114, 4643–4648.

Peter, Alessandra, 2019, Owning up: Closely held firms and wealth inequality, *working paper.*

Pickett, Kate E, and Richard G Wilkinson, 2015, Income inequality and health: a causal review, *Social science & medicine* 128, 316–326.

Pierce, Justin R, and Peter K Schott, 2016, The surprisingly swift decline of us manufacturing employment, *American Economic Review* 106, 1632–62.

Piketty, Thomas, and Emmanuel Saez, 2003, Income inequality in the united states, 1913–1998, *The Quarterly journal of economics* 118, 1–41.

——— , 2006, The evolution of top incomes: a historical and international perspective, *American economic review* 96, 200–205.

Ray, Debraj, and Arthur Robson, 2012, Status, intertemporal choice, and risk-taking, *Econometrica* 80, 1505–1531.

Tucker, Catherine E, and Shuyi Yu, 2019, Does it lead to more equal treatment? an empirical study of the effect of smartphone use on customer complaint resolution, *An Empirical Study of the Effect of Smartphone Use on Customer Complaint Resolution (June 12, 2019).*

Appendix A: Proofs of Lemmas and Propositions

A1. Proof of Lemma 1

Proof. Notice that given the production technology, agents are concerned about the entrepreneurial production technology only because it may affect the expected gains from service auction. We prove by showing that when α is small enough, $\lambda_i = 1$ is agent i's optimal choice regardless the type of agents he matches with.

Step 1: Matched with Failed Entrepreneur

If the other agent is a failed entrepreneur, he won't participate in the auction, then agent i's conditional expected gain from auction is:

$$\frac{2i\alpha}{1+\lambda_i}\log A. \tag{14}$$

It is straightforward to see that agent i's conditional optimal choice is $\lambda_i = 1$.

Step 2: Matched with Worker

If the other agent is a worker, he will bid $\frac{A-1}{A}m$, then agent i's conditional expected gain from auction is:

$$F_1(\lambda_i) = \frac{2i\alpha}{1+\lambda_i}[\alpha\log(A - \frac{A-1}{m^{\lambda_i}}) + (1-\alpha)\log A]. \tag{15}$$

Then the first order derivative is

$$\begin{aligned} F_1'(\lambda_i) =& \frac{2i\alpha}{(1+\lambda_i)^2}\{\alpha[\frac{(1+\lambda_i)(A-1)\log m}{(A-\frac{A-1}{m^{\lambda_i}})m^{\lambda_i}} - \log(A - \frac{A-1}{m^{\lambda_i}})] - (1-\alpha)\log A\} \\ =& \frac{2i\alpha}{(1+\lambda_i)^2}\{\frac{\alpha}{A-\frac{A-1}{m^{\lambda_i}}}[\frac{(1+\lambda_i)(A-1)\log m}{m^{\lambda_i}} - (A-\frac{A-1}{m^{\lambda_i}})\log(A-\frac{A-1}{m^{\lambda_i}})] - (1-\alpha)\log A\}. \end{aligned} \tag{16}$$

Define $G_1(\lambda_i) \equiv \frac{(1+\lambda_i)(A-1)\log m}{m^{\lambda_i}} - (A - \frac{A-1}{m^{\lambda_i}})\log(A - \frac{A-1}{m^{\lambda_i}})$, notice that

$$G_1'(\lambda_i) = -\frac{(A-1)\log m}{m^{\lambda_i}}[log(A - \frac{A-1}{m^{\lambda_i}}) + (1+\lambda_i)\log m] < 0. \tag{17}$$

If $G_1(1) \le 0$, then $F_1'(\lambda_i) \le 0$ for all $\lambda_i \ge 1$. $G_1(1) > 0$, then $\frac{1}{A-\frac{A-1}{m^{\lambda_i}}}G_1(\lambda_i)$ is strictly decreasing in λ_i. Then for $\forall\alpha \le \overline{\alpha}_1 \equiv \frac{\log A}{\frac{1}{A-\frac{A-1}{m}}G_1(1)+\log A}$, one have

$$\begin{aligned} F_1'(\lambda_i) =& \frac{2i\alpha}{(1+\lambda_i)^2}\{\alpha[\frac{(1+\lambda_i)(A-1)\log m}{(A-\frac{A-1}{m^{\lambda_i}})m^{\lambda_i}} - \log(A - \frac{A-1}{m^{\lambda_i}})] - (1-\alpha)\log A\} \\ \le& \frac{2i\alpha}{(1+\lambda_i)^2}\{\alpha[\frac{2(A-1)\log m}{(A-\frac{A-1}{m})m} - \log(A - \frac{A-1}{m})] - (1-\alpha)\log A\} \\ \le& \frac{2i\overline{\alpha}_1}{(1+\lambda_i)^2}\{\overline{\alpha}_1[\frac{2(A-1)\log m}{(A-\frac{A-1}{m})m} - \log(A - \frac{A-1}{m})] - (1-\overline{\alpha}_1)\log A\} \\ =& 0. \end{aligned} \tag{18}$$

So for $\forall \alpha \leq \overline{\alpha}_1$, $F_1(\lambda_i)$ obtains optimal value when $\lambda_i = 1$.

Step 3: Matched with Successful Entrepreneur

If the other agent is a successful entrepreneur with a production technology λ_j, he will bid $\frac{A-1}{A} m^{\lambda_j}$, then agent i's conditional expected gain from auction is:

$$F_2(\lambda_i, \lambda_j) = \frac{2i\alpha}{1+\lambda_i}[\alpha \log(A - \frac{A-1}{m^{\lambda_i - \lambda_j}})\mathbb{1}_{\lambda_i \geq \lambda_j} + (1-\alpha)\log A]. \tag{19}$$

Then the first order derivative is

$$\begin{aligned}
\frac{\partial F_2(\lambda_i, \lambda_j)}{\partial \lambda_i} =& \frac{2i\alpha}{(1+\lambda_i)^2}\{\alpha \mathbb{1}_{\lambda_i \geq \lambda_j}[\frac{(1+\lambda_i)(A-1)\log m}{(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})m^{\lambda_i - \lambda_j}} - \log(A - \frac{A-1}{m^{\lambda_i - \lambda_j}})] - (1-\alpha)\log A\} \\
=& \frac{2i\alpha}{(1+\lambda_i)^2}\{\frac{\alpha \mathbb{1}_{\lambda_i \geq \lambda_j}}{A - \frac{A-1}{m^{\lambda_i-\lambda_j}}}[\frac{(1+\lambda_i)(A-1)\log m}{m^{\lambda_i-\lambda_j}} - (A - \frac{A-1}{m^{\lambda_i-\lambda_j}})\log(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})] \\
&- (1-\alpha)\log A\}.
\end{aligned} \tag{20}$$

Because $\frac{(1+\lambda_i)(A-1)\log m}{(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})m^{\lambda_i-\lambda_j}}$ is decreasing in λ_i, and $\log(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})$ is increasing in λ_i, conditional on $\lambda_i > \lambda_j$, $F_2(\lambda_i, \lambda_j)$ reaches its maximum when the first order condition is satisfied:

$$0 = \alpha[\frac{(1+\lambda_i)(A-1)\log m}{(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})m^{\lambda_i-\lambda_j}} - \log(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})] - (1-\alpha)\log A, \tag{21}$$

substitute the above equation into $F_2(\lambda_i, \lambda_j)$, one obtains the maximum value of $F_2(\lambda_i, \lambda_j)$:

$$\begin{aligned}
&\frac{2i\alpha}{1+\lambda_i}[\alpha \frac{(A-1)(1+\lambda_i)\log m}{(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})m^{\lambda_i-\lambda_j}} - (1-\alpha)\log A + (1-\alpha)\log A] \\
=& 2i\alpha^2 \frac{(A-1)\log m}{(A - \frac{A-1}{m^{\lambda_i-\lambda_j}})m^{\lambda_i-\lambda_j}} \\
<& 2i\alpha^2(A-1)\log m.
\end{aligned} \tag{22}$$

All we need to do is to show that for sufficiently small α, the above maximum value is dominated by $F_2(1, \lambda_j)$. Consider $\overline{\alpha}_2 \equiv \frac{\log A}{2(A-1)\log m + \log A}$, then $\forall \alpha \leq \overline{\alpha}_2$, one have

$$\begin{aligned}
F_2(1, \lambda_j) - 2i\alpha^2(A-1)\log m &\leq 2i\overline{\alpha}_2[(1-\overline{\alpha}_2)\log A - \overline{\alpha}_2(A-1)\log m] \\
&= 0
\end{aligned} \tag{23}$$

Also notice that conditional on $\lambda_i \leq \lambda_j$, then the optimal $\lambda_i = 1$. So $\forall \alpha \leq \overline{\alpha}_2$, the optimal $\lambda_i = 1$.

Step 4: Agent i's Optimal Choice

Given step $1-3$, when $\forall \alpha \leq \overline{\alpha} \equiv \min\{\overline{\alpha}_1, \overline{\alpha}_2\}$, all entrepreneurs choose $\lambda = 1$. □

A2. Proof of Lemma 2

Proof. Lemma 1 suggests that $\lambda = 1$ for all entrepreneurs. For each agent, in equilibrium his career choice would not affect the equilibrium matching probabilities. Being a successful entrepreneur strictly dominates being an worker because he can also enjoy gains from service auction when he is matched with a worker. It is straight forward to see that being a failed entrepreneur is strictly dominated by being a worker. Then, the expected utility for being an entrepreneur is continuous and strictly monotonically increasing in agent's type i. For agent $i = 1$, he always chooses to be an entrepreneur, and for agent $i = 0$ he finds it optimal to become a worker. Then there exists an unique marginal agent $\theta \in (0, 1)$ such that all agents with $i > \theta$ choose to be an entrepreneur and all agents with $i < \theta$ choose to be a worker. $\square$

A3. Proof of Proposition 1

Proof. Given the marginal agent θ, the equilibrium matching probabilities can be computed as $\pi_w(\theta) = \theta$, $\pi_f(\theta) = (1-\theta)\frac{1-\theta}{2}$, and $\pi_s(\theta) = (1-\theta)\frac{1+\theta}{2}$. For the marginal agent θ, let $E_e(\theta)$ be his expected utility gain from service auction if he chooses to be an entrepreneur, and $E_w(\theta)$ be his expected utility gain from service auction if he chooses to be a worker. The difference in expected utility gain from service auction $\Sigma(\theta)$ can be computed as:

$$\begin{aligned}\Sigma(\theta) \equiv &E_e(\theta) - E_w(\theta)\\ =&P_1(T)\theta\alpha^2\left[\pi_w(\theta)\log(A - \frac{A-1}{m}) + \pi_f(\theta)\log A\right] + \theta\alpha\left((1-\alpha)P_1(T) + P_2(T)\right)\log A -\\ &P_1(T)\alpha^2\pi_f(\theta)\log A - \alpha\left((1-\alpha)P_1(T) + P_2(T)\right)\log A\\ =&\alpha P_1(T)\left[\alpha\theta^2\log(A - \frac{A-1}{m}) + (\theta-1)(1-\alpha+\frac{\alpha(1-\theta)^2}{2})\log A\right] + P_2(T)\alpha(\theta-1)\log A.\end{aligned} \tag{24}$$

The first order derivative can be computed as

$$\Sigma'(\theta) = P_1(T)\alpha[2\theta\alpha\log(A - \frac{A-1}{m}) + ((1-\alpha) + \alpha\frac{3(1-\theta)^2}{2})\log A] + P_2(T)\alpha\log A > 0. \tag{25}$$

$\Sigma(\theta)$ is strictly monotonically increasing.

For agent i, let $M(i)$ be the expected utility difference between entrepreneur and worker.

$$M(i) \equiv i\log m^2 - \log m + \Sigma(i). \tag{26}$$

Then $M'(i) = 2\log m + \Sigma'(i) > 0$, and $M(0) < 0 < M(1)$. Hence there exists an unique $\theta \in (0, 1)$ such that

$$M(\theta) = 0. \tag{27}$$

That is to say, there exists a unique equilibrium.

To show that $\theta > i_e$, notice that:

$$\begin{aligned} M(i_e) &= \Sigma(i_e) \\ &= E_e(\frac{1}{2}) - E_w(\frac{1}{2}) \\ &= \alpha P_1(T)[\frac{\alpha}{4}\log(A - \frac{A-1}{m}) - \frac{8-7\alpha}{16}\log A] - \frac{P_2(T)\alpha}{2}\log A. \end{aligned} \tag{28}$$

□

It is easy to see that $M(i_e) < 0 = M(\theta)$ if $\alpha \leq \frac{8}{11}$.

A4. Proof of Corollary 2

Proof. Because $\frac{P_2(T)}{P_1(T)}$ is monotonic increasing, we have $\frac{P_1'(T)}{P_1(T)} \leq \frac{P_2'(T)}{P_2(T)}$. Let $M(i,T)$ and $\Sigma(i,T)$ be functions $M(i)$ and $\Sigma(i)$ for given T, respectively. From Proposition 1, $\theta > i_e$. Then $\Sigma(\theta, T) < 0$.

If $P_1'(T) \geq 0$ and $P_2(T) = 0$, then envelop theorem implies that

$$\begin{aligned} \frac{\partial \Sigma(\theta(T), T)}{\partial T} &= P_1'(T)\alpha\left[\alpha\theta^2\log(A - \frac{A-1}{m}) + (\theta-1)(1-\alpha+\frac{\alpha(1-\theta)^2}{2})\log A\right] \\ &= \frac{P_1'(T)\Sigma(\theta(T),T)}{P_1(T)} < 0. \end{aligned} \tag{29}$$

If $P_1'(T) \geq 0$ and $P_2(T) > 0$, then envelop theorem implies that

$$\begin{aligned} \frac{\partial \Sigma(\theta(T), T)}{\partial T} &= P_1'(T)\alpha\left[\alpha\theta^2\log(A - \frac{A-1}{m}) + (\theta-1)(1-\alpha+\frac{\alpha(1-\theta)^2}{2})\log A\right] + P_2'(T)\alpha(\theta-1)\log A \\ &< \frac{P_1'(T)\Sigma(\theta(T),T)}{P_1(T)} < 0. \end{aligned} \tag{30}$$

If $P_1'(T) < 0$, then $P_2'(T) \geq -P_1'(T)$, and $P_2(T) \geq 0$.

$$\begin{aligned} \frac{\partial \Sigma(\theta, T)}{\partial T} =& P_1'(T)\alpha\left[\alpha\theta^2\log(A - \frac{A-1}{m}) + (\theta-1)(1-\alpha+\frac{\alpha(1-\theta)^2}{2})\log A\right] + P_2'(T)\alpha(\theta-1)\log A \\ =& P_1'(T)\alpha\left[\alpha\theta^2\log(A - \frac{A-1}{m}) + (\theta-1)(1-\alpha+\frac{\alpha(1-\theta)^2}{2})\log A\right] - P_2'(T)\alpha(1-\theta)\log A \\ <& P_1'(T)\alpha\left[\alpha\theta^2\log(A - \frac{A-1}{m}) + (1-\theta)\alpha(1-\frac{(1-\theta)^2}{2})\log A\right] \\ <& 0. \end{aligned} \tag{31}$$

Then $\frac{\partial \Sigma(\theta,T)}{\partial T} < 0$ for $\forall T \in [0,1]$.

To solve $\theta'(T)$, the implicit function theorem implies that

$$\begin{aligned}\theta'(T) &= -\frac{\frac{\partial M(\theta(T),T)}{\partial T}}{\frac{\partial M(\theta(T),T)}{\partial \theta}} \\ &= -\frac{\frac{\partial \Sigma(\theta(T),T)}{\partial T}}{\log m^2 + \frac{\partial \Sigma(\theta(T),T)}{\partial \theta}}.\end{aligned} \tag{32}$$

From proof of Proposition 1, $\frac{\partial \Sigma(\theta(T),T)}{\partial \theta} > 0$. Thus θ is strictly increasing in T. □

A5. Proof of Lemma 3

Proof. In equilibrium, each agent's career choice would not affect the equilibrium price p. Notice that agent $i = 0$ strictly prefers choosing to be a worker, because the (expected) utility of becoming an entrepreneur with any $\lambda \geq 1$ is zero. On the other hand, agent $i = 1$ always can choose to become an entrepreneur with $\lambda = 1$ and it strictly dominates being a worker.

Now, suppose for some $i < j$ and agent j chooses to be a worker. We have:

$$\max_{\lambda \geq 1} E[U_j(\lambda)] \leq U_0. \tag{33}$$

Note that for agent i, the only difference between it choosing to be an entrepreneur and agent j is the probability of success. In other words, in equilibrium if they both choose to be an entrepreneur, then

$$\max_{\lambda \geq 1} E[U_i(\lambda)] = \frac{i}{j} \cdot \max_{\lambda \geq 1} E[U_j(\lambda)] \leq \frac{i}{j} \cdot U_0 < U_0. \tag{34}$$

Agent i will chooses to be worker as well. Similarly, if agent $i < j$ prefers to be an entrepreneur, agent j strictly prefers choosing to be an entrepreneur. By the results of agent 0 and agent 1, we can conclude that there must be some $\theta \in (0, 1)$ such that any agent $< \theta$ prefers becoming a worker and any agent $> \theta$ prefers becoming an entrepreneur. □

A6. Proof of Lemma 4

Proof. Based on equation (9), entrepreneur chooses his production technology λ to maximize hie expected utility:

$$U_i(\lambda, p) = 2i[\frac{\alpha}{1 + \lambda_i} \log(A - \frac{Ap}{m^{1+\lambda_i}}) + \log m]. \tag{35}$$

It is straight forward to see that the optimal choice λ is independent of i. We now proves that the optimal λ is unique. The first order derivative is:

$$\frac{\partial U_i(\lambda, p)}{\partial \lambda} = \frac{1}{(1 + \lambda)^2} \frac{1}{A - \frac{Ap}{m^{1+\lambda}}} [\frac{A(1 + \lambda)p \log m}{m^{1+\lambda}} - (A - \frac{Ap}{m^{1+\lambda}}) \log(A - \frac{Ap}{m^{1+\lambda}})]. \tag{36}$$

Let $G_3(\lambda, p) \equiv \frac{A(1+\lambda)p \log m}{m^{1+\lambda}} - (A - \frac{Ap}{m^{1+\lambda}}) \log(A - \frac{Ap}{m^{1+\lambda}})$. We also have

$$\frac{\partial G_3(\lambda, p)}{\partial \lambda} = -\frac{Ap \log m}{m^{1+\lambda}}[(1+\lambda)\log m + \log(A - \frac{Ap}{m^{1+\lambda}}) + 1] < 0. \tag{37}$$

Then if $G_3(1, p) \leq 0$, then optimal $\lambda = 1$. If $G_3(1, p) > 0$, then there exists a unique optimal $\lambda > 1$.

To prove the existence of $\underline{p}$, notice that $G_3(1, 0) < 0$, and

$$\frac{\partial G_3(\lambda, p)}{\partial p} = \frac{A}{m^{1+\lambda}}[1 + (1+\lambda)\log m + \log(A - \frac{Ap}{m^{1+\lambda}})] > 0. \tag{38}$$

□

A7. Proof of Proposition 3

Proof. To show the existence and uniqueness, one only need to show the monotonicity in $P(T)$. **Part 1:**

Monotonicity of $p(T)$

Let $D(p)$ be the demand of service given price p. We only need to prove the monotonicity of $D(p)$.

Case 1: $p > \underline{p}$

When $p > \underline{p}$, $\lambda > 1$ and the first order condition is binding, then the implicit function theorem implies:

$$\lambda'(p) = -\frac{\frac{\partial G_3(\lambda,p)}{\partial p}}{\frac{\partial G_3(\lambda,p)}{\partial \lambda}} > 0, \tag{39}$$

where the inequality comes from results in the proof of Lemma 4 that $\frac{\partial G_3(\lambda,p)}{\partial p} > 0$ and $\frac{\partial G_3(\lambda,p)}{\partial \lambda} < 0$. Then with a high price, entrepreneurs are taking more risk and are less likely to be successful.

If $\underline{p} > \frac{A-1}{A}m$, then only successful entrepreneurs consume the service. The difference in expected utility gain from service market is $\Sigma(i) = \frac{2i}{1+\lambda_i}\log(A - \frac{Ap}{m^{1+\lambda}})$. It is straightforward to see that $\Sigma(i)$ is decreasing in p. Thus the threshold θ is increasing in p. Because agents are less likely to become an entrepreneur, and are less likely to succeed conditional on being an entrepreneur, $D(p)$ is strictly decreasing in p.

If $\underline{p} \leq \frac{A-1}{A}m$, then both successful entrepreneurs and workers consume the service. The difference in expected utility gain from service market is $\Sigma(i, \lambda, p) = \frac{2i}{1+\lambda_i}\log(A - \frac{Ap}{m^{1+\lambda}}) - \log(Am - AP)$. Then

$$\begin{aligned}
\frac{D\Sigma(i,\lambda,p)}{Dp} &= \frac{\partial\Sigma(i,\lambda,p)}{\partial\lambda}\lambda'(p) + \frac{\partial\Sigma(i,\lambda,p)}{\partial p} \\
&= \frac{\partial\Sigma(i,\lambda,p)}{\partial p} \\
&= -\frac{2i}{1+\lambda}\frac{A}{Am^{1+\lambda} - Ap} + \frac{A}{Am - Ap} \\
&> 0
\end{aligned} \tag{40}$$

So the threshold θ is decreasing in p. Because workers will consume service for sure, while only successful entrepreneurs consume service, a decreasing θ suggests fewer workers. Also, entrepreneurs are taking higher risk and less likely to be successful. Combing these two one obtains $D(p)$ is strictly decreasing in p.

Case 2: $p \leq \underline{p}$

When $p < \underline{p}$, $\lambda = 1$. If $\underline{p} > \frac{A-1}{A}m$, then when $p \in (\frac{A-1}{A}m, \underline{p}]$, only successful entrepreneurs consume service goods. Given fixed $\lambda = 1$, it is straight froward to see that their gains from service market consumption is decreasing in p, suggests θ is increasing in p, suggesting a decreasing demand $D(p)$.

If $\underline{p} \leq \frac{A-1}{A}m$, then both successful entrepreneurs and workers consume service goods. Given the fixed $\lambda = 1$, then

$$\begin{aligned} \frac{D\Sigma(i,\lambda,p)}{Dp} &= -i\frac{A}{Am^2 - Ap} + \frac{A}{Am - Ap} \\ &> 0 \end{aligned} \tag{41}$$

So the threshold θ is decreasing in p. Because workers will consume service for sure, while only successful entrepreneurs consume service, a decreasing θ suggests fewer workers. Combing these two one obtains $D(p)$ is strictly decreasing in p.

Part 2: θ

The proof above shows that $\theta(p)$ is strictly decreasing when $p \leq \frac{A-1}{A}m$, and increasing when $p > \frac{A-1}{A}m$. Also notice that when $p = \frac{A-1}{A}m$, the service market utility for workers is 0, implies $\Sigma(i, \lambda, \frac{A-1}{A}m) > 0$, and $\theta < i_e$. Also notice that when $p \downarrow 0$, $\frac{1}{2}\log(A - \frac{Ap}{m^2}) < \log(A - \frac{Ap}{m})$. Then $\Sigma(i, \lambda, 0) < 0$, and $\theta > i_e$. Also for $\forall p \geq \frac{A-1}{A}m$, only successful entrepreneur can enjoy service, so $\Sigma(i, \lambda, p) > 0$, and $\theta < i_e$. Then there exists a price $p^{i_e} \in (0, \frac{A-1}{A}m)$ such that $\theta < i_e$ if and only if $p > p^{i_e}$.

To solve p_i, notice that at p^{i_e}, $\Sigma(i, \lambda, p^{i_e}) = 0$, then

$$\frac{1}{2}\log(A - \frac{Ap^{i_e}}{m^2}) = \log(A - \frac{Ap^{i_e}}{m}). \tag{42}$$

One obtains $p^{i_e} = m - \frac{\sqrt{4A(m-1)m+1}-1}{2A}$. □

A8. Proof of Corollary 4

Proof. It is straightforward from the proof of Proposition 3. □