Speculative Financial Innovation *

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Abstract

We analyze how speculative financial innovation affects stock prices, risk premiums, market liquidity, and investor welfare in an economy with heterogeneous beliefs. When investors disagree about the covariance of the newly introduced stocks with the original stocks, we show that financial innovation reduces the variance-covariance matrix of the representative investor, which decreases the market portfolio's risk premium. When investors disagree on the expected payoff of the new stocks, the representative investor's expected payoff of the existing stocks can also change due to hedging demands among investors, which causes the stock prices to change. Financial innovation further causes the market liquidity to increase as prices will be less sensitive to supply shocks due to the reduced variance-covariance matrix of the representative investor. Finally, we show that financial innovation could make all investors better off under their own beliefs but worse off under the true beliefs.

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1 Introduction

Financial innovations have increased significantly over the past century. New securities such as futures, options, zero coupon bonds, collateralized mortgage obligations (CMO), collateralized debt obligations (CDO), credit derivatives, cryptocurrencies, and crypto derivatives have been continuously introduced to financial markets (Frame and White (2004), Tufano (2003)). A large literature has analyzed various functions financial innovation performs (Allen and Gale (1994)). These include completing the market, addressing agency concerns and information asymmetry, reducing transaction cost, and responding to taxes and regulations (Miller (1986), Harris and Raviv (1989), Merton (1989), Duffie and Rahi (1995)).

An important but often overlooked factor for financial innovation is that it can facilitate speculative trading on new securities due to heterogeneous beliefs.[1] The 2007–2009 financial crisis due to the subprime mortgage market failure has raised concerns about speculations due to financial innovation. A new debate on how financial innovation affects the prices of stocks and the associated market volatility is now in progress. The popular view has called for more regulation of financial innovation. However, Bernanke (2007) has cautioned against regulations on financial innovation without a consistent approach and objectives. A better understanding of the effects of speculative financial innovation is needed before the regulators rush into action.

While heterogeneous beliefs are widespread in financial markets in general, they could be more important in new securities for two reasons. First, exchanges are more likely to introduce securities on which there are more differences of opinion in order to make more commissions from higher trading volume. Second, investors need time to be familiar with

[1] The recent development of smart contracts on blockchain may further increase speculative trading on a large set of products. Specifically, smart contracts allow traders to write a transaction protocol that is intended to automatically execute. As a result, we argue that the cost of writing a speculative/betting contract is significantly lower with blockchain technology.

and learn about the newly introduced securities which can generate heterogeneous beliefs in the process.[2] When investors disagree about how to value the newly added assets and how these new assets relate to existing assets, they will trade in the newly introduced assets to speculate. In addition, trading in the new assets also affects investors' holdings in the existing assets, which in turn affects the prices of the existing stocks.

In this paper, we analyze how financial innovation driven by speculation affects the demands and prices of existing securities, market liquidity, and investor welfare. To make the analysis tractable, we assume that investors have CARA utility and that the stock payoffs are normally distributed. Investors have homogeneous beliefs about existing stocks. In the absence of financial innovation, the capital asset pricing model (CAPM) holds and stock returns are related to their market risk. However, with financial innovation, they disagree about the expected value of securities to be introduced and their correlation with the existing securities. With these assumptions, we are able to construct a representative agent, whose belief determines the stock prices. We consider various disagreements among investors about the new stocks to be introduced and analyze the effects of financial innovation.

First, when investors agree about the covariance of the new stocks with the existing stocks, financial innovations have no effect on the prices of the existing securities. In this case, there exist a set of portfolios composed of the new stocks and existing stocks such that the economy with the existing stocks and the synthetic portfolio will have the same payoff span as the economy with the original financial innovation. Moreover, the payoffs of the synthetic portfolio are orthogonal to the existing stocks as the risk exposure to the original stocks is hedged away. Because of the absence of wealth effect due to CARA, it follows that the prices of the original stocks are not affected by financial innovation. As a

[2] Many companies that lose money in derivatives often argue that they do not understand the derivatives sold to them by investment banks. (See, for instance, Miller (1986).)

special case, when the covariances of the newly added stocks and the original stocks are zero, financial innovations have no pricing effects on the existing securities.

Second, when investors disagree about the covariance of the new stocks with the original stocks but agree about the expectation of the new stocks, financial innovations decrease the risk premium of the market portfolio. We note that there are no short-selling constraints in our model, yet covariance disagreement leads to inflated prices. The intuition is as follows. The representative investor's expectations of the original stocks are not affected. However, the variance-covariance matrix of the representative investor will be reduced.[3] Intuitively, when investors disagree about the covariance of the new stocks and the original stocks, each investor believes that he can reduce the risk of his portfolio better than the others as he has the correct variance-covariance matrix. It follows that the representative investor's variance-covariance matrix will be smaller. The reduction of the representative investor's perception of risk causes the price of the market portfolio to increase. The effects on individual stock prices are ambiguous although the increase in the price of the market portfolio means that the value-weighted average prices across stocks will increase. Our results about the reduction in risk premium are consistent with the findings of Conrad (1989) and Detemple and Jorion (1990) that financial innovations increase the price of the underlying stock and the market portfolio.

Third, when investors disagree about both the mean of the new stock payoffs and their covariance with the existing stocks, we identify two channels through which financial innovation affects prices: the *expectation* channel and the *risk* channel. The expectation channel refers to the mean while the risk refers to the variance-covariance matrix of the representative investor about the existing stocks. The representative investor's variance-covariance matrix will always reduce but his expectation of stock payoffs can either increase or de-

[3]Notice the ranking of the positive definite matrix is not complete. For two positive definite matrices M and N, M is larger than N when $M - N$ is positive definite.

crease. Consequently, the prices of the original stocks can either increase or decrease with financial innovation.

Since the variance-covariance matrix of the representative investor is smaller, the price for the market portfolio will be less sensitive to supply shocks. As a result, the market becomes more liquid in the presence of financial innovation. Intuitively, investors will perceive less risk as they believe that they can shift the risk to others by trading in the new stocks. As a result, the perceived risk is smaller and the market is more liquid. Our prediction is consistent with Damodaran and Lim (1991), and Fedenia and Grammatikos (1992), who found market liquidity increases after options listing.

As investors agree on the distribution of the original stocks, they would achieve the Pareto optimal allocation without financial innovation. Consequently, in the current model, financial innovation cannot make a Pareto improvement under the true probability measure. Interestingly, we show that it is possible that all investors are worse off in the presence of financial innovation under the true probability measure while each investor believes that he is better off in the presence of financial innovation given his own beliefs.

The market portfolio holds a special position in this economy as it is the portfolio that all investors would hold in the absence of financial innovation. Financial innovation affects investors' holdings such that investors will not hold the market portfolio in general. However, when investors agree on the markets of the newly introduced stocks, the price of the market portfolio and the liquidity of the market portfolio are not affected. When investors agree on the markets of all stocks, each stock can be decomposed into the market component and the idiosyncratic component. Disagreements about the idiosyncratic component will not affect investors' holdings in the market portfolio and thus the price of the market portfolio is not affected.

While we assume in most parts of the paper that investors share the same belief on the original stocks, we show that our results extend to the case with heterogeneous beliefs on the original stocks as well. Financial innovation always reduces the representative investor's variance-covariance matrix about the original stocks. The effects on the prices of stocks are mixed but the risk premium on the market portfolio always reduces. As shown earlier, financial innovation always increases market liquidity. Finally, there will be no pricing effects when investors share the same belief on β's (regression coefficients of the new stock on the original market portfolio).

Related Literature. A sizable literature has analyzed various rationales for financial innovation. Duffie and Rahi (1995) survey the theoretical literature on market incompleteness and financial innovation. A key message is that investors can share risk better and are made better off in a complete market.[4] Our paper studies a different channel by arguing that financial innovation can facilitate speculative trading due to heterogeneous beliefs. This viewpoint is gaining momentum, particularly in the wake of the 2007-2009 financial crisis. First, Simsek (2013a,b) show that financial innovations can increase portfolio risks by generating new bets and amplifying traders' existing bets.[5] His papers focus on first-moment disagreement while in our paper, the reduction of the representative investor's variance-covariance matrix is driven by second-moment disagreement. Second, another strand of literature (e.g., Fostel and Geanakoplos (2016); Ellis, Piccione, and Zhang (2022); Shen, Yan, and Zhang (2014)) shows that speculative financial innovation can endogenously arise to help with resolving frictions when agents have diverse beliefs. Our paper complements these papers in the sense that we focus on asset prices and market liquidity under second-moment disagreement. Our key insight is that investors demand a lower risk premium on the market portfolio after the introduction of speculative financial innovations. This point relates us to Iachan, Nenov, and Simsek (2021), who shows that financial innovation can re-

[4]Recent contributions include Cong and Xu (2016); Huang, O'Hara, and Zhong (2021) among others.

[5]Similarly, Brock, Hommes, and Wagener (2009) show that introducing new assets can destabilize the financial market because of the hedge-more/bet-more channel.

duce the risk premium through greater participation. In our model, there is no participation decision, and our results are purely driven by disagreement.

Our paper contributes to the large literature that studies disagreement in financial markets. Most papers in this field (e.g., Miller (1977) among many others) study the implications of first-moment disagreement with short-selling frictions. There are two sub-strands of literature we contribute to. First, our result on inflated prices is related to several recent contributions that relax the short-selling constraints in models with heterogeneous beliefs. Disagreement can be modeled as disagreement over parameters in a parametrized distribution or disagreement over moments in a non-parametric setup. On the parametric side, Martin and Papadimitriou (2022) offer a tractable dynamic asset pricing model where uncertainty evolves on a binomial tree. Disagreement matters in their model because of risk aversion and binary distribution.[6] On the non-parametric side, Goulding, Santosh, and Zhang (2022) observe that skewness can make investors' demand schedule nonlinear, and as a result, trades due to disagreement may not cancel out even without short-selling constraints. To the best of our knowledge, our result on covariance disagreement leading to inflated prices is new to the literature.

Second, our paper highlights the interesting asset pricing implications of second-moment disagreement. While existing research argues that the variance-covariance matrix can be accurately estimated in a continuous time setting with Brownian motion using quadratic variation, we contend that this method is inadequate due to jumps coming from, for example, the daily closure of financial markets. Introducing jumps to the continuous time setting poses challenges for estimating second moments. In fact, the high trading volume on VIX indicates that there is a lack of consensus among investors regarding the second moment. Finally, we argue that second-moment disagreement can arise in situations where there is a dearth of data, which is particularly pertinent when a new asset is first introduced. Re-

[6] Earlier work that makes a similar point includes Harris and Raviv (1993) among others.

cently, several papers (Huang, Lunawat, and Wang (2021) and Banerjee, Davis, and Gondhi (2022) among others) study the financial market in a setting where investors disagree about the precision of exogenous signals, in which case there is second-moment disagreement. By contrast, we analyze a general structure featuring both the first and second moments of disagreement.

Empirically, Black (1986) shows a relationship between a new contract's viability and its ability to complete markets. Grinblatt and Longstaff (2000) find that new Treasury STRIPS are created primarily to make markets more complete as STRIPS are created when it is most difficult to synthesize the discount bonds from existing coupon instruments. Harris and Raviv (1989) and Allen and Gale (1994) review the literature on how contracts can be written to better align the interests of different parties. Conflict of interests and asymmetric information between different parties lead the firm to issue a multiplicity of securities. Lerner and Tufano (1993), and Beatty, Chamberlain, and Magliolo (1995) have used this motive to explain the embedded options in some innovative R&D financing. Merton (1989) discusses how transaction costs provide a critical role in financial innovation. Mada and Soubra (1991) examine how financial intermediaries design multiple products to maximize their revenues net of marketing costs. Finally, Miller (1986) argues that the major impulses to successful innovation are regulation and taxes.

Zapatero (1998) analyzes the effects of financial innovation on the interest rate in a continuous time model with intertemporal consumption. He considers the case of a single stock and there are disagreements only about the mean. The stock price does not change but the interest rate is affected by financial innovation. In a multiple stock model, Calvet, Gonzalez-Eiras, and Sodini (2004) propose that the introduction of non-redundant assets can endogenously modify trader participation in financial markets, which can lead to a lower market premium. While Calvet, Gonzalez-Eiras, and Sodini (2004) also demonstrate a lower market risk premium, the underlying mechanism is very different from our model.

In their economy, financial innovation increases the benefits of market participation and causes more investors to hold stocks. Risk is shared among more investors and thus market premium reduces. In our model, the risk premium drops because the representative investor's variance-covariance matrix is smaller.

Road Map. The rest of this paper is organized as follows. Section 2 first describes the basic model without financial innovation, then we introduce financial innovation. Section 3 analyzes financial innovation's effect on the prices of stocks, market liquidity, and investor welfare. Section 4 allows investors to have different priors about existing stocks and nontradable assets. Section 5 concludes the paper. The appendix contains technical proofs.

2 Model

2.1 Benchmark Model

We consider an endowment economy with two periods, $t \in \{0, 1\}$. There is one risk-free asset and N risky stocks available for trading. It is assumed that the financial market is populated by investors with the population size normalized to one, each indexed by i where $i \in [0, 1]$. At time 0, we assume that each investor is endowed with X_V units of the stock and zero units of the bond. The per capita supply of the stock is also X_V. We assume that X_V is a positive vector. Without loss of generality, the net interest rate is taken to be zero. The stock payoff at time 1 is V. The demand for the stocks for investor i is denoted by $N \times 1$ vector D_{Vi}.

To obtain closed form solutions, we assume that each investor i in the market has a negative exponential utility function, $-\exp(-\gamma W_i)$, where γ is his risk aversion coefficient and W_i is his terminal wealth or consumption. We assume that V is normally distributed and that all investors believe that the unconditional mean of V is μ_V, the unconditional variance-covariance matrix of V is Σ_V, and the precision matrix of V is $\Pi_V = \Sigma_V^{-1}$.

The following well-known lemma finds the equilibrium demands and prices for the stock.

Lemma 1. *There exists a unique equilibrium in which the demands and prices are:*

$$P_V = \mu_V - \gamma \Sigma_V X_V, \quad (1)$$

$$D_{Vi} = \frac{1}{\gamma} \Pi_V (\mu_V - P_V). \quad (2)$$

In the equilibrium, the stock prices equal the investor's expectation minus the risk premium, which is proportional to the product of risk aversion, variance-covariance matrix, and the supply. It should be noted that the Capital Asset Pricing Model holds in this economy and we have the familiar security market line:

$$E[r_i] - r_f = \beta_i (E[r_M] - r_f) \quad (3)$$

where r_i is the return of stock i and r_M is the return of the market portfolio, $r_f = 0$ by assumption, and β_i is the market risk of stock i.[7]

2.2 Illustrative Example

In this subsection, we present a simple example to understand the effect of financial innovation on the original asset. The key result is that introducing a zero-supply new asset weakly reduces the representative investor's variance-covariance matrix. As a result, disagreement about covariance between the old asset and the new asset leads to an inflated price of the existing asset, in a setting without any frictions like short-selling constraints.

For ease of exposition, let $N = 1$ so there is only one risky asset in Section 2.1. The risky asset pays a random liquidating dividend of V. With some abuse of notation, V is a scalar and no longer a vector in this section. Now consider introducing a zero-supply new asset (e.g., financial innovation), whose liquidating payoff is denoted by F. We study

[7] We remark that the return can be either the percent return or the dollar return.

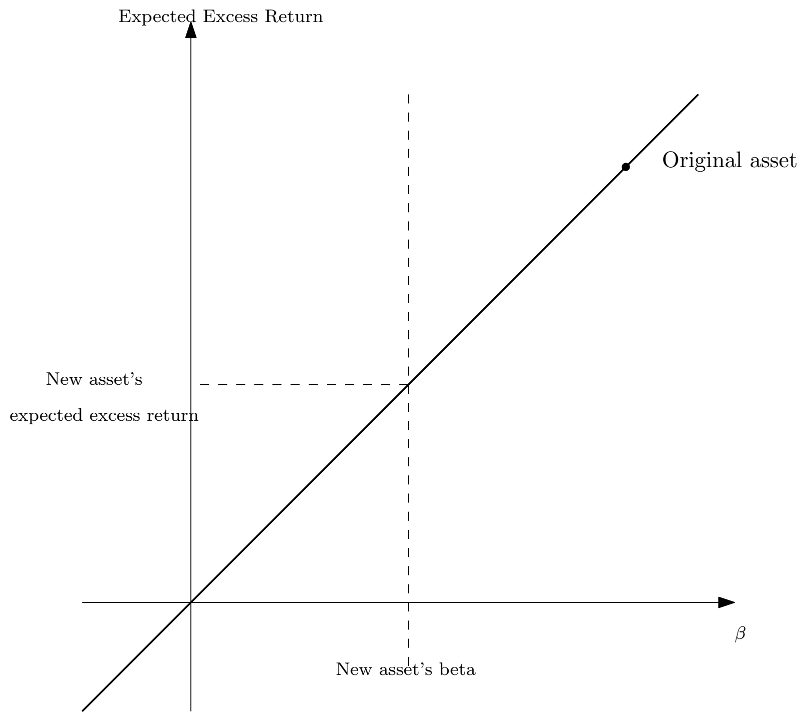

FIGURE I
This figure illustrates that the price of the original asset is unchanged if investors agree about the joint distribution of (V, F).

the effect of this new asset on the original asset under two scenarios. First, suppose all investors agree about the joint distribution of (V, F), which is assumed to be multivariate Gaussian. The capital asset market pricing model (CAPM) obtains (c.f. Huang and Litzenberger (1988)). Figure I illustrates the result. Since the new asset has zero supply, the aggregate/market portfolio is formed by holding the original asset. Since the representative investor does not change her portfolio allocation, the price of the original asset is unaffected after introducing the new asset. From the joint distribution of (V, F), we can compute the new asset's beta, and from the security market line (SML), we can find the new asset's expected excess return, equivalently its price.

Second, suppose half of the investors (Group 1) believes that the new asset is independent of the original asset while the other half (Group 2) believe that the new asset has a positive covariance with the original asset. We maintain the assumption that all investors agree about the expected payoff (i.e., the first moment) of the new asset. For a concrete example, suppose Group 1's belief is

$$\begin{bmatrix} V \\ F \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_V \\ \mu_F \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right),$$

and Group 2's belief is

$$\begin{bmatrix} V \\ F \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_V \\ \mu_F \end{bmatrix}, \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}\right).$$

We write $(D_{V,j}, D_{F,j})$ as Group j investors' demand in the original asset and the new asset and (p_V, p_F) as the prices. The CARA and normality setup implies that demand schedules are linear in prices:

$$\gamma \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} D_{V,1} \\ D_{F,1} \end{bmatrix} = \begin{bmatrix} \mu_V - p_V \\ \mu_F - p_F \end{bmatrix},$$
$$\gamma \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} D_{V,2} \\ D_{F,2} \end{bmatrix} = \begin{bmatrix} \mu_V - p_V \\ \mu_F - p_F \end{bmatrix}.$$

Inverting investors' variance-covariance matrix, their demand schedules are given by

$$\begin{bmatrix} D_{V,1} \\ D_{F,1} \end{bmatrix} = \gamma^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_V - p_V \\ \mu_F - p_F \end{bmatrix},$$
$$\begin{bmatrix} D_{V,2} \\ D_{F,2} \end{bmatrix} = \gamma^{-1} \frac{4}{3} \begin{bmatrix} 1 & -1/2 \\ -1/2 & 1 \end{bmatrix} \begin{bmatrix} \mu_V - p_V \\ \mu_F - p_F \end{bmatrix}.$$

Recall market's clearing requires that

$$\frac{1}{2} \begin{bmatrix} D_{V,1} \\ D_{F,1} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} D_{V,2} \\ D_{F,2} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Aggregating the demand schedules,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \gamma^{-1} \begin{bmatrix} 7/6 & -1/3 \\ -1/3 & 7/6 \end{bmatrix} \begin{bmatrix} \mu_V - p_V \\ \mu_F - p_F \end{bmatrix}.$$

We can find the risk premiums of the original asset and the new asset:

$$\begin{bmatrix} \mu_V - p_V \\ \mu_F - p_F \end{bmatrix} = \gamma \begin{bmatrix} 7/6 & -1/3 \\ -1/3 & 7/6 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \gamma \begin{bmatrix} 14/15 & 4/15 \\ 4/15 & 14/15 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

So the original asset's risk premium is $\mu_V - p_V = \gamma \frac{14}{15} < \gamma$. Recall the risk premium had the new asset been absent is given by $\gamma * 1 = \gamma$. In other words, introducing the new asset decreases the risk premium of the original asset, equivalently, the price of the original asset goes up. Furthermore, what this example shows is that there exists a representative investor whose variance-covariance matrix is given by $\begin{bmatrix} 14/15 & 4/15 \\ 4/15 & 14/15 \end{bmatrix}$ and this investor's Euler's equations determine the prices. An interesting observation is that even though all investors agree that the original asset's variance is 1, the representative investor believes the variance is $14/15$, which is strictly smaller than 1. We will show that this observation holds in the general model.

We provide a graphical illustration using the beta-pricing idea. From the equilibrium allocation in the benchmark model, Group 1 investors and Group 2 investors disagree about the new asset's β but agree about its expected excess return. Since the new asset is in zero supply, the new asset's expected excess return has to be positive, because otherwise, all investors would short the new asset, violating the zero-supply condition. As Figure II shows, Group 1 investors long the new asset because they believe it earns a higher risk-adjusted return than the original asset. As a result, Group 1 investors would tilt their portfolio toward the new asset by reducing their holdings in the original asset. Recall Group 1 investors believe the two assets are independent, so their portfolio allocation problem is separable. Since they hold fewer shares in the original assets, they demand a lower risk premium on the original assets, consistent with our numerical calculations.

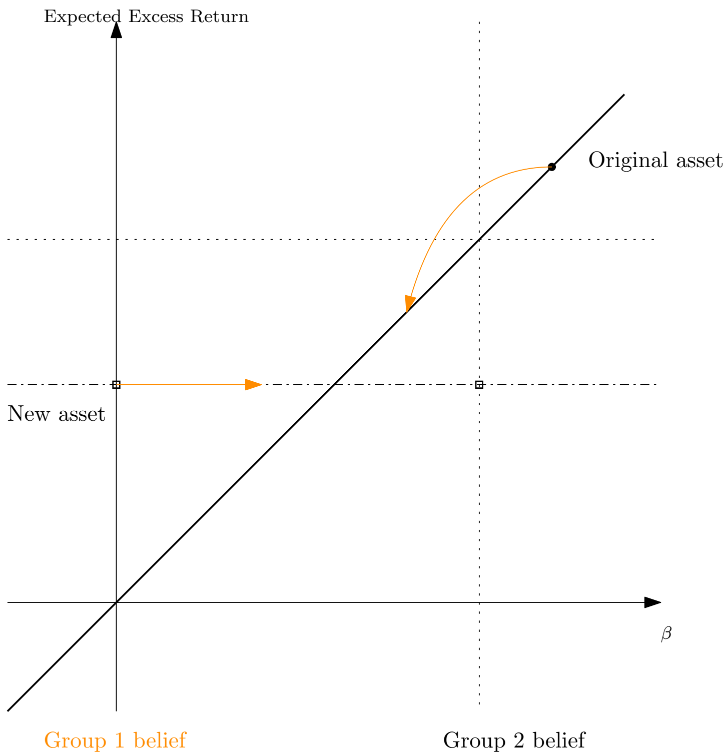

FIGURE II
This figure illustrates that covariance disagreement reduces the risk premium of the original asset.

2.3 Model with Financial Innovation

Having shown the example with one new asset, in this section, we formally introduce the model with financial innovation. We add M new stocks in zero net supply to the economy. The payoffs of the newly introduced stocks are denoted by a random vector F. Investors disagree about the distribution of F and its covariance with the original stocks in the economy V. Let Z denote the joint random vector $\begin{pmatrix} V \\ F \end{pmatrix}$, which is assumed to follow a multivariate Gaussian distribution. Let X_Z denote the net supply of Z in the market. Let μ_{Fi} denote investor i's expectation of F. Let μ_{Zi} denote investor i's expectation of Z, Σ_{Zi} his variance-covariance matrix of Z, and Π_{Zi} his precision matrix. Investor i's demand for Z is denoted by a $(N + M) \times 1$ vector D_{Zi}.

We remark that the financial market is not complete, so the classical proof in Wilson (1968) and Brennan and Cao (1996) that show the existence of a representative investor does not apply. However, as illustrated in Section 2.2, demand schedules are linear, therefore we can aggregate the demands and find a representative investor. The following proposition summarizes the results.

Proposition 1. *In the equilibrium with financial innovation, there exists a representative investor, whose precision matrix is given by the average precision matrix across investors and whose expectation of Z is the precision matrix weighted average across investors.*

$$\Pi_{Zr} = \int_i \Pi_{Zi}\, di, \tag{4}$$

$$\mu_{Zr} = \Pi_{Zr}^{-1} \int_i \Pi_{Zi}\mu_{Zi}\, di. \tag{5}$$

Interestingly, even though all investors agree on the expectation and variance-covariance matrix of the payoffs of the original stocks, V, the representative investor generically has a different expectation and variance-covariance matrix from all investors regarding V. We have the following results with respect to the effects of financial innovation on the expectation and variance-covariance matrix of the representative investor.

Proposition 2. *Financial innovation changes the expectations and reduces the variance-covariance matrix of the representative investor in the sense that $\Sigma_V - \Sigma_{Vr}$ is positive semidefinite.*

We make a remark on the reduction of the variance-covariance matrix of the representative investor. This result comes from the matrix version of harmonic-mean-arithmetic-mean (HM-AM) inequality. Recall that the traditional HM-AM states that for any positive numbers a and b, it follows that $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \frac{a+b}{2}$. The applied mathematics literature (See Ando (1983); Mond and Pečărić (1996) among others) has shown that there is an analogous matrix version of the inequality. That is

$$\left(\frac{A^{-1}+B^{-1}}{2}\right)^{-1} \leq \frac{A+B}{2}, \text{ for positve definite } A, B,$$

where the operator $\leq$ means that the difference between RHS and LHS is a positive semidefinite matrix. In our context, applying the above matrix inequality, it follows that

$$\underbrace{\left(\int_i \Sigma_{Zi}^{-1}\, di\right)^{-1}}_{\substack{\text{inverse of representative} \\ \text{investor's precision matrix}}} \leq \int_i \Sigma_{Zi}\, di.$$

We note that the LHS is the inverse of the representative investor's precision matrix. Looking at the upper-left $N \times N$ sub-matrix, we obtain $\Sigma_{Vr} \leq \Sigma_V$. In words, financial innovation reduces the variance-covariance matrix of the representative investor.

With the construction of the representative investor, the prices of the securities are determined by the representative investor's beliefs, yielding the following equilibrium demand and prices.

Theorem 1. *There exists a unique equilibrium in which the demands and prices are:*

$$P_Z = \mu_{Zr} - \gamma \Sigma_{Zr} X_Z, \tag{6}$$

$$D_{Zi} = \frac{1}{\gamma} \Pi_{Zi}(\mu_{Zi} - P_Z). \tag{7}$$

The prices have similar expressions to those in the economy without financial innovation, except that the expectation and the variance-covariance matrix are those of the representative investor's about Z.

3 Effects of Speculative Financial Innovation

Given the equilibrium prices and demands in Theorem 1, we discuss the effects of financial innovation on security prices, market liquidity, and investor welfare below.

3.1 Stock Price

In the presence of financial innovation, the prices of the original stocks depend on the representative investor's expectation and variance-covariance matrix. We can now compare the prices of the original stocks V in this economy with financial innovation to those in the economy without financial innovation. Notice that the supply of the newly introduced stocks is zero.

Let $\hat{P}_V$ denote the prices of the original stocks in the presence of financial innovation, we have

$$\hat{P}_V = \mu_{Vr} - \gamma \Sigma_{Vr} X_V. \tag{8}$$

Comparing (8) with (1), the effects of financial innovation on the prices of stocks can be decomposed into two parts. The first part is through the effect on the expectation of the representative investor. The second part is through the effect on the variance-covariance matrix of the representative investor. With financial innovation, the Capital Asset Pricing Model breaks down for individual investors but holds for the representative investor. We have

$$E_r[r_{ri}] - r_f = \beta_{ri}(E_r[r_{rM}] - r_f) \tag{9}$$

where r_{ri}, r_{rM} are returns of stock i and the market portfolio and β_{ri} is the market risk with respect to the representative investor's belief. Since the representative investor's belief with respect to the original stocks will in general be different from that of individual investors, the traditional CAPM no longer holds in the presence of financial innovation.

We remark that the fact that CAPM breaks down from an individual investor's perspective is surprising in light of the classical references (e.g., Lintner (1969); Rubinstein (1974) among others). As nicely summarized in Fama and French (2007), the existing work suggests if the relevant weighted averages of investor assessments are equal to the true expected values and the true covariance matrix, asset prices are set as if there is complete agreement and CAPM follows. Our paper shows that even if all investors completely agree on the original assets, the traditional CAPM breaks down.

We consider three cases of disagreement below and analyze the effects of financial innovation on the prices of stocks in detail.

3.1.1 Case I: Covariance Disagreement

Investors agree on the expected payoff of the new stocks F but disagree about the covariances of F and V. In this case, all investors agree on the expectation of Z and the representative investor's expectation of V is simply $\mu_{Vr} = \mu_V$. This assumption can be justified when econometricians are averaging across events in different periods and investors' expectation averaged through different events converges. The effect of financial innovation exerts only through its effect on the variance-covariance matrix of the representative investor. As we have shown earlier that financial innovation always reduces the variance-covariance matrix of the representative investor, i.e. $\Sigma_V - \Sigma_{Vr}$ is positive definite. The reduction of the variance-covariance matrix implies that the price of the market portfolio decreases. We have

$$X_V' \hat{P}_V = X_V' \mu_{Vr} - \gamma X_V' \Sigma_{Vr} X_V > X_V' \mu_V - \gamma X_V' \Sigma_V X_V = X_V' P_V.$$

The effects on the risk premium for individual stocks are ambiguous as a positive definite matrix does not have to be positive in all entries. However, the fact that the risk premium on the market portfolio has reduced means that the overall risk of the market has reduced. When investors disagree about variance-covariance matrix, each investor believes that he can shift the risk to other investors and thus the perceived risk is smaller on average, which causes the market risk premium to drop.

Consider the following numerical example with two types of investors type I and type II of equal proportions. There is one existing stock and one new stock due to financial innovation. $\mu_{Vi} = 1$, $\mu_{Fi} = 0$ for $i =$ I, II. The supply of the original stock V is 0.2 and the risk aversion coefficient is 2. The variance-covariance matrices are:

$$\Sigma_{ZI} = \begin{bmatrix} 1 & -1/2 \\ -1/2 & 1 \end{bmatrix}, \qquad \Sigma_{ZII} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}.$$

Accordingly, the precision matrices of investors are

$$\Pi_{ZI} = \begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 4/3 \end{bmatrix}, \qquad \Pi_{ZII} = \begin{bmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{bmatrix}.$$

Consequently, the precision matrix and variance-covariance matrix of the representative investor is:

$$\Pi_{Zr} = \begin{bmatrix} 4/3 & 0 \\ 0 & 4/3 \end{bmatrix}, \qquad \Sigma_{Zr} = \begin{bmatrix} 3/4 & 0 \\ 0 & 3/4 \end{bmatrix}.$$

The mean of the representative investor is the same as that of individual investors. However, the variance of the representative investor for the stock payoff is smaller. The price without financial innovation is P_V = 3/5 and that with financial innovation is $\hat{P}_V$ = 7/10, or 17% higher. Group I will buy the new stock to hedge risk as they perceive a negative correlation between the stocks while group II will sell to hedge risk as they perceive a positive correlation. From this example, it is clear that both groups perceive risk reduction through hedging using the new financial instrument. Therefore, the representative investor's variance about the original stock diminishes.

3.1.2 Case II: First-Moment Disagreement

Investors disagree about the expected payoffs of the new stocks but agree about the covariance of the new stocks with the original stocks. In this case, the representative investor's expected payoff and variance-covariance matrix of V is the same as the economy without financial innovation. Thus there is no price effect on the original stocks. When investors agree on the covariance of F and V, we can rewrite

$$F = BV + \hat{F},$$

where B is the regression coefficient of F on V that all investors agree on, as they agree on the covariance of F and V. Consequently, introducing F to the economy is equivalent to introducing $\hat{F}$ to the economy. As $\hat{F}$ is independent of V for all investors and investors have CARA utility, the ability to trade on $\hat{F}$ has no effect on the trading in the original stock and thus there are no pricing effects due to financial innovations.

We consider the following example with two types of investors type I and type II of equal proportions, $\mu_{Vi} = \mu_V = 1$, for $i = I, II$ and $\mu_{FI} = 0$, $\mu_{FII} = 1$. The supply of the stock is 0.2 and the risk aversion coefficient is 2. The variance-covariance matrices of Z are:

$$\Sigma_{ZI} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}, \qquad \Sigma_{ZII} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 3/4 \end{bmatrix},$$
$$\Pi_{ZI} = \begin{bmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{bmatrix}, \qquad \Pi_{ZII} = \begin{bmatrix} 3/2 & -1 \\ -1 & 2 \end{bmatrix}.$$

The precision matrix and variance-covariance matrix of the representative investor in the dynamic economy are shown below:

$$\Pi_{Zr} = \begin{bmatrix} 17/12 & -5/6 \\ -5/6 & 5/3 \end{bmatrix}, \qquad \Sigma_{Zr} = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 17/20 \end{bmatrix}.$$

The mean of the representative investor is given by

$$\mu_{Zr} = \Sigma_{Zr} \left(\frac{1}{2} \Pi_{ZI} \mu_{ZI} + \frac{1}{2} \Pi_{ZII} \mu_{ZII} \right) = \begin{bmatrix} 1 \\ 3/5 \end{bmatrix}.$$

The representative investor has the same variance, the expectation of the original stock, and the covariance of the two stocks as those of individual investors. Individual investors and the representative investor differ only on the expectation and variance of the new stock.

As a result, the prices of the original securities remain the same. In this example, we have $P_V = \hat{P}_V = 3/5$.

3.1.3 Case III: First- and Second-Moment Disagreement

Investors disagree about both the expected payoffs of the new stocks and their covariance with the original stocks. In this general scenario, the representative investor's expected payoffs and variance-covariance matrix of the original stocks are both affected by financial innovation. Financial innovation affects stock prices through both channels, the expectation channel and the risk channel.

The expectation of the representative investor for an original stock k tends to be high when investors, who have low covariances of the stock with the newly introduced stocks F, also have high expectations of F. Intuitively, investors who have high expectations of F tend to buy more of F. In addition, since F has low covariances with stock k, they tend to buy more of k to reduce risk. The purchase of stock k pushes up the price of the stock and the expectation of the representative investor for the stock. Similarly, the expectation of the representative investor for an original stock k tends to be low when investors who have low covariances of the stock with the newly introduced stocks F also have low expectations of F. Thus the effects of financial innovation on the prices of the original stocks through the expectations channel can go both ways. As discussed before, financial innovation always reduces the variance-covariance matrix of the representative investor. Thus the pricing effects through the risk channel tend to reduce the risk premium and increase the stock price. This holds strictly for the market portfolio. The net effects of financial innovation on stock prices depend on the tradeoff of these two channels.

Consider the following numerical example with two types of investors type I and type II of equal proportions, $\mu_{Vi} = \mu_V = 1$, for $i = I, II$. The mean of F is μ_{FI} for type I and 0 for type II investors. The supply of the stock is 0.2 and the risk aversion coefficient is 2.

The variance-covariance matrices are:

$$\Sigma_{ZI} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \Sigma_{ZII} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix},$$
$$\Pi_{ZI} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \Pi_{ZII} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}.$$

The precision matrix and variance-covariance matrix of the representative investor is

$$\Pi_{Zr} = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 1 \end{bmatrix}, \qquad \Sigma_{Zr} = \begin{bmatrix} 4/5 & 2/5 \\ 2/5 & 6/5 \end{bmatrix}.$$

The stock price without financial innovation is $P_V = 3/5$ and that with financial innovation is $\hat{P}_V = 17/25 + \mu_{FI}/5$. In the economy with financial innovation, the representative investor's expectation of the original stock payoff now depends on type I investors' expectation of the new stock. When μ_{FI} is positive, combined with the lower variance of the stock, the price of the original stock will be higher. When μ_{FI} is very negative, it can offset the increase in the stock price due to a lower risk premium. How financial innovation affects the price of the original stock will depend on which of the expectation effect and the risk effect would dominate. The next proposition summarizes the results.

Proposition 3. *We have the following results regarding the pricing effects of the financial innovation on the original stocks.*

(i) When investors agree about the expected payoff of the new stocks but disagree about the covariance between the new stocks and the original stocks, financial innovation reduces the variance-covariance matrix of the original stocks for the representative investor. As a result, the risk premium of the market portfolio is reduced.

(ii) When investors disagree about the expected payoff of the new stocks but agree about the covariance between the new stocks and the original stocks, financial innovation has no effect on stock prices.

(iii) More generally, when investors disagree about both the expected payoff of the new stocks and the covariance between the new stocks and the original stocks, financial innovation affects the stock price through both the expectations channel and the risk channel.

The market portfolio holds a special position in this economy since all investors would hold the market portfolio in the absence of financial innovation. In fact, even in the original economy, the market can be viewed as the market portfolio plus $N-1$ financial innovations with zero supplies. We have the following corollary regarding the effects of financial innovations and the market portfolio.

Corollary 1. *When investors agree on the expected payoffs of new stocks but disagree about the covariance of the new stocks with the market portfolio, the price of the market portfolio will increase. When investors disagree on the expected payoffs of the new stocks but agree about the covariance (or market β) of the new stocks, the price of the market portfolio remains the same.*

3.2 Market Liquidity

Clearly, financial innovation increases liquidity by making nontradable assets tradable. However, it would be interesting to measure the effects of financial innovation on the liquidity of existing stocks. We measure market liquidity by how sensitive the prices are to the changes in the supply of the stock. Differentiating the stock prices with respect to the supply of the stock, in the absence of financial innovation, we have

$$-\frac{\partial P_V}{\partial X} = \gamma \Sigma_V.$$

In the presence of financial innovation, we have

$$-\frac{\partial \hat{P}_V}{\partial X} = \gamma \Sigma_{Vr}.$$

Since $\Sigma_{Vr} < \Sigma_V$, the diagonal elements of Σ_{Vr} is smaller than those of Σ_V, we have the following result:

Proposition 4. *Financial innovation increases the market liquidity of all stocks.*

In the presence of financial innovation, investors perceive less risk in holding stocks and the market liquidity increases as a result.

3.3 Investor Welfare

Since investors have achieved Pareto optimal allocation in the absence of financial innovation, it is impossible for the investors to achieve a Pareto improvement with financial innovation. However, we can show that financial innovation can make all investors worse off under the true probability belief. Let investors have the same covariance matrix between an old stock and Pareto optimal allocation. Let all expectations for the original stocks be zero. There are two groups of investors. One group's expectation of the new stocks is $\bar{F}$, and the other group's expectation of the new stocks is $-\bar{F}$. The two groups have equal masses.

We assume that under the true distribution, we have $\bar{F} = 0$ while the variance-covariance matrix of Z is the same as that of individual investors. Since investors all agree on the variance-covariance matrix of Z, financial innovation has no effect on the prices of the original stocks. As a result, each investor must be strictly better off under his own belief as he now has more trading opportunities. However, in this case, as investors are symmetric, the gains from trading in the presence of financial innovation under the true distribution are the same for all individual investors. As investors have already achieved Pareto optimal allocation in the absence of financial innovations, the gains of trading due to financial innovation must be negative. We have the following proposition.

Proposition 5. *Investors could be worse off with financial innovation even though each investor perceives gains from financial innovation.*

4 Extensions

We consider two extensions here. The first extension allows for differential priors regarding the original stocks and the second extension includes nontradable endowments in the analysis.

4.1 Differential Priors

In previous sections, we considered homogeneous priors about the stock payoffs. In this section, we extend the model to heterogeneous priors and show that the modified version of our results still goes through. We assume that investor i's belief about the mean and variance-covariance matrix of V and Z are μ_{Vi}, μ_{Zi} and Σ_{Vi}, Σ_{Zi}, respectively. Moreover, Π_{Vi}, Π_{Zi} denote the precision matrix of investor i for V, Z respectively. The expectation, variance-covariance matrix, and precision matrix of the representative investor with and without financial innovation are given by

$$\mu_{Vr} = \Pi_{Vr}^{-1} \int_i \Pi_{Vi} \mu_{Vi} \, di, \tag{10}$$

$$\Pi_{Vr} = \int_i \Pi_{Vi} \, di, \tag{11}$$

$$\Sigma_{Vr} = \Pi_{Vr}^{-1}, \tag{12}$$

$$\mu_{Zr} = \Pi_{Zr}^{-1} \int_i \Pi_{Zi} \mu_{Zi} \, di, \tag{13}$$

$$\Pi_{Zr} = \int_i \Pi_{Zi} \, di, \tag{14}$$

$$\Sigma_{Zr} = \Pi_{Zr}^{-1}, \tag{15}$$

We have the following results.

Theorem 2. *In the absence of financial innovation, there exists an equilibrium in which demands and asset prices are given below:*

$$P_V = \mu_{Vr} - \gamma \Sigma_{Vr} X_V, \tag{16}$$

$$D_{Vi} = \frac{1}{\gamma} \Pi_{Vi} (\mu_{Vi} - P_V). \tag{17}$$

In the presence of financial innovation, there exists an equilibrium in which demands and stock prices are given below:

$$P_Z = \mu_{Zr} - \gamma \Sigma_{Zr} X_Z, \tag{18}$$

$$D_{Zi} = \frac{1}{\gamma} \Pi_{Zi} (\mu_{Zi} - P_Z). \tag{19}$$

Similarly, we have the following proposition:

Proposition 6. *Financial innovation has the following effects:*

(i) The variance-covariance matrix of the representative investor always reduces;

(ii) When investors agree about the expectation of both the new and the original stocks in the market, the risk premium of the market portfolio will reduce and thus the price of the market portfolio will increase;

(iii) When investors agree on the regression coefficient matrix of the new stocks on the old stocks, the prices of stocks and market liquidity are not affected;

(iv) When investors agree on the market beta of all stocks, the price of the market portfolio and the market liquidity on the market portfolio are not affected;

(v) Market liquidity will always increase.

Even with differential priors, the same results still go through. The mutual insurance effect due to differential beliefs still makes investors feel safer holding the stocks and they can benefit from the mistakes made by others in the new stocks. Consequently, the perceived risk is lower. Lower perceived risk also implies that the market will be more liquid. The effect on the mean of the representative investor is the same as before. When investors with bullish expectations of F also have low covariances of F with the existing stocks, they are going to buy more of F. The purchases of the new stocks further prompt the purchases of the current stocks to hedge risk. As a result, the prices of the existing stocks will increase and the representative investor's expectation of the existing stocks will increase. For there to be no price effect, we require investors to agree on the regression coefficient of F on V. In this special case, there exists a linear combination of F and V that is orthogonal to V for all investors, which implies no pricing effect. Finally, the results on the market portfolio hold as the economy can be viewed as the market portfolio plus financial innovation with zero supplies.

4.2 Nontradable Endowments

In this subsection, we analyze the effects of nontradable endowments. The economy is the same as Section 4.1 but investors here are endowed with nontradable assets. We assume investor i is endowed with a nontradable asset e_i which is jointly normally distributed with Z. Investor i's expectation of e_i is denoted by $\bar{e}_i$. His variance of e_i is denoted by σ_{ei}^2, and his covariances of e_i with V and Z are denoted by a $N \times 1$ vector Σ_{Vei}, and a $(N+M) \times 1$ vector Σ_{Zei}, respectively.

Notice that when V are the traded stocks in the economy, we can decompose e_i into hedgeable and non-hedgeable components:

$$e_i = \bar{e}_i + \Sigma'_{Vei}\Sigma_{Vi}^{-1}(V - \mu_{Vi}) + \epsilon_{Vei}, \tag{20}$$

where ϵ_{Vei} is the non-hedgeable residual and does not affect equilibrium demands and prices. Thus the prices and demands in the economy with nontradable endowments e_i are equivalent to that in an economy in which each investor i holds additional $\Sigma_{Vi}^{-1}\Sigma_{Vei}$ shares of the traded stocks V. Similarly, when Z are traded stocks, the prices, and demands in the economy with nontradable endowments e_i are equivalent to that in an economy in which each investor i holds additional $\Sigma_{Zi}^{-1}\Sigma_{Zei}$ shares of the traded stocks Z. We have the following results.

Theorem 3. *In the absence of financial innovation, there exists an equilibrium in which asset demands and prices are given below:*

$$P_V = \mu_{Vr} - \gamma\Sigma_{Vr}\left(X_V + \int_i \Sigma_{Vi}^{-1}\Sigma_{Vei}\,di\right), \tag{21}$$

$$D_{Vi} = \frac{1}{\gamma}\Pi_{Vi}(\mu_{Vi} - P_V) - \Sigma_{Vi}^{-1}\Sigma_{Vei}. \tag{22}$$

In the presence of financial innovation, there exists an equilibrium in which demands and prices are given below:

$$P_Z = \mu_{Zr} - \gamma\Sigma_{Zr}\left(X_Z + \int_i \Sigma_{Zi}^{-1}\Sigma_{Zei}\,di\right), \tag{23}$$

$$D_{Zi} = \frac{1}{\gamma}\Pi_{Zi}(\mu_{Zi} - P_Z) - \Sigma_{Zi}^{-1}\Sigma_{Zei}. \tag{24}$$

It should be noted that in the presence of nontradable endowments, the stock prices can be decomposed into three components corresponding to expectations, risk, and hedging. Consequently, financial innovation affects stock prices through all three channels. Moreover, the effective market portfolio is no longer the market portfolio of traded stocks, instead, it should include nontradable assets as well. Consequently, our earlier results with respect to the market portfolio no longer hold. We have the following proposition:

Proposition 7. *Financial innovation has the following effects on the prices and liquidity of existing securities.*

(i) The variance-covariance matrix of the representative investor always reduces;

(ii) When investors agree on the regression coefficient matrix of the new stocks on the old stocks, the prices of stocks and market liquidity are not affected;

(iii) Market liquidity always increases.

5 Conclusion

We have abstracted from considerations of market incompleteness, asymmetric information, transaction cost, and taxes and focus on the effects of financial innovation due to speculation. In a market with heterogeneous beliefs about the newly introduced stocks with zero net supplies, we show that the CAPM no longer holds even though investors have homogeneous expectations about the original stocks. Financial innovation has the following effects on the existing securities: (i) The variance-covariance matrix of the representative investor always reduces; (ii) When investors agree about the expectations of stocks traded in the market, the risk premium of the market portfolio will reduce and thus the price of the market portfolio will increase; (iii) When investors agree on the regression coefficient matrix of the new stocks on the old stocks, the prices of stocks and market liquidity are not affected; (iv) When investors agree on the market beta of all stocks, the price of the market portfolio and the market liquidity on the market portfolio are not affected; (v) Market liquidity will increase. We then extend the model to allow for differential priors on the original stocks and nontradable endowments. With differential priors, our results still hold. With nontradable endowments, the effective market portfolio is different from the traded market portfolio and the results with respect to the market portfolio will not stand while other results still hold.

The increase in the stock price and liquidity makes companies more willing to expand their business as they can sell their shares at attractive prices. The reduction in the cost of capital also makes it cheaper to raise funds to finance new projects. Consequently, financial innovation can stimulate economic growth. Financial innovations also allow investors to share the risk due to nontradable endowments better. However, we also show that when all investors have wrong beliefs about the new securities, they could all be made worse off under the true belief although each investor perceives gains from financial innovation using their own beliefs. Thus any regulation about financial innovation should balance these effects, better risk sharing, and the reduction of the risk premium that facilitates economic growth versus welfare reducing frivolous trading.

A Proofs

Proof of Lemma 1: For investor i, his maximization problem reduces to the familiar mean-variance maximization problem

$$\max_{D_i} D_i'(\mu_V - P_V) - \frac{\gamma}{2} D_i' \Sigma_V D_i. \tag{25}$$

Differentiating with respect to D_i, we get

$$D_i = \frac{1}{\gamma} \Sigma_V^{-1} (\mu_V - P_V). \tag{26}$$

Aggregating the demands, we have

$$X_V = \frac{1}{\gamma} \Sigma_V^{-1} (\mu_V - P_V), \tag{27}$$

$$P_V = \mu_V - \gamma \Sigma_V X_V. \tag{28}$$

It is easy to check that in this equilibrium, investors achieve Pareto optimal allocation. □

Proof of Proposition 1: Following the proof of Lemma 1, each investor's demand is

$$X_{Zi} = \frac{1}{\gamma} \Pi_{Zi} (\mu_{Zi} - P_Z)$$

Aggregating the demand,

$$X_Z = \int \frac{1}{\gamma} \Pi_{Zi} (\mu_{Zi} - P_Z)$$

Write

$$\Pi_{Zr} = \int_i \Pi_{Zi} \, di, \tag{29}$$

$$\mu_{Zr} = \Pi_{Zr} \int_i \Pi_{Zi} \mu_{Zi} \, di. \tag{30}$$

The market's clearing condition becomes

$$X_Z = \frac{1}{\gamma}\Pi_{Zr}(\mu_{Zr} - P_Z).$$

The representative investor's information set contains a multivariate normal distribution of Z, with his precision matrix being the average precision matrix and his expectation of Z being the precision weighted average across individual investors. □

Proof of Proposition 2: The first part follows from the expression of the representative investor's expectation of V as it now depends on individual investors' expectation of F. We have

$$Var_r[V] = \left[\Pi_{Zr}(V) - \Pi_{Zr}(VF)\Pi_{Zr}(F)^{-1}\Pi_{Zr}(FV)\right]^{-1}, \quad (31)$$

$$\Sigma_V = \left[\int_i (\Pi_{Zi}(V) - \Pi_{Zi}(VF)\Pi_{Zi}(F)^{-1}\Pi_{Zi}(FV))di\right]^{-1}. \quad (32)$$

Notice that

$$\int_i \Pi_{Zi}(V)di = \Pi_{Zr}(V)$$

Moreover, let $dw(i)$ be an $M-$dimensional standard Wiener process, and define

$$A \equiv \int_i \Pi_{Zi}(VF)\Pi_{Zi}(F)^{-1/2}dw(i), \quad (33)$$

$$B \equiv \int_i \Pi_{Zr}(VF)\Pi_{Zr}(F)^{-1}\Pi_{Zi}(F)^{-1/2}dw(i), \quad (34)$$

$$C \equiv A - B \quad (35)$$

we have that B and C are orthogonal and

$$\int_i (\Pi_{Zi}(V) - \Pi_{Zi}(VF)\Pi_{Zi}(F)^{-1}\Pi_{Zi}(FV))di \tag{36}$$

$$= Var[A] \tag{37}$$

$$= Var[B] + Var[C] \tag{38}$$

$$> Var[B] \tag{39}$$

$$= \Pi_{Zr}(V) - \Pi_{Zr}(VF)\Pi_{Zr}(F)^{-1}\Pi_{Zr}(FV) \tag{40}$$

Therefore, we have

$$\Sigma_{Vr} < \Sigma_V.$$

□

Proof of Theorem 2: It follows directly from the proof of Proposition 1. □

Proof of Proposition 3: Part (i) holds and the variance-covariance matrix decreases when financial innovations are introduced. Part (ii) holds as in this case financial innovations do not affect the beliefs of the representative investor about V. Let $F = BV + \epsilon$, where B is the common regression coefficient matrix of F on V for all investors. Then ϵ is orthogonal to V for all investors. Let $\mu_{\epsilon i}$ the expectation of investor i of ϵ, then the representative investor's precision of ϵ is given by

$$\Pi_{\epsilon r} = \int_i \Pi_{\epsilon i}\, di, \tag{41}$$

$$\mu_{\epsilon r} = \Pi_{\epsilon r}^{-1} \int_i \Pi_{\epsilon i}\mu_{\epsilon i}\, di, \tag{42}$$

and the representative investor's expectation of V remains to be μ_V and his variance-covariance matrix of V remains to be Σ_V. As a result, the prices of stocks and market liquidity are not affected by financial innovation. Part (iii) holds as in this generic case, the expectation and variance-covariance matrix of V would change when financial innovations are introduced in general. □

Proof of Corollary 1: Notice that the securities markets can be viewed as the market portfolio plus a set of financial innovations with zero supplies. The results here follow from those of Proposition 3, if we compare prices in an economy in which only the market portfolio is traded with the two economies with V and Z traded respectively. □

Proof of Propositions 4-7 and Theorem 2-3: The proofs are similar to those of Theorem 1, Proposition 3, and Corollary 1 and thus are omitted here. □

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