A Model of Pro-Cyclical Exchange Rates*

Qiushi Huang
Shanghai Advanced Institute of Finance

Leonid Kogan
MIT Sloan School of Management and NBER

Dimitris Papanikolaou
Kellogg School of Management and NBER

Abstract

Exchange rates in the standard macro-finance model with a representative agent are counter-cyclical. The reason is that exchange rates are equal to the ratio of marginal utilities of consumption of the representative investor in each country. This prediction is counterfactual: across a variety of countries, (real) exchange rates are, on average, positively correlated with output and consumption growth. We provide a model in which the cyclical behavior of exchange rates varies with the source of the economic shocks. A key feature of our model is incomplete markets, which introduces a wedge between aggregate consumption and the marginal utility of the average investor. We introduce a minimal deviation from the standard endowment economy model of exchange rate depreciation: in a boom, new trees are created, but they are randomly distributed to a small part of the population. As a result, the marginal utility of the average investor can rise, leading to an appreciation of the real exchange rate. Our calibrated model does a good job replicating key features of the data, specifically, the joint dynamics of exchange rates, stock returns, real output and consumption growth, and trade flows.

Keywords: Exchange rates, Incomplete markets, Backus-Smith puzzle

*Qiushi Huang: qshuang@saif.sjtu.edu.cn; Leonid Kogan: lkogan@mit.edu; Dimitris Papanikolaou: d-papanikolaou@kellogg.northwestern.edu. We are grateful to Nikolai Roussanov for helpful comments and discussions.

1 Introduction

Standard macro-finance models with a representative agent predict counter-cyclical exchange rates. In these models, home currency is expected to depreciate during domestic economic booms due to a decline in the marginal utility of consumption. Yet, this prediction is at odds with the data – the Backus-Smith puzzle (Backus and Smith (1993); Kollmann (1995)). If anything, exchange rates tend to be on average pro-cyclical, that is periods of economic growth and/or high consumption growth are associated with an appreciation of the (real) exchange rate. In addition, interest rate differentials do not predict changes in exchange rates with the right sign – the UIP puzzle Fama (1984), and real exchange rates are not sufficiently volatile when confronted with the evidence from asset prices – the volatility puzzle Brandt, Cochrane, and Santa-Clara (2006). Several studies introduce asset market frictions to rationalize these anomalies. Lustig and Verdelhan (2019) and Jiang, Krishnamurthy, and Lustig (2023) analyze the potential limitation of these models in explaining these three puzzles, particularly under the assumption that marginal utility is low during economic booms.

Our goal in this paper is to provide a model in which exchange rates are potentially pro-cyclical and thus can help rationalize these three puzzles. In particular, marginal utility could be high even during periods of economic booms. To do so, we move beyond the representative agent paradigm and introduce a wedge between aggregate consumption growth and the marginal utility of the average household. Following Gârleanu, Panageas, Papanikolaou, and Yu (2016), we introduce a minimal deviation to the standard endowment economy model: in addition to the standard endowment shock in each country, countries can now each experience displacive shocks that reallocate output among agents. This mechanism is a reduced-form version of a model of endogenous production and creative destruction; in such models periods of economic growth can be associated with significant reallocation (see, e.g., Kogan, Papanikolaou, and Stoffman, 2020; Huang, Kogan, and Papanikolaou, 2023). If the benefits of economic growth do not accrue symmetrically across all agents—for instance, if a positive endowment shock is also associated with economic reallocation or displacement, then marginal utility—and hence exchange rates—can rise even as aggregate output and consumption grow.

We apply this idea to exchange rates by incorporate reallocation shocks to an otherwise standard two-country endowment economy model with recursive preferences (largely based on Colacito and Croce, 2013). Under the assumption that periods of economic growth are associated with a sufficient degree of economic reallocation, the model can deliver pro-cyclical exchange rates. Further, to the extent that the strength of reallocation in relation to growth can vary across countries or periods, possibly due to the underlying source of economic shocks, the model can deliver heterogeneity in the degree of cyclical behavior of currencies, both across countries as well as over time.

In fact, if such forces are behind the real exchange rate dynamics, we would expect to see that periods of technological progress correlate with an appreciation of the real exchange rate. An illustration of this pattern is presented in Figure 1, which plots annual U.S. innovation intensity, measured by the average economic value per patent each year (Kogan, Papanikolaou, Seru, and Stoffman (2017)), and the U.S. real trade-weighted exchange rate, both in deviations from a Hodrick–Prescott (HP) filtered trend. The swings in the dollar in real terms are strongly associated with movements of the technological improvements[1].

To build intuition, we start with a set of minimal ingredients: a two-country endowment model in which each household has logarithmic preferences and home bias in consumption. In addition to a shock to the aggregate endowment, each country experiences a shock constructed to mimic the properties of creative destruction as in Gârleanu et al. (2016) and Huang et al. (2023). In particular, growth is partially driven by the arrival of new projects (firms) that potentially displace the incumbents. The key feature of the model is incomplete markets: ownership of the new projects does not accrue to the shareholders but are instead randomly allocated to a (measure zero) subset of the population. The key friction is that households cannot sell claims on their future potential endowment of these new projects. As a result, shocks to the relative profitability of new projects lead to the redistribution of wealth from the existing firms owners to the new entrepreneurs. This wealth redistribution increases the cross-sectional dispersion of consumption growth—the majority of households incur small losses while a fortunate few experience substantial increases in their wealth. Since households' marginal utility is convex, the displacement shock raises the stochastic discount factor and therefore leads to an appreciation of the real exchange rate (this mechanism is similar in spirit to Constantinides and Duffie, 1996).

Our model generates pro-cyclical exchange rates with a minimal deviation from the standard setting. Depending on the correlation between economic growth and the displacive shock, the correlation between aggregate consumption and output can be positive or negative. In addition, this simple model has some testable implications. Even though obtaining a direct proxy for the displacement shock across countries is challenging, in the model a positive displacement shock increases income inequality. As a result, the model implies a

[1] Figure A.1 reproduces this pattern using an alternative innovation index, which is equal to the sum of value of patents granted in each year scaled by the level of aggregate output.

positive correlation between exchange rates and changes in (relative) income inequality. This prediction is consistent with the data. In a panel regression of 11 countries covering the post-Bretten Woods era, we find a positive and statistically significant correlation between changes in bilateral exchange rates and changes in relative income inequality.[2] For instance, focusing on the coefficients from the pooled regression, a one-standard deviation increase in income inequality in a foreign country relative to the United States is associated with a 0.012 log point appreciation of its currency relative to the US dollar. For comparison, a one-standard deviation increase in the relative consumption growth in the foreign country is associated with a 0.011 log point appreciation of its currency against the dollar.

We then explore the ability of our mechanism to quantitatively account for the key correlations in the data. To do so, we extend the model along several dimensions, specifically we allow for recursive preferences over relative consumption and relax the assumption of extreme inequality—a positive measure set of households receive new projects. In addition, we allow for the distribution of the displacement shock to vary over time. Though these modifications are not needed to qualitatively explain the key patterns in the data, they help the model deliver realistic quantitative predictions. We calibrate the model to the data by choosing parameters that minimize the distance between the data and model-implied statistics, essentially a form of the simulated method of moments (SMM).

Our model successfully replicates the first two moments of aggregate consumption and output growth, exchange rates, and stock returns, while generating low and relatively smooth risk-free rates. Our model replicates the three key 'anomalies' in the exchange rate literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of the uncovered interest rate parity (UIP). Key to replicating the failure of the UIP is the time-varying distribution of the displacement shock. In addition, the model also replicates the cyclical properties of trade flows: in both the model and in the data, net exports are counter-cyclical.

Importantly, the model can simultaneously deliver a positive correlation between consumption (or output) growth and exchange rates and a negative correlation between exchange rates and stock market returns, an empirical pattern that is hard for existing models to replicate. In the model, a positive displacement shock is positively correlated with aggregate consumption but leads to a decline in the stock market. This feature, in addition to replicating the failure of the consumption CAPM (CCAPM) in the data, helps the model jointly match the dynamics of stock returns, exchange rates and consumption growth.

[2]The sample covers the 1971 to 2019 period and combination of G-10 currency countries and G-7 countries: Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland, United Kingdom, France, Italy and the United States.

The quantitative success of the model does not come at the cost of unrealistic parameters. In terms of preference parameters, the model calibration requires a degree of relative risk aversion of 6.5 and an elasticity of inter-temporal substitution (EIS) equal to 1.8, which are largely in line with the literature. The preference weight on relative consumption is rather high (0.82) though it comes with a high standard error, implying that the model solution is not very sensitive to this particular value. Further, the model requires a highly persistent and risk-skewed displacement shock. To ensure that the magnitudes of displacement shocks are realistic, we also target the mean level of observed income inequality as part of the model calibration, which helps discipline the volatility of the displacement shock. Last, just like most existing models (Colacito and Croce, 2013), our calibration requires a high degree of home bias in household preferences (0.990). A high degree of home bias is needed in order to generate sufficiently volatile exchange rates given the high level of correlation in consumption growth across countries.

Relation to the prior literature

In this paper we develop a quantitative general equilibrium model that successfully replicates the joint dynamics of exchange rates, consumption growth, trade flows, and stock returns. Our model thus contributes to a voluminous literature studying the determination of exchange rates in two-country equilibrium models (see, e.g. Chari, Kehoe, and McGrattan, 2002; Alvarez, Atkeson, and Kehoe, 2002; Corsetti, Dedola, and Leduc, 2008; Pavlova and Rigobon, 2007; Alvarez, Atkeson, and Kehoe, 2009).

Similar to Colacito and Croce (2011, 2013); Colacito, Croce, Gavazzoni, and Ready (2018), our model features households with recursive preferences. However, our model does not rely on the highly persistent consumption growth rates, as endowment shocks in our framework need not be persistent (they are i.i.d. in the basic version of the model), and the main mechanism is imperfect consumption risk sharing. Farhi and Gabaix (2016) introduce a small probability of extreme disasters, which helps decouple exchange rates and consumption growth in samples without a consumption disaster.

The weak correlation between exchange rates and fundamentals (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2001) has led to the development of models with segmented asset markets (e.g. Farhi and Werning (2014); Itskhoki and Mukhin (2021); Fukui, Nakamura, and Steinsson (2023)). The goal of this literature is to link international movements in capital flows to exchange rates. Hau and Rey (2006); Camanho, Hau, and Rey (2020) study a model of segmentation in equity markets that links portfolio flows in equities to exchange rates. Greenwood, Hanson, Stein, and Sunderam (2020); Gourinchas, Ray, and Vayanos (2020) focus on the bond market. Gabaix and Maggiori (2015) develop a tractable general-equilibrium model in which portfolio-rebalancing motives drive exchange rate movements. Bacchetta and Van Wincoop (2006) examines the implications of exchange rates of agents who infrequently rebalance their bond portfolios. Lilley, Maggiori, Neiman, and Schreger (2019) provide supporting evidence that links the purchase of US bonds by foreigners to the dollar exchange rate. Lustig and Verdelhan (2019) discuss the limitation of market incompleteness in resolving the volatility, cyclicality, and risk premium puzzles. Jiang et al. (2023) highlight the role of bond euler equation in understanding exchange rate dynamics.

Although our model also features incomplete markets, the mechanisms are quite distinct. In these models, capital flows (rebalancing needs) are in general exogenously assumed; these models focus instead on how these capital flows can impact exchange rates in the absence of complete risk sharing. By contrast, our model does not feature any exogenous movements in capital flows; rather, capital flows are determined in equilibrium. That said, in our model, shifts in the degree of technological innovation across countries also generate movements in financial flows due to diversification motives, together with a positive correlation between capital inflows and currency appreciation. In particular, a positive displacement shock in the home country is associated with the creation of new firms (projects) which are initially owned by a small subset of households (entrepreneurs). Entrepreneurs sell their shares to diversify their holdings and foreign investors buy some of these shares to rebalance their portfolio as the share of the home country in the world market portfolio increases. The net effect is that the home country experiences net capital inflows and its currency appreciates, which is consistent with the finding of Hau and Rey (2006); Camanho et al. (2020).

The failure of the uncovered interest rate parity (UIP) has been an important topic in the international finance literature. Verdelhan (2010) provides a model that reproduces the UIP as a result of counter-cyclical risk aversion arising from preferences with external habits. Ready, Roussanov, and Ward (2017) develop a model where countries specialize in intermediate or final goods. The model delivers an endogenous relation between interest rate levels and currency risks. Jiang, Krishnamurthy, and Lustig (2021); Jiang, Krishnamurthy, Lustig, and Sun (2021) focus on convenience yield earned on dollar safe assets and their implications for exchange rates.

The existence of common risk factors in exchange rates has been the subject of considerable debate (Lustig, Roussanov, and Verdelhan, 2011; Verdelhan, 2018; Jiang, 2023). Richmond (2019); Lustig and Richmond (2019); Jiang and Richmond (2019) emphasize the importance of international trade linkages in generating comovement across currencies. Our model can potentially speak to these facts as well. In particular, a key driver of bilateral exchange rate dynamics is the difference between the two countries' rates of creative destruction. Over the recent decades, many the major innovations took place in the US. To the extent that the degree of creative destruction is more pronounced in the United States, either because of easier firm entry or a less progressive tax system, one potential source of the "dollar factor" in exchange rates could be US technological innovation. Further, to the extent that technology spillovers are correlated with trade flows, our framework provides a new perspective on the importance of trade network linkages.

The main mechanism in our paper is closely related to Huang et al. (2023), Kogan et al. (2020) and Gârleanu et al. (2016). Kogan et al. (2020) build a general equilibrium model with capital embodied technology shocks in which benefits of innovation are distributed asymmetrically across the economy. The key friction is that potential innovators cannot contract ex ante to share the economic rents that their ideas generate. As a result, financial market participants capture only part of the benefits, despite bearing all of the costs of creative destruction. The reallocative impact on household wealth implies that improvements in technology can reduce household indirect utility. This displacive effect on indirect utility is amplified when households care about their consumption relative to the economy-wide average, since household dislike being 'left behind'. Kogan et al. (2020) show that the resulting displacement risk can lead to increased demand for insurance (an increase in the stochastic discount factor) and can help rationalize certain cross-sectional features of asset returns. Huang et al. (2023) examine this mechanism in a multi-region model of a monetary union and study its implications for regional inflation dynamics. Gârleanu et al. (2016) embed a reduced-form of this mechanism in a standard endowment model and study its implications for the equity risk premium. Kogan, Papanikolaou, Schmidt, and Song (2020) presents complementary evidence that surges in innovation correlate with higher labor income risks for incumbent workers, leading to a stronger demand for insurance.

At a broader level, our mechanism can also be re-interpreted through the lens of the Balassa-Samuelson hypothesis (Balassa, 1964; Samuelson, 1964). The Balassa-Samuelson effect is that, if productivity increases in the tradable sector tend to be higher than those in the nontradable sector, then the conventionally constructed real exchage rates—that is, using a price index of a combination of both tradable and non-tradable prices as the price deflator— will comove with the cross-country differences in the relative speed of productivity increases between tradable and non-tradable sectors. This effect is neutralized by risk sharing if financial markets are complete. Specifically, as long as foreign investors can invest in the home country's tradable sector, an increase in the productivity of the domestic tradable sector will raise the price of non-tradeable goods (relative to the tradable goods) at both the home and the foreign country. With imperfect risk sharing, some of the productivity gains in the home tradeable sector cannot be shared with foreigners, which provides one potential mechanism for the Balassa-Samuelson effect in equilibrium.

2 Exchange Rates and Aggregate Quantities

We begin by showing that exchange rates are, on average, pro-cyclical in the data. This analysis essentially replicates the Backus-Smith puzzle, specifically that exchange rates are only weakly correlated with relative consumption growth, and the sign often goes in the opposite direction than predicted by the standard model with complete markets and time-separable preferences.

2.1 Data Sources

We obtain data on nominal exchange rate, consumption, GDP and net exports from the World Bank, specifically, the World Development Indicators. We use households final consumption expenditure for consumption series, and the difference between the indices of export of goods and imports of goods and services as our net export series. Both consumption and GDP are real, PPP-adjusted. Inflation rates are calculated using Consumer Price Index (CPI) from the World Bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country. Data on interest rates comes from Global Financial Data. Real interest rates are constructed using three-month T-bills yields from the Global Financial Data, adjusting for realized inflation using annual changes in CPI. Data on equity index returns (MSCI series) is obtained from Datastream. We measure income inequality using the top 1% percentage income share; we obtain data from the World Inequality Database. See the Appendix D.1 for more details.

Our sample is dictated by data availability and consists of a combination of G-10 currency countries and G-7 countries. Specifically, it includes Australia, Canada, Germany, Japan, Norway, New Zealand, Sweden, Switzerland, United Kingdom, France, Italy and the United States. We take the domestic country to be the United States and define the exchange rate as the units of foreign currency per dollar. The sample period covers the post-Bretten Woods era. After restricting attention to years for which income inequality data is available, the sample covers the 1971 to 2019 period.

2.2 Exchange Rates are Pro-cyclical

We begin by showing that exchange rates are on average pro-cyclical. To do so, we estimate the following specification,

$$\log e_{F,t+1} - \log e_{F,t} = \alpha + \beta \left(\log \frac{x_{t+1}^{US}}{x_{t+1}^{F}} - \log \frac{x_{t}^{US}}{x_{t}^{F}} \right) + \gamma \log e_{F,t} + \delta \log \frac{x_{t}^{US}}{x_{t}^{F}} + \log \varepsilon_{F,t}. \quad (1)$$

The dependent variable is the growth in the bilateral exchange rate e between the foreign country F and the US. Here, $x^c = \in \{C, Y\}$ refers to consumption and output in country c.

Our main coefficient of interest in β. An increase in e corresponds to an appreciation for the US dollar relative to the foreign currency; thus, a positive slope coefficient β indicates a pro-cyclical exchange rate. Recall that in the standard model with time-separable preferences, the slope coefficient is negative and equal to the coefficient of relative risk aversion (in the case where x refers to consumption). That is, an increase in the relative consumption growth of the foreign country relative to the US should lead be associated with a depreciation of the foreign currency.

By contrast, in the data, the slope coefficient is largely positive, as we see in Tables 1 and 2. Table 1 reports results for relative consumption growth. In a panel regression, a one-standard deviation increase in consumption growth differential of the foreign country is associated with a 0.011 log point appreciation of its currency against the dollar. Estimating equation (1) separately for each country, we see that the point estimates are positive in 8 out of the 11 cases; in addition, the negative estimates are not statistically different from zero.

Table 2 shows that the estimates for output are similar: the estimated coefficient β in the panel regression is equal to 0.011 with a standard error of 0.004. When estimating equation (1) separately for each country, we again see that the point estimates are positive in 9 out of the 11 cases, while none of the 2 negative estimates are statistically different from zero.

2.3 Exchange Rates and Stock Returns

We next also examine the correlation between exchange rates and stock returns.

$$\log e_{F,t+1} - \log e_{F,t} = \alpha + \beta \left(\log \frac{S_{t+1}^{US}}{S_{t}^{US}} - \log \frac{S_{t+1}^{F}}{S_{t}^{F}} \right) + \gamma \log e_{F,t} + \varepsilon_{F,t}. \quad (2)$$

Here, S refers to the (cum dividend) stock market index. As before, a positive slope coefficient β indicates that the US (foreign) currency appreciates when the US (foreign) stock market appreciates in value.

Table 3 reports the results. Focusing on the panel regression result, we note that estimated slope coefficient is negative. That is, positive stock market returns are often associated with a depreciation of the currency. In terms of magnitudes, a one-standard deviation increase in stock market returns in the foreign country is associated with a 0.017 log point depreciation of its currency against the dollar. Examining results for individual countries, we note that the correlation is negative in 9 out of the 11 countries. These findings are consistent with Hau and Rey (2006), who document a negative correlation.

In brief, the correlation between exchange rates and stock returns is negative. Though this fact is largely consistent with the standard model, that model cannot jointly generate a positive correlation between exchange rates and consumption (or output) growth and a negative correlation with stock returns. To understand such relation, we examine the following correlation between exchange rates and stock returns and consumption growth:

$$\log e_{F,t+1} - \log e_{F,t} = \alpha + \beta_r \left(\log \frac{S_{t+1}^{US}}{S_t^{US}} - \log \frac{S_{t+1}^{F}}{S_t^{F}} \right) + \beta_c \left(\log \frac{C_{t+1}^{US}}{C_t^{US}} - \log \frac{C_{t+1}^{F}}{C_t^{F}} \right) + \gamma \log e_{F,t} + \varepsilon_{F,t}. \tag{3}$$

Table 4 reports the results. We note that the estimated slope coefficient on consumption is positive while the coefficient on stock market returns is negative. In terms of magnitudes, a one-standard deviation increase in stock market returns in the foreign country is associated with 0.016 log point depreciation of its currency against the dollar. A one-standard deviation increase in consumption growth differential is associated with 0.010 log point appreciation of its currency against dollar. Results with output are similar, as we can see in Appendix Table A.1.

To generate such a pattern, one needs to introduce a distinction between aggregate consumption growth and the stochastic discount factor. Our model does exactly that; the model mechanism relies on incomplete markets and the unequal distribution of benefits from innovation.

3 A Simple Model

To fix ideas, we begin our analysis with the minimal set of modelling ingredients that are necessary. As a result, our goal in this section is to provide some analytic intuition for the main mechanism in the paper. Section 4 presents a more general model that can be calibrated to fit the data.

3.1 Setup

We begin by a discussion of the modeling setup. The economy consists of two countries, home (H) and foreign (F), and two goods, X and Y. Time is discrete and is indexed by t.

3.1.1 Firms

There is a continuum of productive units in each country that produce output. We term these production units firms, but that definition is somewhat arbitrary since firm boundaries are ill-defined. We can also think of these as individual projects.

Firms in each respective country only produce the local good. That is, the firms in the home country only produce the X good, while foreign firms only produce the Y good. There is an expanding measure of firms in each country, indexed by (i, s, c) where s denotes the date at which the firm is created, $i \in [0, 1]$ denotes the index of the firm within its cohort in each country, and $c \in \{H, F\}$ denotes the country.

A firm characterized by (i, s, H) produces a flow of output $x_{t,s}^{i,H}$ at time t according to

$$x_{t,s}^{i,H} = a_{t,s}^{i,H} X_t \tag{4}$$

The setup is symmetric in both the home and foreign country; hence, a firm in the foreign country (i, s, F) produces output $y_{t,s}^{i,F}$

$$y_{t,s}^{i,F} = a_{t,s}^{i,F} Y_t \tag{5}$$

Here, $a_{t,s}^{i,H}, a_{t,s}^{i,F} \in [0, 1]$ denote the fraction of aggregate output accruing to a firm i in located in the home and foreign country, respectively. By construction, these shares add to one

$$\sum_{s \leq t} \int_{i \in [0,1]} a_{t,s}^{i,c} = 1, \quad c \in \{H, F\} \tag{6}$$

The model has an element of creative destruction, in which new productive units displace existing ones. We model this in reduced form, following Gârleanu et al. (2016). Each period a new set of firms arrive exogenously in each country. These new firms, indexed by $i \in [0, 1]$, are heterogeneous in their productivity. The productivity of a newly arriving firm i in country $c \in \{H, F\}$ satisfies

$$a_{t,t}^{i,c} = (1 - e^{-u_t^c}) dL_t^{i,c} \tag{7}$$

where u_t^H, u_t^F are random, non-negative, shocks in home and foreign countries, affecting all firms in each country at time t. The components $L_t^{i,H}, L_t^{i,F}$ denotes cross-sectional measures and its increment $dL_t^{i,H}, dL_t^{i,F}$ are random, non-negative, idiosyncratic productivity components, which are determined at time t and satisfies $\int_{i\in[0,1]} dL_t^{i,H} = 1$ and $\int_{i\in[0,1]} dL_t^{i,H} = 1$. It follows that the total fraction of output produced by the cohort of firms born at time t is equal to

$$\frac{\int_{i\in[0,1]} x_{t,t}^{i,H}}{X_t} = 1 - e^{-u_t^H} \tag{8}$$

$$\frac{\int_{i\in[0,1]} y_{t,t}^{i,F}}{Y_t} = 1 - e^{-u_t^F} \tag{9}$$

The random shocks u_t^c reallocate revenue from incumbents to new entrants. Collectively, the fraction of output produced by existing firms is $e^{-u_t^H}$ for home and $e^{-u_t^F}$ for foreign. Specifically, the output share of an incumbent firm created at a time $s < t$ in country $c \in \{H, F\}$ is given by

$$a_{t,s}^{i,c} = a_{s,s}^{i,c} e^{-\sum_{n=s+1}^{t} u_n^c} \tag{10}$$

3.1.2 Aggregate Output

The aggregate output in each country evolves exogenously according to

$$\Delta \log X_{t+1} = \mu + \varepsilon_{t+1}^H + \delta u_{t+1}^H \tag{11}$$

$$\Delta \log Y_{t+1} = \mu + \varepsilon_{t+1}^F + \delta u_{t+1}^F \tag{12}$$

Notice that each output process is driven by two country-specific shocks, ε and u. The first shock, ε, affects the output (and dividends) all firms symmetrically. The second shock, u, is the 'displacive' shock discussed above, which reallocates market share from existing to new firms. We allow this shock to affect aggregate output—motivated by standard models of endogenous growth—and parameterize it impact by $\delta \in (0, 1)$.

3.1.3 Households

Each country is populated by a unit measure of infinitely-lived agents, indexed by (i, c) where $i \in [0, 1]$ and $c \in \{H, F\}$ denotes their country. At time zero, households are equally endowed with all firms in existence at that time. Households have access to financial market and maximize their expected utility of consumption

$$U_{i,t}^c = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^s \log(C_{i,s}^c). \tag{13}$$

Household consumption C_t^c is an aggregate of the two goods produced by the home (X) and foreign country. Importantly, households exhibit 'home bias', that is, they tilt their consumption basket to the domestically produced good. That is, the consumption basket of each household living in country $c \in \{H, F\}$ at date t is given by

$$C_t^H = (x_t^H)^{\alpha}(y_t^H)^{1-\alpha} \tag{14}$$

$$C_t^F = (x_t^F)^{1-\alpha}(y_t^F)^{\alpha}. \tag{15}$$

Here, x_t^c and y_t^c denote the consumption of good X and good Y in country $c \in \{H, F\}$ at date t. The parameter $\alpha \in (\frac{1}{2}, 1)$ captures the degree of home bias in household preferences.

Last, we normalize the price of the home consumption good (the numeraire) to one; hence,

$$\alpha p_{x,t} + (1-\alpha)p_{y,t} = 1 \tag{16}$$

where $p_{x,t}, p_{y,t}$ are the price of the two goods X and Y, respectively. We denote the price of X and Y as $p_{x,t}, p_{y,t}$. The numeraire is α units of X good and $(1-\alpha)$ units of Y good, i.e.,

3.1.4 Creative Destruction and New Firms

Each period, households innovate with some probability. Successful innovation leads to the creation of a new firm. The key feature of the model is that households cannot share this risk ex-ante, that is, they cannot sell claims against their future endowment of these new firms, as in Kogan et al. (2020). As a result, a shock to the relative profitability of new firms u leads to the redistribution of wealth from the owners of existing firms to the new entrepreneurs.

In particular, at time zero, agents are equally endowed with all firms in existence at that time. From that point onward, agent (i, c) where $i \in [0, 1]$ and $c \in \{H, F\}$ receives firm (i, t, c) at time t, i.e., a new firm with productivity proportional to $a_{t,t}^{i,c}$. For tractability, we closely follow Gârleanu et al. (2016) and focus on the limiting case in which firm creation generates extreme inequality. Specifically, we assume that only a set of measure zero of firms manage to produce non-zero profits; by contrast, the vast majority of new firms are worthless.[3] Consequently, when making consumption and saving decisions, households attach

[3] More formally, we assume that, for every t, the distribution of idiosyncratic productivity $dL_t^{i,c}$ consists

zero probability to the event they receive a profitable firm.[4]

3.1.5 Financial Markets

Households can trade a complete set of securities contingent on the realization of aggregate shocks. That is, they can trade equity claims on existing firms and risk-less, zero-net-supply bonds in either country. Consumers can also trade claims to the realizations of the displacement shocks (u^H_{t+1}, u^F_{t+1}) and output growth (X_{t+1}, Y_{t+1}). Importantly, however, a key market is missing: consumers cannot enter contracts that are contingent on the realized value of their future endowments of new firms.

This market incompleteness is a key part of the mechanism, as it introduces a wedge between aggregate consumption growth and the marginal utility of the average investor.

3.2 Equilibrium

Our definition of equilibrium is standard. An equilibrium is a set of price processes, consumption choices, and asset allocations such that (a) consumers maximize expected utility over consumption and asset choices subject to their dynamic budget constraint, (b) all asset and goods market clear.

Markets are incomplete, hence households' marginal utilities are not equalized across states. To solve for the competitive equilibrium, we construct a representative agent whose preferences are a weighted average of household utility in each country

$$\max_{\{x^H_t, y^H_t, x^F_t, y^F_t\}} \sum_t \beta^t \left(\log C^H_t + \lambda_t \log C^F_t \right) \tag{17}$$

Importantly, the Pareto-Neigishi weight λ_t is stochastic in our model. This representative agent maximizes her utility subject to the following resource constraints,

$$x^H_t + x^F_t = X_t \tag{18}$$

$$y^H_t + y^F_t = Y_t \tag{19}$$

exclusively of point masses. That is, we assume that L^c_t is a discrete measure on $[0, 1]$, so that it is an increasing right-continuous, left-limits process that is constant on $[0, 1]$ except on a countable set, where it is discontinuous. Both the magnitudes of the jumps of L_t, and the locations of the points of discontinuity is random. This assumption ensures that only a set of measure zero of consumers obtain the profitable new firms.

[4] More precisely, what matters for household portfolio decisions is the physical probability of obtaining a new firm times the marginal utility of consumption in that state. Not only is the physical probability of receiving a new firm equal to zero, but also so is the marginal utility of wealth (and consumption) since each firm is extremely valuable.

along with the consumption aggregator in (14).

Here, we note that even though households in both countries are heterogeneous in their wealth, consumption-wealth ratios are equalized within each country which facilitates aggregation. Hence, the representative consumer in each country solves the same optimization problem. That said, it is important to emphasize that even though we construct the preferences of each representative household as a function of the country-level consumption variables C_t^H and C_t^F, no household actually consumes that amount as markets are incomplete. Given our assumption, the effect of market incompleteness collapses into a scaling factor λ_t^c—and without loss of generality we have normalized $\lambda_t^H = 1$. See Appendix A.2 for more details.

In brief, λ_t is the time-varying ratio of marginal utilities of either good of the two countries at time t, and varies over time as the result of market incompleteness

$$\lambda_t = \frac{W_{F,t}}{W_{H,t}} \tag{20}$$

where $W_{c,t} = \int_{i\in[0,1],c} w_t^{i,c}$ is the total wealth of households in country $c \in \{H, F\}$. In equilibrium, the ratio of wealth λ_t between the foreign and the home country affects both real allocations as well as the terms of trade. For example, the relative price of the foreign good Y in units of the domestic good X is equal to

$$p_t \equiv \frac{p_{y,t}}{p_{x,t}} = \frac{X_t}{Y_t}\frac{1-\alpha+\alpha\lambda_t}{\alpha+(1-\alpha)\lambda_t}, \tag{21}$$

and depends not on only on aggregate quantities Y_t and X_t, but also on the countries' relative wealth λ_t.

3.3 Displacement Risk and the SDF

The presence of displacement risk introduces a wedge between aggregate consumption growth and the stochastic discount factor. To understand why this is the case, note that, because of incomplete markets, the marginal utility of the 'representative' household is not only determined by aggregate consumption, but also by the realization of the displacement shock.

To see this, consider the following simplified version of the model, in which a) households have extreme home bias preferences $\alpha = 1$ (or equivalently single-country version of the model) and b) the value of all new firms is equally and randomly allocated to a measure π of the population. In this case, we can divide all households at each point in time into two groups, those that receive profitable new firms and those that do not. Agents have a constant consumption to wealth ratio, hence their consumption process is directly linked to the dividends of the firms they own. Hence, the equilibrium stochastic discount factor can be written as

$$\frac{M_{t+1}^H}{M_t^H} = \beta \left(\frac{X_{t+1}}{X_t}\right)^{-1} \left((1-\pi)\, e^{u_{t+1}^H} + \pi \left(\frac{1-e^{-u_{t+1}^H}}{\pi}\right)^{-1}\right). \tag{22}$$

Recall that we have assumed that income inequality is extreme, that is, L_t^i is comprised of point masses or equivalently $\pi \to 0$. In this case, the expression for the SDF simplifies into

$$\frac{M_{t+1}^H}{M_t^H} = \beta \left(\frac{X_{t+1}}{X_t}\right)^{-1} e^{u_{t+1}}. \tag{23}$$

In brief, we see that incomplete markets introduce a wedge between our stochastic discount factor and the one arising in a standard, Lucas-tree endowment economy. This additional term, given by $e^{u_{t+1}}$ adjusts for the fact that not all households experience the same growth rate in consumption; a set of measure zero experiences a dramatic increase as they receive new firms. Since marginal utility is a convex function of consumption, an increase in the dispersion of consumption growth raises the stochastic discount factor, similar in spirit to Constantinides and Duffie (1996).

In our model, the dynamics of the stochastic discount factor in each country is given by

$$\frac{M_{t+1}^H}{M_t^H} = \beta \frac{C_t^H}{C_{t+1}^H} \frac{1}{b_{H,t+1}} \quad \text{and} \quad \frac{M_{t+1}^F}{M_t^F} = \beta \frac{C_t^F}{C_{t+1}^F} \frac{1}{b_{F,t+1}}, \tag{24}$$

where $b_{H,t+1}$ and $b_{F,t+1}$ are the wealth shares of the people in home and foreign country who did not receive profitable firms at $t+1$,

$$b_{H,t+1} = \frac{\int_{i\in[0,1],a_{t+1,t+1}^{i,H}=0} w_{t+1}^{i,H}}{\int_{i\in[0,1]} w_t^{i,H}} \quad \text{and} \quad b_{F,t+1} = \frac{\int_{i\in[0,1],a_{t+1,t+1}^{i,F}=0} w_{t+1}^{i,F}}{\int_{i\in[0,1]} w_t^{i,F}} \tag{25}$$

The difference between (24) and equation (23) above is due to the fact that households own both domestic as well as foreign stocks, which implies that $b_{H,t+1}$ depends on both the domestic as well as the foreign displacement shocks u_H and u_F. That said, the relation between b and u depends on the state of the economy, specifically, the relative wealth of the two countries, as captured by λ. For instance, when λ is high then country F is richer than country H. In this case, a small u_H shock will likely lead to a larger change in b_H than would be the case if country H were richer than F—since the new trees created in country H constitute a large share of wealth relative to the wealth of H households.

Overall, these movements in the stochastic discount factors of the home and foreign country in response to the displacement shocks u_t^c have direct implications for exchange rates, which we explore next.

3.4 Exchange Rates

We next characterize the behavior of exchange rates in the model. Because financial markets are integrated between the two countries, absence of arbitrage implies that the value of the exchange rate-is equal to the ratio of the two countries stochastic discount factors,

$$e_t = \frac{M_{t+1}^H}{M_{t+1}^F}. \tag{26}$$

The change in the exchange rate (in logs) can be written as

$$\Delta \log e_{t+1} = \Delta \log C_{t+1}^F - \Delta \log C_{t+1}^H + \log b_{F,t+1} - \log b_{H,t+1}. \tag{27}$$

Equation (27) summarizes the main result in this paper. In the case of log utility, if markets were complete, λ would a constant. In that case, bilateral exchange rate movements are purely determined by movements in the relative consumption growth between the home and foreign country. More generally, the ratio λ_t could vary over time, but its movements would still be determined by movements in relative consumption growth (either in the short run or in the long run). As a result, these models imply that exchange rates are *counter-cyclical*: an economic boom in the home country (an increase in X_t and thus, due to home-bias, C_t^H) leads to a decline in e, that is , a *depreciation* of the home currency relative to the foreign currency.

By contrast, in our model, there is an additional factor in play that arises due to market incompleteness: displacement risk, which is captured by $b_{H,t+1}$ and $b_{F,t+1}$. To obtain some intuition, we can approximate the evolution of λ_t around its long-run mean using a first-order Taylor expansion,

$$\log \frac{\lambda_{t+1}}{\lambda_t} = \log \frac{b_{H,t+1}}{b_{F,t+1}} \approx u_{t+1}^F - u_{t+1}^H. \tag{28}$$

Consistent with the discussion above, the wealth share λ_t varies over time as a result of incomplete markets and the displacement shock. A positive realization of u_{t+1}^F implies that a measure-zero of households in the foreign country received claims to new firms. Due to the limited risk-sharing, these households were not able to share these claims with the other households—either in the foreign or the domestic country. As a result, the relative wealth of the foreign country rises. See Section A.5 of the Appendix for more details on the derivation of (28).

As a result, the log growth rate of exchange rate can be approximated as

$$\begin{aligned}\Delta e_{t+1} &\approx \Delta c^F_{t+1} - \Delta c^H_{t+1} + u^H_{t+1} - u^F_{t+1} \\ &\approx \underbrace{(2\alpha - 1)(1-\delta)}_{>0}(u^H_{t+1} - u^F_{t+1}) + (1-2\alpha)(\varepsilon^H_{t+1} - \varepsilon^F_{t+1}).\end{aligned} \tag{29}$$

Consistent with the discussion so far, a positive displacement shock u^H_{t+1} will lead to an appreciation of the exchange rate, while a positive 'neutral' shock ε^H_{t+1} will cause the exchange rate to depreciate. Since country output and consumption depend on both shocks, exchange rates in the model can be either positively or negatively correlated with consumption or output growth.

To see how the model can generate pro-cyclical exchange rates, consider the log growth in the relative country output,

$$\Delta x_{t+1} - \Delta y_{t+1} = \delta(u^H_{t+1} - u^F_{t+1}) + \varepsilon^H - \varepsilon^F \tag{30}$$

which is increasing in both u^H_{t+1} and ε^H_{t+1}. Similarly, the growth in relative consumption can be written as

$$\Delta c^H_{t+1} - \Delta c^F_{t+1} \approx (1-2\alpha)(1+\delta-2\alpha)(u^H_{t+1} - u^F_{t+1}) + (2\alpha-1)(\varepsilon^H - \varepsilon^F). \tag{31}$$

Importantly, assuming that

$$\delta < 2\alpha - 1 \tag{32}$$

implies that aggregate consumption growth in the home country is positively correlated with the displacive shock in that country, u^H_{t+1}.

Examining equations (29) and (30), we can see that the presence of the neutral shock ε tends to make exchange rates counter-cyclical, just like the standard model. By contrast, as long as (32) holds, the displacement shock u leads to positive co-movement between exchange rates, aggregate output and consumption. Thus, the unconditional correlation between exchange rates, country output and consumption depends on model parameters, for instance, the relative variance of the two aggregate shocks.

3.5 The Stock Market

The previous section illustrates that the model can generate a pro-cyclical exchange rate. But if that is the case, can the model also simultaneously generate a negative correlation between a country's exchange rate and its local stock market? The answer is that it can, and the reason again is due to the disconnect between the value of the stock market, that is, claims to existing firms, and aggregate consumption growth.

In particular, consider the value of existing trees in each country (the stock market) in country $c \in \{H, F\}$,

$$S_t^H = p_{x,t}X_t + \mathrm{E}_t[M_{t,t+1}^H(S_{t+1}^H e^{-u_{t+1}^H})] = p_{x,t}X_t(1 + pd_t^H) \tag{33}$$

$$S_t^F = p_{y,t}Y_t + \mathrm{E}_t[M_{t,t+1}^F(S_{t+1}^F e^{-u_{t+1}^F})] = p_{y,t}Y_t(1 + pd_t^F) \tag{34}$$

Take home country for example, the log return of holding the market portfolio is

$$\begin{aligned} r_{t+1}^H &= \log(\frac{X_{t+1}e^{-u_{t+1}^H}}{X_t}\frac{1 + pd_{t+1}^H}{pd_t^H}) \\ &= \mu + (\delta - 1)u_{t+1}^H + \varepsilon_{t+1}^H + \log(\frac{1 + pd_{t+1}^H}{pd_t^H}) \end{aligned} \tag{35}$$

(35) highlights an important feature of our model: the distinction between aggregate dividend grwoth X_t and the growth of dividends that accruing to the stock market portfolio. The reason for this distinction is that aggregate dividends do not constitute the gains from holding the stock market: investing in the stock market at time t only generate $X_{t+1}e^{-u_{t+1}^H}$ dividends at $t + 1$. A positive displacement shock increases the aggregate dividends by introducing new firms, but also dilutes the shares of the existing firms. On the other hand, following a positive displacement shock the price-dividend ratio also decreases. As a result, a positive displacement shock leads to a decline in the stock market returns.

3.6 Exchange Rates and the Growth of Top Incomes

The presence of the displacement shock u captures the idea that the benefits of economics growth are not shared equally. In the model, u captures the reallocation (creative destruction) that occurs between owners of existing trees and those who create new firms (entrepreneurs). Specifically, each period, a measure zero of the population receives profitable new firms. If we were to treat this transfer as capital income, fluctuations in u would translate into fluctuations into income inequality in the model.

Here, we develop this idea further and connect the displacive shock u in the model to an observable quantity, the top 1% share of income. In particular, the top 1% income consist of two groups of households.

The first group is the households that receive new firms in the current period. The total capital gains from new firms in home country, as a fraction of total income is

$$I_{H,t}^{capital} = \frac{S_t^H(1-e^{-u_t^H})}{W_{H,t}\xi + (p_{x,t}X_t e^{-u_t^H} + p_{y,t}Y_t e^{-u_t^F})\frac{W'_{H,t}}{W'_{H,t}+W'_{F,t}} + S_t^H(1-e^{-u_t^H})}. \tag{36}$$

Where $W'_{H,t}$ and $W'_{F,t}$ are the total wealth of the two countries excluding new projects, that is

$$W'_{H,t} = W_{H,t} - (1-e^{-u_t^H})S_t^H$$
$$W'_{F,t} = W_{F,t} - (1-e^{-u_t^F})S_t^F$$

We can see that the size of u shock at time t determines the amount of wealth that is transferred from existing firms to the new firms. The value of these new firms constitutes a capital gain for the successful entrepreneurs, and they are randomly distributed to a small part of the population. Hence, some of it is part of the income share of the top 1%.

The second group is the households who have had received projects in the past and consequently earn a large capital income on those wealth. These households derive capital income equal to

$$W_{H,t}\xi + (p_{x,t}X_t e^{-u_t^H} + p_{y,t}Y_t e^{-u_t^F})\frac{W'_{H,t}}{W'_{H,t}+W'_{F,t}}. \tag{37}$$

These capital gains and annuity income are proportionally distributed to all the population. Therefore, the top income inequality is a function of both the current displacement shock and the current wealth inequality. The wealth inequality, in turn, is a function of past displacement shocks.

The above discussion illustrates how the joint dynamics of income inequality and exchange rates can inform us about the quantitative impact of displacement shock. In particular, recall equation (27), which states that exchange rate growth is determined by relative consumption growth and changes in the wealth share of households that are displaced in each country b_H and b_F, which are primarily driven by the displacive shock u_H and u_F, respectively. To the extent that income inequality is a useful proxy for the u shock in the model, the correlation between exchange rates and income inequality would be revealing of the importance of the displacive shock u as a driver of exchange rates.

To explore this idea further, we estimate the following specification,

$$\log e_{F,t+1} - \log e_{F,t} = \alpha + \beta_{ineq}\left(\log \frac{Q_{t+1}^{US}}{Q_t^{US}} - \log \frac{Q_{t+1}^{F}}{Q_t^{F}}\right) + \beta_x\left(\log \frac{x_{t+1}^{US}}{x_t^{US}} - \log \frac{x_{t+1}^{F}}{x_t^{F}}\right) + \log e_{F,t} + \varepsilon_{F,t}. \tag{38}$$

Here, Q_{US}, Q_F are inequality measures in the United states and country F, respectively. As our baseline case, we take Q to refer to the income share of the top 1%. We estimate equation (38) and Table 5 presents the results for the panel regression (38) together with country-by-country estimates.

Focusing on the panel regression in the top row, we see that the estimated coefficient β is positive and statistically significant in case with consumption growth. That is, increases in income inequality in the foreign country are associated with an appreciation of its currency relative to the US. In terms of magnitudes, a one-standard deviation increase in income inequality in the foreign country is associated with a 0.014 log point appreciation of its currency relative to the US dollar. Similarly, a one-standard increase in consumption growth in the foreign country is associated with a 0.014 log point appreciation of its currency against the dollar. Examining the country-level regressions, we note a consistent pattern. The individually estimated β_{ineq} coefficients are in general positive (8 out of 11) though not always statistically significant (2 out of 11).

We examine the robustness of these findings using alternative measures of inequality and estimating equation (38) without consumption term. Appendix Tables A.2 show these results are also similar if we estimate the univariate regression. Appendix Table A.3 shows that we obtain similar findings if we measure inequality as the top 0.1% share of income.

In brief, the correlation between income inequality and exchange rates is comparable in magnitude to the correlation between exchange rates and consumption growth. Here, we note that even after controlling for income inequality, the estimated coefficient for consumption is positive and statistically significant in the panel regression. This is not particularly surprising: even under the null of the model, income inequality is likely to be a noisy proxy for the displacive shock u as it is affected by other quantities as equation (36) shows. Nevertheless, when calibrating the model, we will take these positive correlations into account: we will include the estimated slope coefficients in (38), specifically β_{ineq} and the slope coefficient on consumption growth β_c in our calibration targets.

4 The Full Model

So far, we have presented a stylized model that allows us to highlight the key mechanism in the paper. Though transparent, however, the model is not rich enough to quantitatively capture all the interesting aspects of the data. Here, we introduce several additional features and aim for a full quantitative exploration of the mechanism.

4.1 Setup

To conserve space, we only highlight the differences with the simpler model in the previous section.

4.1.1 Agents' Preferences and Demographics

We make three changes relative to the previous setup.

First, we introduce finite lives. This helps ensure that the level of inequality in each country remains stationary. For simplicity, the size of population in each country is normalized to one. At each date a mass ξ of agents, chosen randomly, die, and a mass of ξ of agents are born, so that the population remains constant. There is an annuity market so that households who do not die receives ξ of their wealth from the annuity. For the wealth of people who dies at period t, $1-\xi$ fraction will be used to finance the annuity within the country, and the remaining ξ fraction will be distributed uniformly to the new borne agents. This way, we don't need to keep track of the time at which the agents are born.

Second, we modify household preferences. Agents have non-time separable preferences; in addition, they care about both their own absolute level of consumption but also their consumption relative to an index. In particular, households' continuation utility at time t is given by

$$U_{i,t}^{c} = \left[(1-\beta)(\hat{C}_{i,t}^{c})^{1-\frac{1}{\psi}} + \beta\,\mathrm{E}_t\left[(U_{i,t+1}^{c})^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}. \tag{39}$$

That is, households have recursive preferences of the Epstein-Zin form. The parameters γ and ψ measure the relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS), respectively. The coefficient β is the effective time-preference parameter, which also incorporates the probability of death, that is, $\beta = \tilde{\beta}(1-\xi)$ where ξ is the probability of death and $\tilde{\beta}$ is the households' subjective time discount factor.

In addition, $\hat{C}_{i,t}^{c}$ refers to a composite good that depends both on the households' own consumption $C_{i,t}^c$ but also its level relative to the aggregate $\bar{C}_t^c$ in their country,

$$\hat{C}_{i,t}^c = \left(C_{i,t}^c\right)^h \left(\frac{C_{i,t}^c}{\bar{C}_t^c}\right)^{1-h}. \tag{40}$$

Here, $C_{i,t}^c$ is the agent i's own consumption bundle in country $c \in \{H, F\}$—defined in (14)—which is comprised of both home and foreign goods. The parameter h denotes the strength of the relative preference effect. When $h = 1$, these preferences specialize to the standard Epstein-Zin preferences. In general, for $h \in [0, 1]$ agents place a weight h on their own consumption and a weight $1 - h$ on their consumption relative to average consumption in country $c \in \{H, F\}$.

Households can hedge their mortality risk using a competitive annuity market. Households are risk averse, hence they all purchase annuities. The annuity issuer collects the wealth of deceased households ξW and distributes the proceeds to the surviving population and the newly born agents.

Finally, we relax the assumption of extreme inequality, by assuming that the measure of population that receives the value of new firms is non-negligible, that is, $\pi > 0$. Though this modification makes the model significantly less tractable, it helps the model match the observed patterns of inequality in the data.

4.1.2 Aggregate Output

The evolution of aggregate output in each country is still given by equations (11) and (12). We next make distributional assumptions about these shocks.

First, we allow for the displacement shocks in each country to be correlated, possibly due to technology spillovers. That is, the effective displacement shock in each country u is a weighted average of each country's idiosyncratic displacement shock $\bar{u}$,

$$\begin{aligned} u_{t+1}^H &= (1 - \rho_u)\, \bar{u}_{t+1}^H + \rho_u\, \bar{u}_{t+1}^F \\ u_{t+1}^F &= (1 - \rho_u)\, \bar{u}_{t+1}^F + \rho_u\, \bar{u}_{t+1}^H. \end{aligned}$$

The idiosyncratic displacement shocks in each country $\bar{u}_t^c, c \in \{H, F\}$ follow a Markov chain with three states $[u_1, u_2, u_3]$ and transition matrix given by

$$T = \begin{bmatrix} \nu_{1,1} & \nu_{1,2} & \nu_{1,3} \\ \nu_{2,1} & \nu_{2,2} & \nu_{2,3} \\ \nu_{3,1} & \nu_{3,2} & \nu_{3,3} \end{bmatrix}, \qquad \sum_{j=1}^{3} \nu_{i,j} = 1. \tag{41}$$

Second, we assume that the 'neutral' shocks are i.i.d. and jointly normally distributed $[\varepsilon^h, \varepsilon^f] \in N(0, \Sigma)$, where

$$\Sigma = \begin{bmatrix} \sigma_e^2 & \rho_e \sigma_e^2 \\ \rho_e \sigma_e^2 & \sigma_e^2 \end{bmatrix} \tag{42}$$

$\rho_e > 0$ is the correlation between the neutral shocks between two countries.

4.2 Equilibrium

The equilibrium in the full model is largely similar to the one in the simplified model, even as the algebra is somewhat more involved.

Given our assumptions, the stochastic discount factor in country c is given by (see Appendix B.3 for derivation)

$$\frac{M_{t+1}^c}{M_t^c} = \beta \left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}} \right)^{-\frac{h}{\psi}+h-1} \tilde{b}_{c,t+1} \left(\frac{U_{c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{43}$$

where

$$\tilde{b}_{c,t+1} = \pi \left(\frac{b_{c,t+1}\pi + 1 - b_{c,t+1}}{\pi} \right)^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}} + (1-\pi) b_{c,t+1}^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}} \tag{44}$$

As before, exchange rates are equal to the ratio of stochastic discount factors. We have

$$\begin{aligned} \Delta \log e_{t+1} &= \Delta \log M_{t+1}^H - \Delta \log M_{t+1}^F \\ &= \left(\frac{h}{\psi} + 1 - h \right) \left(\Delta \log C_{t+1}^F - \Delta \log C_{t+1}^H \right) + \left(\log(\tilde{b}_{F,t+1}) - \log(\tilde{b}_{H,t+1}) \right) \\ &\quad + \frac{1/\psi - \gamma}{1-\gamma} \left(\log \frac{U_{H,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{H,t+1}^{1-\gamma}]} - \log \frac{U_{F,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{F,t+1}^{1-\gamma}]} \right) \end{aligned} \tag{45}$$

Examining (45), we note the similarities with the log utility case—equation (27). That is, exchange rate dynamics are still driven by relative consumption growth in the two countries, as well as the relative degree of displacement in the current period ($b_{H,t+1}$ and $b_{F,t+1}$). The key difference with the time-separable case is that now the shocks to the future distribution of these variables matters, as encoded into households' continuation utility $U_{H,t+1}$ and $U_{F,t+1}$.

4.3 Estimation

In this section, we describe how we calibrate the model to the data. Given the degree of non-linearity in our model, solution methods that are based on log-linearizations around the steady state are not necessarily reliable. As such, we solve for the global solution of the model by discretizing the state-space and using a combination of value and policy function iteration. See Appendix C for a brief description of our numerical procedure.

To reduce the number of parameters, we make a number of simplifying restrictions on the dynamics of u shocks. First, we assume that $u_1 = u_2$. Hence, a transition from u_1 to u_2 only affects the future distribution of u (as the transition probabilities change) rather than the current level of displacement. Second, we assume that the matrix T corresponds to transition matrix of a discretized AR(1) process, so that it could be parameterized by only two parameters—the corresponding autocorrelation parameter p and q. Specifically, we assume that the transition matrix has the following form

$$T = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix} \tag{46}$$

Where p^2 is the probability of staying in the lowest state once already there and q^2 is the probability of staying in the highest state once there [5].

After restricting the evolution of u, the full model has a total of 16 parameters. We estimate the parameters of the model using an indirect inference method (Lee and Ingram, 1991). Specifically, given a vector of X of target statistics in the data, we obtain parameter estimates by

$$\hat{p} = \arg\min_{p \in \mathcal{P}} \left(X - \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p) \right)' W \left(X - \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p) \right) \tag{47}$$

Where $\hat{X}_i(p)$ is the vector of statistics computed in one simulation of the model.

The matrix W determines the importance of each statistic to the distance criterion to be minimized. In general, we choose to penalize proportional deviations of the model statistics from their empirical counterparts, so $W = diag(XX')^{-1} I_W$. The diagonal matrix I_W allows us to introduce some exceptions to this criterion based on the importance the existing literature

[5] Conversely, $(1-p)^2$ is the probability of transitioning from the lowest to the highest state and $(1-q)^2$ is the probability of transiting from the highest to the lowest. When $p \neq q$, there is conditional heteroscedasticity in the shocks. For the case when $p = q$, the discrete process has the first-order persistence as q.

places on matching certain features of the data—but also moments that are revealing of our model mechanism. As such, we apply a factor of 10 on the UIP coefficient and the volatility of exchange rate. The remaining elements on the diagonal of I_W are normalized to one.

Our estimation targets are reported in the first column of Table 6. They include a combination of first and second moments of aggregate quantities, asset prices and exchange rates. In additional to these standard international moments in the literature, we also target a set of correlations between exchange rate and real variables. The neutral shock and displacement shock have different implications for the cyclicality of the exchange rates. Thus, the set of correlation between exchanges rates and consumption, output and stock market, together with the set of bilateral correlations, are informative about the relative magnitude of these two shocks. In addition, we target the average top 1% income inequality of the United States and the estimated coefficients of bi-variate regressions (38) and (3). In the model, we consider the stock market as a levered claim of domestic consumption goods by a factor of 2. See Appendix D or more details on the construction of the target moments.

4.4 Model Fit

Table 6 shows that the baseline model fits data reasonably well. Our model reproduces the realist patterns of both aggregate consumption and output growth. On the asset pricing side, the model generates the realistic levels of equity risk premium and volatility of the stock market. The volatility of the realized interest rate in the data is more volatile than the simulated data, but this may be largely driven by the high inflation around 1980s.

On the international side, our model successfully replicates the three key anomalies in the literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of the UIP. Specifically, the model generates pro-cyclical exchange rates – the displacement shock produces co-movement between exchange rate and aggregate quantities. This is because the u shock is not only positively correlated to the aggregate consumption and output, but also carrying a negative risk premium due to imperfect risk sharing. The key to the replication of the UIP anomaly is the time-varying volatility—more precisely, the time-varying distribution of the effective size of the u-shock—that endogenously arises in the equilibrium. Despite the fact that consumption, output and stock market are highly correlated, the exchanges rate in our model is as volatile as in the data due to a high level of home-bias. Finally, net exports in our model are counter-cyclical, as in the data.

In addition, the replication of international puzzles does not require a unrealistic magnitude of technology shocks. In fact, our model generates a realistic level of income inequality. Given that most of the dynamics in our model are driven by the displacement shock, whose magnitude is directly linked to the observed income inequality, this result is reassuring. Further, our model reproduces the two positive estimated coefficients of the bi-variate regression of exchange rates growth on consumption and inequality growth. That is, the consumption series in the data should not absorb all the impact of displacement shocks on the exchange rates – a feature in the data that our model can also replicate.

4.5 Parameter Estimates

Table 7 reports the parameter estimates of the model. Examining the set of parameter estimates, there are several points worth making. In terms of preference parameters, the model calibration requires a reasonable set of preference parameters: degree of relative risk aversion (6.5) and the elasticity of inter-temporal substitution (1.8). That said, the standard errors in both parameters are quite high, which implies that the model solution is not particularly sensitive to these parameters. In addition, the model requires a very high level of home bias (0.990), similar to Colacito and Croce (2013), in order to generate volatile exchange rates. In addition, the preference weight on relative consumption is rather high (0.82) though again, the relatively high standard error implies the model solution is not very sensitive to this particular value. Second, in terms of the distribution of shocks, we see that the calibration requires a highly persistent ($p = 0.93, q = 0.83$) and right-skewed displacement shock to fit the data.

To get a deeper understanding of how these parameters are identified from the data, we also compute the Gentzkow and Shapiro (2014) measure of elasticity of parameters to moments. To conserve space, we only briefly discuss these results here, and relegate the full set of results to Appendix Figures 6 to 11.

In terms of technology shock, the mean μ and volatility σ_e of the neutral shock is identified by the first two moments of consumption and output growth. The distribution of the displacive shock u is primarily identified by the level of top 1% income share (since it directly affects the average top 1% income share in the model), as well as the volatility of exchange rate and the stock market (since the spread between u_1 and u_3 affects the volatility of the SDF in the model). The parameter governing the importance of the displacement shock to output δ is also primarily identified by the correlation between output growth and exchange rate and the volatility of exchange rates, since it determines the joint dynamics of the SDF and output growth. The two parameters governing the correlation between the home/foreign shocks (ρ_e and ρ_u) are primarily identified by the correlation of home and foreign consumption growth, output, and the stock market. Last, the persistence of the displacive shock u is primarily identified by the equity premium (since the u shock is a key driver of stock returns) and the correlation between inequality and exchange rates.

In terms of preference parameters, the degree of home bias α is identified primarily by the volatility of the exchange rate and its correlation with output growth. The coefficient of relative risk aversion γ is identified from the mean and volatility of stock returns. The subjective discount factor β and the probability of death ξ are jointly mainly identified by the mean of the risk-free rate and the level of inequality. In general, these two parameters play a similar role in most model quantities, with the exception of inequality: higher β implies a higher price-dividend ratio and therefore a lower share of top income from "accumulated wealthy" people; by contrast, higher death rate ξ implies less concentration of wealth and dividend income which lowers income inequality. As they generate somewhat different implications for the relation between top income share and u shock, the coefficient on inequality helps determines these two parameters. The weight on own h consumption is primarily identified by the volatility of exchange rate and the correlation between net export growth and consumption growth: as h falls, households place a higher weight on relative consumption and thus place higher importance on the displacive shock which is the primary driver of the volatility of SDF and a counter-cyclical trade surplus. The elasticity of intertemporal substitution (EIS) affects the volatility of interest rates and hence is primarily identified by the volatility of excess returns.

5 Model Implications

Here, we examine the model's implications. First, we focus on the key mechanisms in the model, that is, how the key quantities in the model respond to the two exogenous shocks u and ε. Second, we examine the forces that allow the model can replicate some of the stylized facts in the literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of UIP.

5.1 Model Mechanism

Figures 2 present the response of key model quantities to the two shocks in the model: the neutral shock ε (Panel A) and the displacement shock u (Panel B). For brevity, we examine responses to shocks in the home country only; shocks to the foreign country are exactly symmetric.

5.1.1 Quantities

First, consider Figure 2. The first two columns shows the response of the exchange rate and consumption growth to the two shocks. As we can see, a positive ε shock in the home country leads to a depreciation of the currency and an increase in consumption growth. This is the standard shock in most models and the reason why exchange rates are counter-cyclical. By contrast, a positive u shock leads to an appreciation of the exchange rate as well as an increase in consumption growth.

The next two columns of Figure 2 illustrate why the exchange rate appreciates in response to a positive u shock. Recall equation (45) in the full model. Columns three and four of the Figure 2 illustrate how the last two terms of the equation respond to the shocks in the model. Specifically, an increase in u_H leads to a decline in the wealth share of the owners of incumbent firms in the home country b_H and therefore to an appreciation of the exchange rate. Similarly, the first column shows that an increase in u_H leads to a decline in the continuation utility of households in the home country U_H, which also contributes to the appreciation of the home currency. Put differently, both of these latter forces lead to an increase in the stochastic discount factor in the home country, as can be seen from equation (43), which causes the home currency to appreciate.

Last, this figure illustrates why exchange rates are in general counter-cyclical even in models with recursive preferences (e.g., Colacito and Croce, 2013). As we can see in the top right panel, a positive shock to ε leads to an increase in households' continuation utility, which contributes to the home currency depreciation. Though the neutral shock ε is i.i.d. in the model, this result is much more general: any shock which increases households' continuation utility will lead to a depreciation of the currency. Persistent shocks to consumption growth (long-run risk) fall into this category.

5.1.2 Financial Assets

Next, we examine the impact of two shocks on financial assets. Figure 3 plots impulse responses for log-SDF, stock market return r_{ex}, risk-free rate r_f and volatility of log-SDF for both countries. The first column shows that the neutral shock and the displacement shock have an opposite effect on the growth of log-SDF: a positive displacement (neutral) shock leads to an increase (decrease) of the log-SDF growth. This means that the displacement shock u has a negative risk premium while the neutral shock carries a positive risk premium.

Consistent with the analyses above, the difference in how the SDF responds to two shocks stems primarily from how the benefits of technological progress are shared among households.

Both shocks u and ε lead to an increase in the aggregate output, which causes SDF to fall. However, in case of displacement shock, the fall in consumption and continuation utility due to unequal sharing of technological progress is sufficiently large to offset the benefits of higher aggregate consumption.

The third column depicts the response of stock market and highlights an important feature of our model: the difference between aggregate dividend growth X_t and the growth of dividends accruing to the investment in the stock market. The reason for this difference is that aggregate dividends do not constitute the gains from holding the stock market: investing in the stock market at time t only generate $X_{t+1}e^{-u^H_{t+1}}$ dividends at time $t+1$. A positive displacement shock increases the aggregate dividends by introducing new firms, but also dilutes the shares of the existing firms. As a result, a positive displacement shock leads to a decline in the stock market returns.

5.1.3 Trade and Capital Flows

Finally, we examine the impact of two shocks on aggregate output and international flows. Figure 4 plots impulse responses for consumption share λ_t, wealth share w_t, output growth $\Delta \log(X)$ and $\Delta \log(Y)$, net export scaled by output and net international investment position scaled by country's wealth. The second column shows that both the neutral shock and displacement shock contribute to an increase in the aggregate output.

In the model, the net export as a fraction of total output is

$$\frac{NX_t^H}{X_t} = \frac{p_{x,t}X_t - p_{x,t}x_t^H - p_{y,t}y_t^H}{p_{x,t}X_t} = 1 - \frac{1}{\alpha + (1-\alpha)\lambda_t} \tag{48}$$

$$\frac{NX_t^F}{Y_t} = \frac{p_{y,t}Y_t - p_{y,t}Y_t^F - p_{y,t}x_t^H}{p_{y,t}Y_t} = 1 - \frac{\lambda_t}{1-\alpha+\alpha\lambda_t} \tag{49}$$

And the net international investment position (NIIP) scaled by the country's wealth is

$$\frac{A_t^H}{W_t^H} = \frac{W_t^H - S_t^H}{W_t^H} \tag{50}$$

$$\frac{A_t^F}{W_t^F} = \frac{W_t^F - S_t^F}{W_t^F} \tag{51}$$

The third and the fourth column of Figure 4 show that the dynamics of the international flows are mostly driven by the displacement shocks.

Specifically, the third column shows that following a positive displacement shock, the net export declines and the country becomes an importer. We can see from (48) that the balance of trade is purely determined by λ. In the model, the large country is the net importer and the small country is the net exporter. As λ_t decreases, home country becomes wealthier and its households want to consume more. Therefore, home country exports less of domestic goods and imports more of the foreign goods. Home country's balance of trade deteriorates and home currency appreciates. Thus, the model is able to reproduce the counter-cyclical net export.

The fourth column shows that a positive displacement shock leads to capital inflows. Recall that each period, investors who hold the market portfolio needs to pay to acquire the new firms that enter the market. When the home country receives a larger displacement shock than the foreign country, there are more new firms in home country than the foreign. Households receiving these new firms (entrepreneurs) are motivated to sell their stakes to rebalance their portfolio. Part of these firms are acquired by foreigners who wish to rebalance their portfolio. The net result is that foreign demand for home assets increases relative to home demand for foreign assets, and therefore the home country experiences net capital inflows as its wealth increases. These inflows are associated with currency appreciation in the model, consistent with the evidence in Camanho et al. (2020); Hau and Rey (2006).

5.2 Implications for Exchange Rate 'Puzzles'

Here we examine the extent to which our model can resolve some of the existing puzzles in the literature.

5.2.1 Aggregate quantities and the Backus-Smith Puzzle

Given the analyses in the first part of Section 5.1, it directly follows that the model is able to generate a positive correlation between countries' differences in consumption growth and exchange rate growth, resolving the Backus-Smith anomaly. The replication of this correlation requires that the impact of displacement shock dominates that of neutral shock. The final quantitative impact of the displacement shock depends on the calibration of its displacement effect δ and households relative preference h, as well as the relative magnitude between two shocks.

5.2.2 The Forward Premium Anomaly

Uncovered interest rate parity (UIP) states that the expected change in exchange rates should be equal to the interest rate differential between two countries, and that the currency with lower interest rate tends to appreciate. Therefore, the regression coefficient of future exchange rates growth on interest rate differential should be equal to one. Empirically, the coefficient is much smaller than one and even negative. The violation of the UIP is often referred to as the forward premium puzzle. Fama (1984) and Backus, Foresi, and Telmer (2001) note that time-varying volatility of the SDFs is a necessary condition for the replication of this anomaly. We next show that in our model the failure of the UIP is an endogenous equilibrium outcome.

The bottom panels (Panel B) of Figures 2 through 4 shows the responses of key model quantities to a displacive shock (the economy moves from u_2 to u_3). Figure 5 shows the responses of exchange rate, log-SDF, risk-free rate and the volatility of log-SDF following a shock from u_1 to u_3 and a shock from u_1 to u_2.

Upon the realization of a positive displacement shock to the home country ($u_1 \to u_3, u_2 \to u_3$, or $u_1 \to u_2$), home currency appreciates and the total wealth of the home country increases relative to the foreign country. Recalling the discussion of the mean-reversion of λ, in the future the effective size of the u-shock in the foreign country is expected to be greater than that of the home country. As a result, the foreign currency is subsequently expected to appreciate (Figure 5, column 1).

Turning our attention to the risk-free rate, we see two forces in opposite directions. On the one hand, due to the difference in effective size of displacement shocks, foreign households expect a lower consumption growth than home households. Without the endogenous time-varying higher moments that arise in equilibrium, foreign interest rate will be lower than domestic interest rate and that the UIP coefficient would be exactly one.

However, home households face a higher level of uncertainty than foreign households. Column 3 of Figure 5 shows that following a positive shock, the volatility of domestic log-SDF will be higher than that of foreign in the following periods. Since X-good denominated assets are more valuable than that of the foreign country, home displacement shock has a larger impact on foreign households than the impact of foreign displacement shock on home households. Taking u shocks from both countries into consideration, foreign households' uncertainty about future displacement impact is smaller than that of home country. This leads to a lower interest rate at home country[6]. In sum, the time-varying λ_t gives rise to the time-varying conditional distribution of the effective size of future u shocks. This, in turn, implies a time-varying volatility of SDF, which weakens the UIP. Depending on which effect dominates, the home interest rate could be either lower or higher than the foreign interest rate.

In addition, when the model is close to the steady state, the relative size of two countries

[6]For example, the foreign households expect a larger but less volatile displacement shock, as u shock on home country also leads to a big displacement effect. In contrast, domestic households expect a small but skewed distribution of u–only a big displacement shock at home has sizable impact.

are similar. Thus, the second and higher moments variation of two countries' SDF dominates their first-moment difference. As a result, UIP fails to hold. When the model is far away from the steady state, the difference between the effective size of u shock–which drives the first moment of log-SDF – becomes so large that it outweighs the higher order variations. Consequently, UIP strengthens.

6 Conclusion

Overall, we provide a quantitative general equilibrium model that successfully replicates the joint dynamics of exchange rates, consumption growth, trade flows, and stock returns. A robust implication of the standard macro-finance model with a representative agent is counter-cyclical exchange rates. Our goal in this paper is to provide a model in which the exchange rates can potentially be pro-cyclical. We introduce a minimal deviation to the standard endowment economy model: in addition to the standard endowment shock in each country, countries can now each experience displacive shocks that reallocate output among agents. This minimal deviation from the standard model is sufficient to generate pro-cyclical exchange rates. Depending on the correlation between economic growth and the displacive shock, the correlation between aggregate consumption and output can be positive or negative.

Our calibrated model successfully replicates the first two moments of aggregate consumption and output growth, exchange rates, and stock returns while generating low and relatively smooth risk-free rates. Our model replicate the three key 'anomalies' in the exchange rate literature: the volatility puzzle of Brandt et al. (2006), the Backus-Smith correlation puzzle, and the violation of the uncovered interest rate parity (UIP). Key to replicating the failure of the UIP is the time-varying distribution of the displacement shock. In addition, the model also replicates the cyclical properties of trade flows: in both the model and in the data, net exports are counter-cyclical. Importantly, the model can simultaneously deliver a positive correlation between consumption (or output) growth and exchange rates and a negative correlation between exchange rates and stock market returns, an empirical pattern that is hard for existing models to replicate. In the model, a positive displacement shock is positively correlated with aggregate consumption but leads to a decline in the stock market. This feature, in addition to replicating the failure of the consumption CAPM (CCAPM) in the data, helps the model jointly match the dynamics of stock returns, exchange rates and consumption growth.

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Figures and Tables

The real dollar index and the US innovation

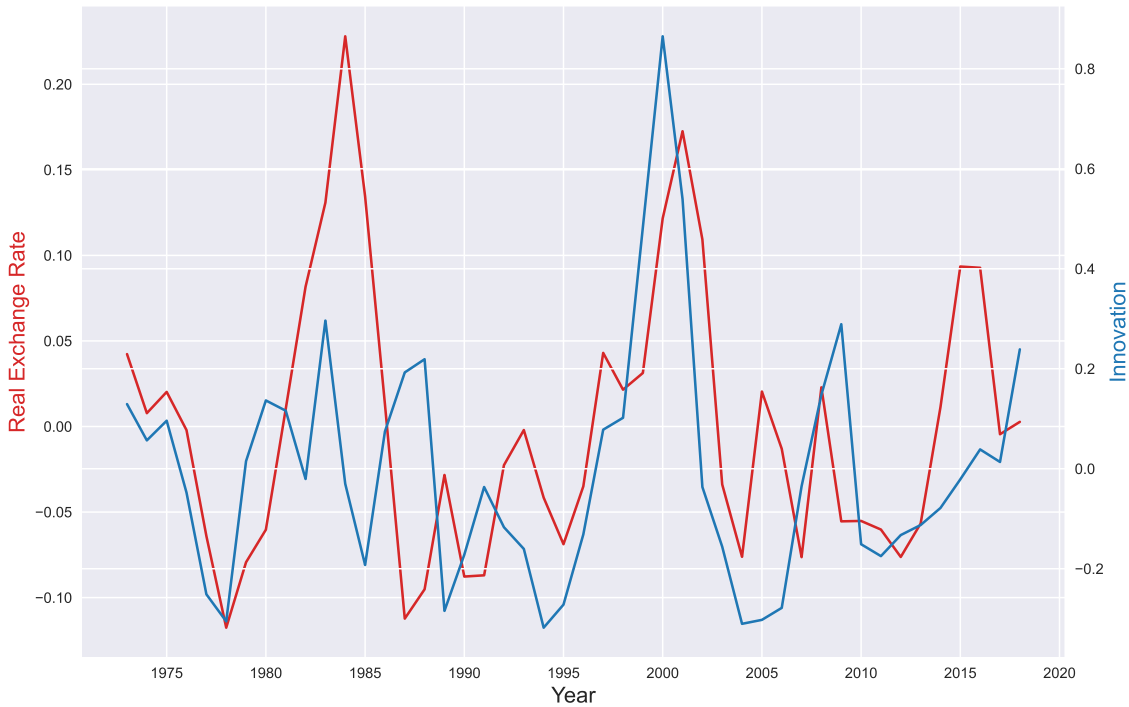


Figure 1: This figure plots the real dollar index and the US innovation. The dollar index in red is the traded-weighted real advanced foreign economy dollar (AFE) indexes, calculated by the Fed. The US innovation series in blue plots the average real value per patent each year (adjusted using CPI, in logs), using methodology in Kogan et al. (2017).

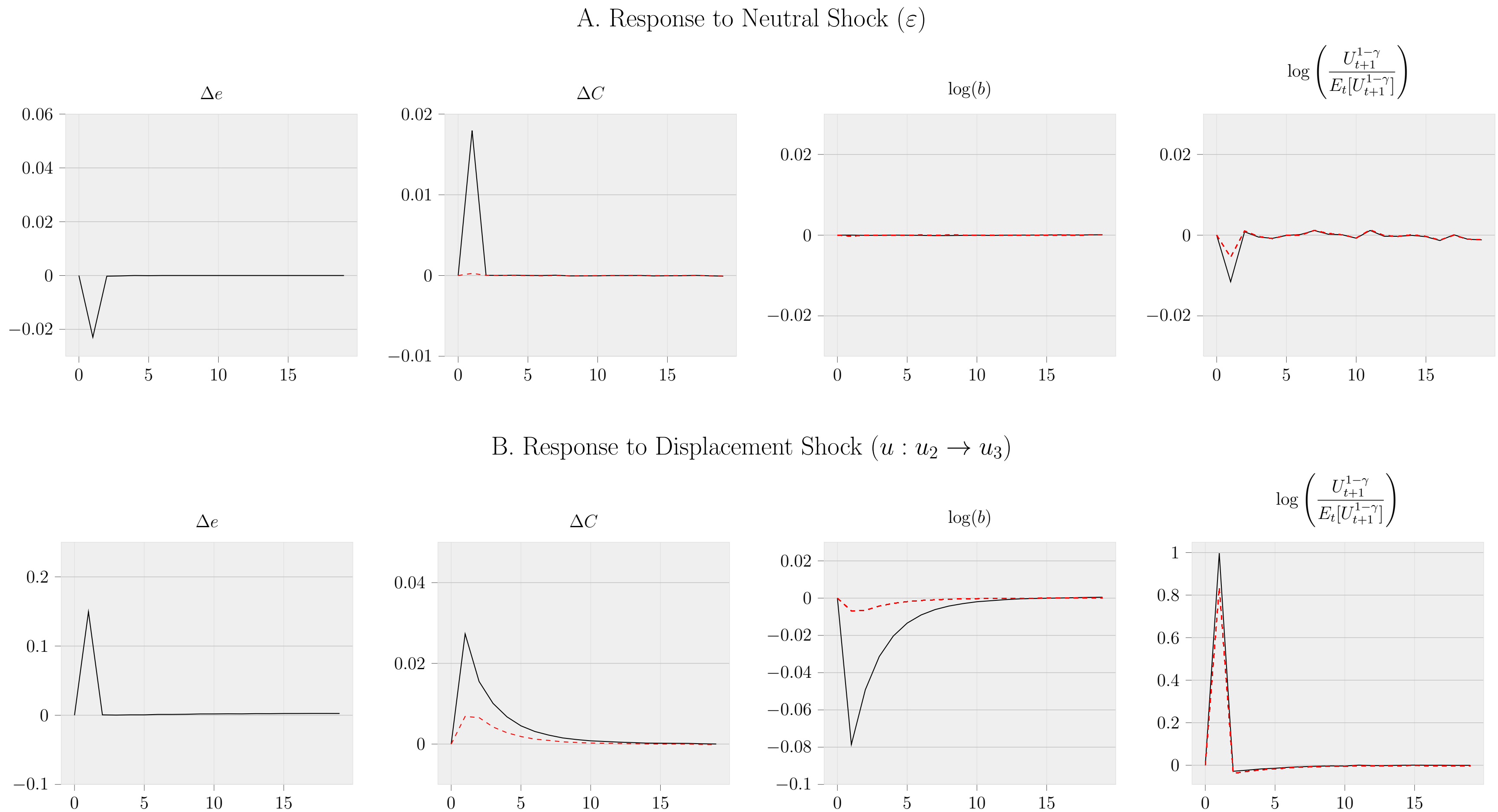


Figure 2: This figure plots the impulse response of variables to a shock to the home country (ε in Panel A and u in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 7. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse response are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

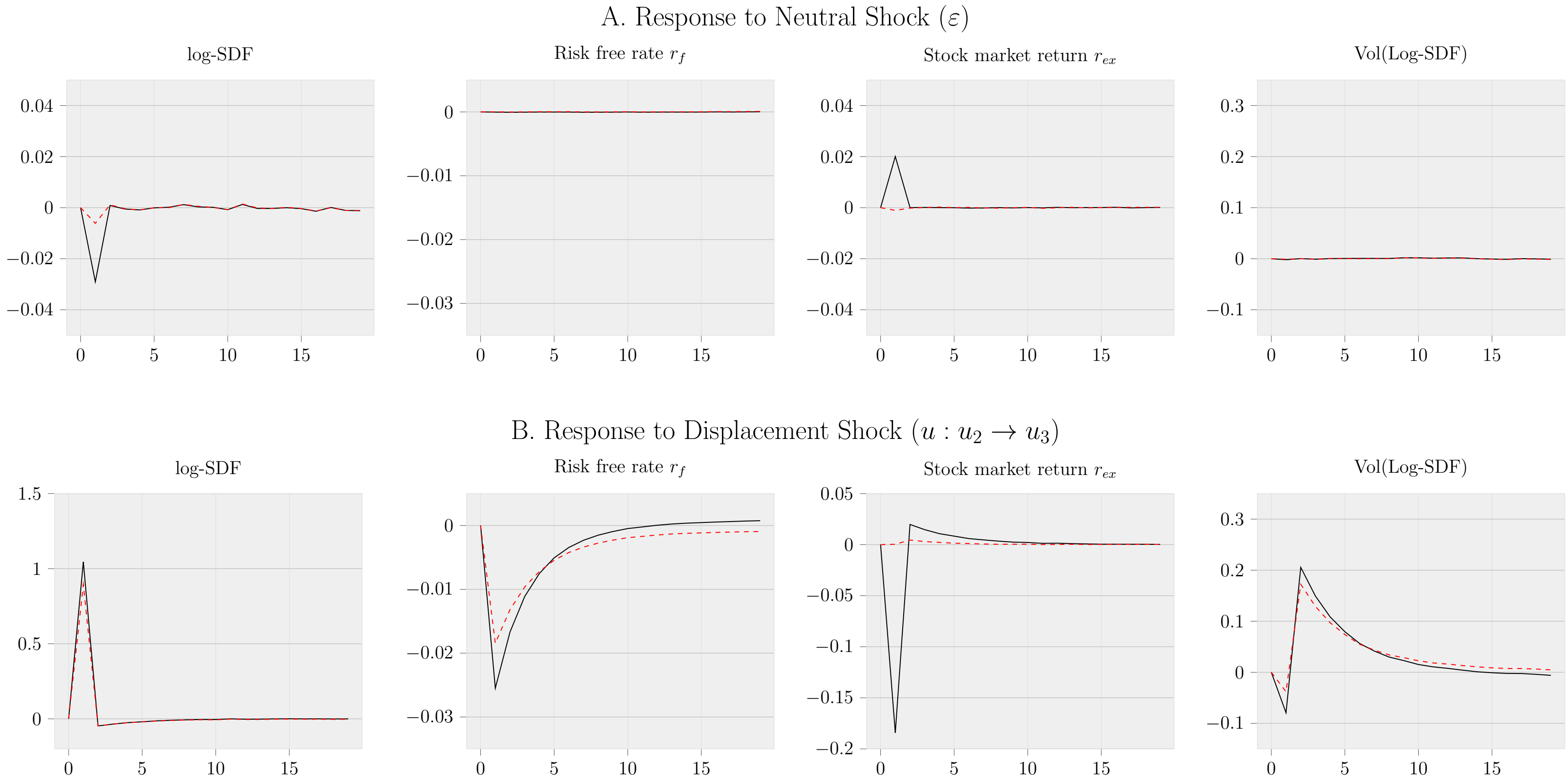


Figure 3: This figure plots the impulse response of variables to a shock to the home country (ε in Panel A and u in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 7. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse response are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

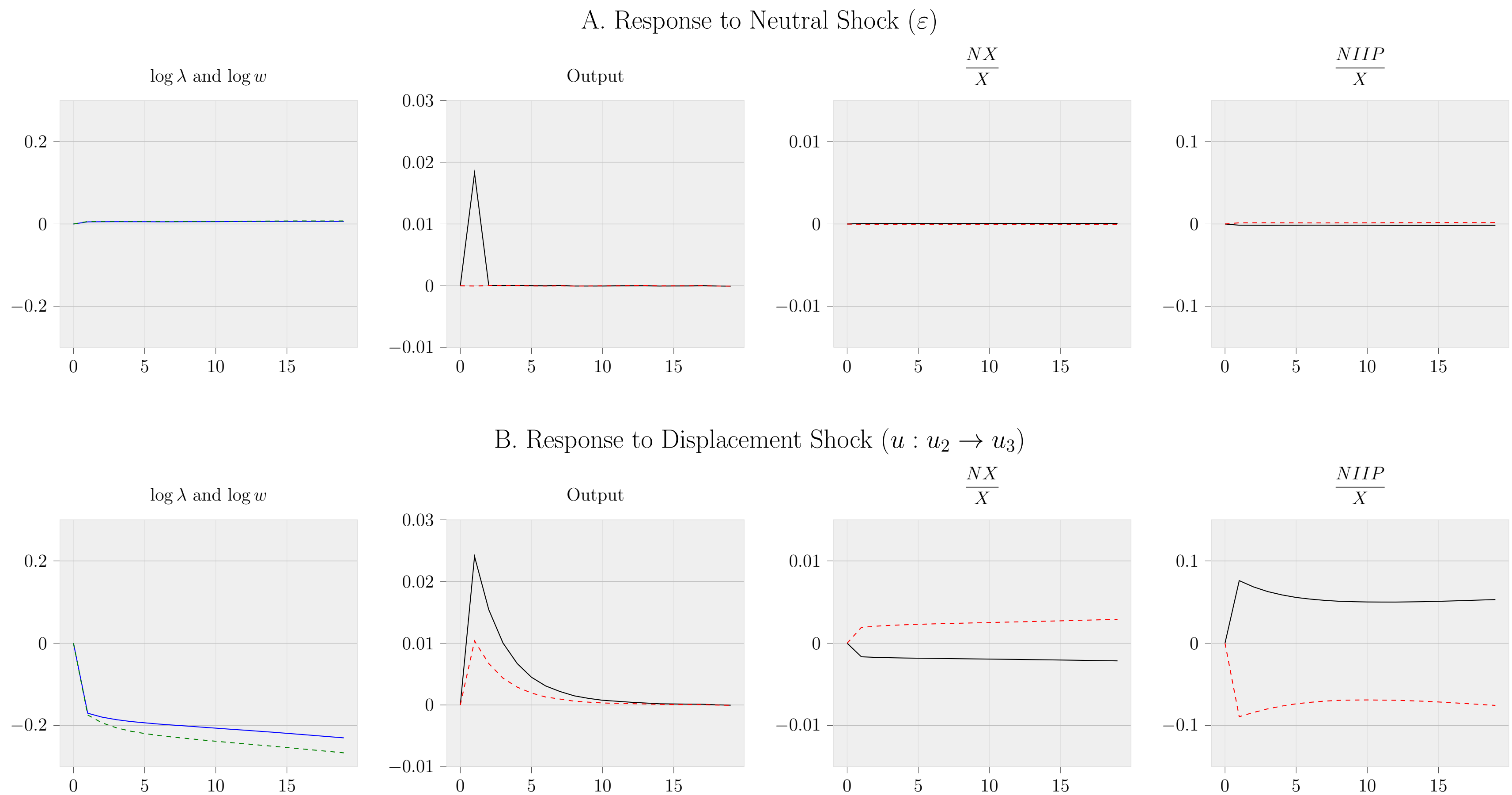


Figure 4: This figure plots the impulse response of variables to a shock to the home country (ε in Panel A and u in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 7. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse response are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

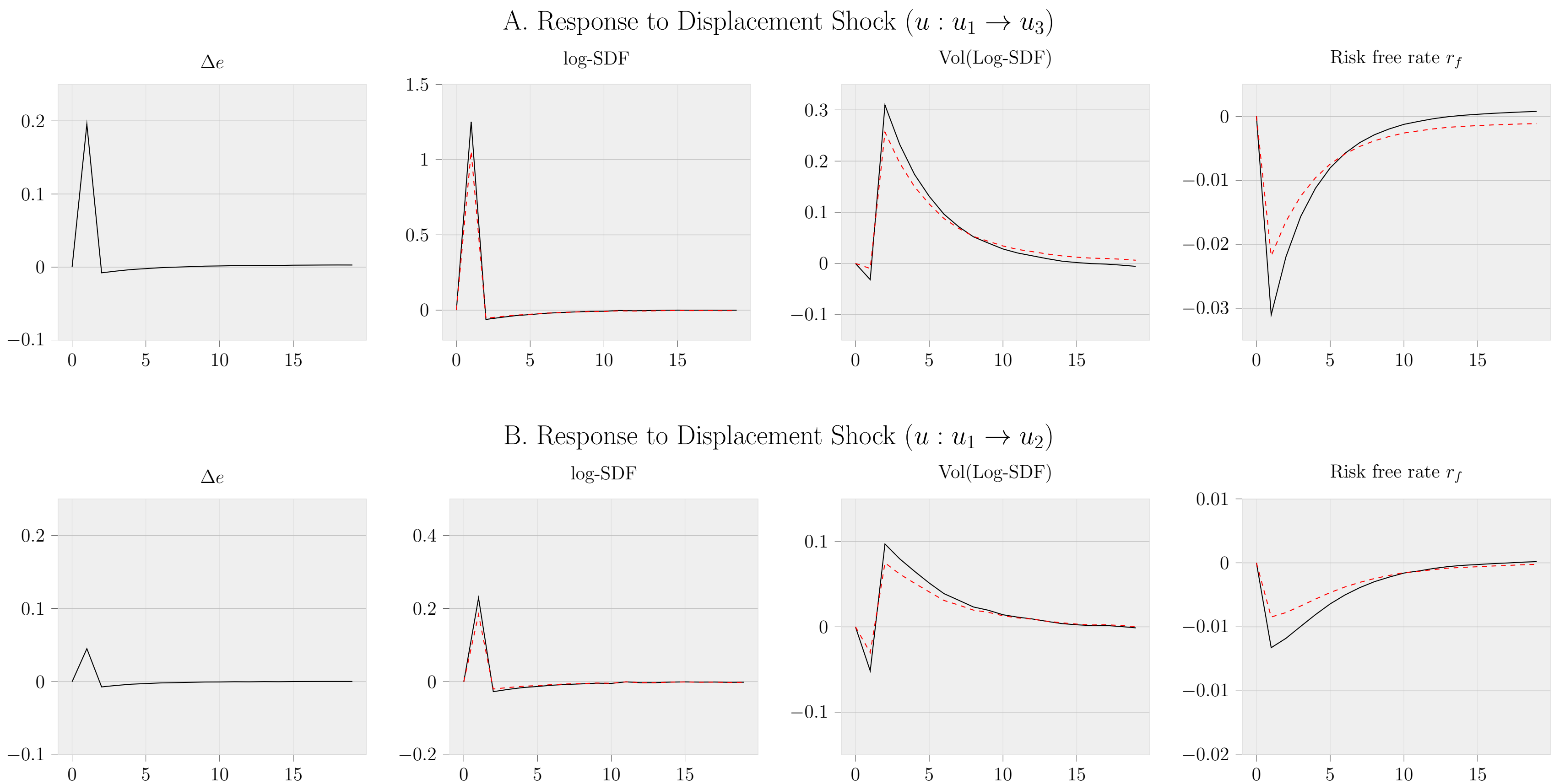


Figure 5: This figure plots the impulse response of variables to a shock to the home country ($u_1 \rightarrow u_3$ in Panel A and $u_1 \rightarrow u_3$ in Panel B), for both the home country (the solid line) and the foreign country (the dashed line). All parameters are calibrated to the values reported in Table 7. We construct the impulse responses by introducing an additional one-standard deviation shock at time t=1 without altering the realization of future shocks. The impulse response are computed at the symmetric steady state. Neutral shock is orthogonalized, i.e., ignoring the correlation when introducing the shock.

Table 1: Exchange rate growth and consumption growth

	Consumption growth	$R^2(\%)$	Observations
Panel	0.011**	14.85	476
	(0.004)		
Australia	0.000	12.46	49
	(0.014)		
Canada	0.021*	16.88	49
	(0.012)		
France	0.025	20.82	28
	(0.022)		
Germany	-0.004	17.75	28
	(0.016)		
Italy	0.010	15.05	28
	(0.017)		
Japan	-0.018	17.53	49
	(0.014)		
New Zealand	0.020	16.26	49
	(0.015)		
Norway	0.029*	21.00	49
	(0.016)		
Sweden	0.009	6.19	49
	(0.019)		
Switzerland	0.008	21.55	49
	(0.021)		
United Kingdom	0.005	19.62	49
	(0.01)		

Notes: The table reports regression results of the growth of log exchange rate on log consumption growth ratio:

$$\log e_{t+1} - \log e_t = \beta \Delta \log C_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 2: Exchange rate growth and gdp growth

	GDP growth	R^2(%)	Observations
Panel	0.011** (0.004)	14.86	476
Australia	-0.001 (0.011)	12.48	49
Canada	0.020** (0.008)	16.14	49
France	0.030 (0.018)	22.70	28
Germany	0.026 (0.018)	21.76	28
Italy	0.022 (0.021)	17.92	28
Japan	-0.011 (0.015)	16.20	49
New Zealand	0.015 (0.017)	15.21	49
Norway	0.009 (0.011)	15.18	49
Sweden	0.005 (0.017)	5.83	49
Switzerland	0.006 (0.021)	21.44	49
United Kingdom	0.026 (0.018)	24.03	49

Notes: The table reports regression results of the growth of log exchange rate on log gdp growth ratio.

$$\log e_{t+1} - \log e_t = \beta \Delta \log Y_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 3: Exchange rate growth and stock market returns

	Stock market returns	R^2(%)	Observations
Panel	-0.017**	15.01	460
	(0.007)		
Australia	-0.011	13.40	49
	(0.016)		
Canada	0.002	9.26	49
	(0.008)		
France	-0.020	19.39	28
	(0.028)		
Germany	-0.037*	26.19	28
	(0.019)		
Italy	-0.018	16.71	28
	(0.019)		
Japan	-0.011	16.08	49
	(0.016)		
New Zealand	-0.028	15.42	42
	(0.02)		
Norway	0.008	15.01	49
	(0.012)		
Sweden	-0.039***	15.54	49
	(0.014)		
Switzerland	-0.050***	32.36	40
	(0.014)		
United Kingdom	-0.024	23.38	49
	(0.022)		

Notes: The table reports regression results of the growth of log exchange rate on stock market returns.

$$\log e_{t+1} - \log e_t = \beta \Delta r_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. Stock market returns (MSCI Indexes) data are from Datastream. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 4: Consumption growth, stock market returns and exchange rate growth

	Consumption growth	Stock market returns	R^2(%)	Observations
Panel	0.010*	-0.016**	15.60	460
	(0.005)	(0.007)		
Australia	0.000	-0.011	13.40	49
	(0.014)	(0.016)		
Canada	0.021*	0.001	16.89	49
	(0.012)	(0.009)		
France	0.020	-0.012	21.67	28
	(0.025)	(0.03)		
Germany	-0.011	-0.039*	26.86	28
	(0.014)	(0.02)		
Italy	0.012	-0.019	17.72	28
	(0.015)	(0.019)		
Japan	-0.018	-0.010	18.17	49
	(0.014)	(0.017)		
New Zealand	0.033*	-0.023	21.75	42
	(0.019)	(0.022)		
Norway	0.034**	0.018	23.41	49
	(0.017)	(0.014)		
Sweden	0.016	-0.042***	17.25	49
	(0.016)	(0.012)		
Switzerland	-0.007	-0.051***	32.68	40
	(0.024)	(0.013)		
United Kingdom	0.002	-0.023	23.40	49
	(0.01)	(0.023)		

Notes: The table reports regression results of the growth of log exchange rate on both 1-year log consumption growth (first column) and stock market returns (second column)

$$\log e_{t+1} - \log e_t = \beta_1 \Delta \log C_{t,t+1} + \beta_2 \Delta r_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. Stock market returns (MSCI Indexes) data are from Datastream. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 5: Consumption growth, stock market returns and exchange rate growth

	Consumption growth	Inequality growth	$R^2(\%)$	Observations
Panel	0.014**	0.014**	16.23	406
	(0.005)	(0.006)		
Australia	0.000	0.049***	45.59	49
	(0.01)	(0.012)		
Canada	0.027*	-0.010	21.15	49
	(0.015)	(0.008)		
France	0.011	0.007	38.92	28
	(0.025)	(0.012)		
Germany	-0.004	-0.017	45.10	18
	(0.029)	(0.036)		
Italy	0.003	0.033	48.77	18
	(0.026)	(0.038)		
Japan	-0.036	0.023	15.67	39
	(0.027)	(0.018)		
New Zealand	0.016	0.014	19.41	49
	(0.017)	(0.017)		
Norway	0.036*	-0.000	22.47	39
	(0.021)	(0.02)		
Sweden	0.007	0.024	19.03	39
	(0.027)	(0.024)		
Switzerland	0.014	0.008	21.66	39
	(0.015)	(0.012)		
United Kingdom	0.025	0.033*	38.97	39
	(0.016)	(0.018)		

Notes: The table reports regression results of the growth of log exchange rate on both 1-year log consumption growth (first column) and growth of log top 1% income share ratio (second column)

$$\log e_{t+1} - \log e_t = \beta_1 \Delta \log C_{t,t+1} + \beta_2 \Delta I_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. Stock market returns (MSCI Indexes) data are from Datastream. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 6: Moments used in Model Estimation

	Data	Model		
		Median	5%	95%
Aggregate Quantities				
Consumption growth, mean	0.016	0.015	0.008	0.022
Consumption growth, volatility	0.022	0.020	0.018	0.035
Output growth, mean	0.016	0.015	0.008	0.021
Output growth, volatility	0.021	0.021	0.018	0.023
Mean top 1% income share	0.158	0.216	0.150	0.289
Asset prices				
Risk-free rate, mean	0.014	0.025	-0.005	0.033
Risk-free rate, volatility	0.033	0.011	0.004	0.046
Excess stock returns, mean	0.049	0.035	0.013	0.095
Excess stock returns, volatility	0.232	0.111	0.058	0.246
Exchange rate, volatility	0.104	0.077	0.036	0.182
Correlations (regression slopes)				
Exchange rate and				
—relative consumption growth	0.011	0.038	-0.005	0.155
—relative output growth	0.011	0.005	-0.035	0.049
Bi-variate correlations (regression slopes)				
Exchange rate and				
—relative c-growth	0.014	0.022	-0.009	0.126
—relative growth in top 1% income shares	0.014	0.038	0.000	0.076
Exchange rate and				
—relative c-growth	0.010	0.015	-0.006	0.124
—relative difference in stock returns	-0.016	-0.036	-0.084	0.000
Correlations				
Consumption growth (H and F)	0.337	0.804	0.333	0.923
Output growth (H and F)	0.449	0.862	0.735	0.954
Stock Returns (H and F)	0.541	0.275	-0.087	0.646
Trade surplus (as % of output) growth and c-growth	-0.472	-0.148	-0.852	0.267
Uncovered Interest Parity				
UIP slope	-0.572	-0.506	-6.104	2.225

Notes: This table reports both empirical moments computed using the G-7 & G-10 data set and simulated moments from the model. All the parameters are estimated as in Table 7.

Table 7: Parameter Estimates

Description	Symbol	Value	SE
Preferences:			
Home bias	α	0.990	0.149
Preference for own consumption	h	0.174	0.728
Subjective discount rate	β	1.057	0.064
Risk aversion	γ	6.501	6.325
Elasticity of intertemporal substitution	ψ	1.762	2.568
Death	ξ	0.078	0.035
Endowments:			
Displacement shock productivity	δ	0.269	0.758
Measure of projects-receiver	π	0.086	0.423
Mean of output growth	μ	0.012	0.007
Displacement shock low state	u_1	0.001	0.018
Displacement shock high state	u_3	0.137	0.096
Persistence of displacement shock			
	p	0.930	0.098
— low state persistence			
	q	0.830	0.417
— high state persistence			
Volatility of neutral shock	σ_e	0.019	0.015
Technology spillover	ρ_u	0.698	0.221
Correlation of neutral shock	ρ_e	0.872	0.253

Notes: This table reports the estimated parameters of the model. See the main text and the Appendix D for details on the estimation of the model.

A Solution to the Simplified Model

A.1 Representative agents

A.1.1 The representative agent in each country

First, we show that within a country, finding optimal solutions for heterogeneous agents is equivalent to finding optimal solution for a representative agent.

In each country, even though agents are heterogeneous in their wealth, because of homethetic preference consumption-wealth ratios are equalized.

Consider H country for example, we define the representative agent as

$$U_t^H = \int_{i \in [0,1]} U_{i,t} w_t^{i,H}$$

where $U_{i,t}$ and $w^{i,t}$ are the utility and wealth share of household i. That is, the representative agent takes the country-level endowment and the wealth distribution as given and maximizes the wealth-weighted utility.

Because all agents within a country are solving the same optimization problem up to their wealth, so is the wealth-weighted representative agent. Put differently, the representative agent behaves the same way as the individual agent, but scaled up to a wealth that is equal to the country's aggregate wealth. Thus, solving for the equilibrium solutions for heterogeneous agents within a country is equivalent to finding the optimal solution for the representative agent.

Denote C_t^c as the country-level aggregate consumption, the utility of representative agent can be written as

$$U_t^H = \lambda_t^H U(C_t^H)$$

Where $U(x)$ is the utility function for individual household – $U(x) = \log(x)$ in this case. That is, the utility of representative agent is proportional to an fictitious agent who consumes country-level aggregate consumption. The time-vaying scaling factor λ_t^H reflects the change of wealth distribution $w_t^{i,c}$ within the country. If market is complete, wealth distribution is invariant and λ_t^H would be a constant.

Now the equilibrium allocation problem reduces to a problem with two (representative) agents and an incomplete markets.

A.1.2 Aggregation with log preference

The H's representative agent's utility can be written as

$$U_t^H = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s^H$$

With incomplete markets [7], the usual construction of a planner's utility as a weighted sum, with constant weights, of individual representative utility function is not possible. Instead, we are going to employ a fictitious planner with stochastic weights (we use the results from Cuoco and He (2001)).

This fictitious representative agent maximizes his utility subject to the resource constraints:

$$\begin{aligned}
\max_{\{x_t^H, y_t^F, x_t^F, y_t^F\}, t=0,1,2,..} \sum_t \beta^t \left(\log C_t^H + \lambda_t \log C_t^F\right) \\
\text{s.t.} \qquad x_t^F + x_t^F &= X_t \\
y_t^H + y_t^F &= Y_t \\
C_t^H &= (x_t^H)^{\alpha} (y_t^H)^{1-\alpha} \\
C_t^F &= (x_t^F)^{1-\alpha} (y_t^F)^{\alpha}
\end{aligned}$$

where we have normalized the weight on the Home representative agent to be equal to one and assigned the weight λ to the foreign representative agent. λ_t is the marginal utilities of either good of the two countries.

A.2 Allocations

For concreteness, we focus on the exposition on the Home consumer. First, at each t, we derive the consumer's demands for X and Y goods, keeping overall consumption expenditure $\mathcal{C}_H$ fixed.

$$\max_{\{x_t^H, x_t^H\}} \alpha \log x_t^H + (1-\alpha) \log y_t^H \tag{52}$$

$$\text{s.t.} \quad p_{x,t} x_t^H + p_{y,t} y_t^H = \mathcal{C}_H \tag{53}$$

We obtain the following demands

$$x_t^H = \frac{\alpha \mathcal{C}_H}{p_{x,t}}, y_t^H = \frac{(1-\alpha)\mathcal{C}_H}{p_{y,t}} \tag{54}$$

[7] the argument here follows as in Pavlova and Rigobon (2011)

The indirect utility function defined as $U_H(\mathcal{C}_H, p_{x,t}, p_{y,t})$ is then given by

$$U_H(\mathcal{C}_H, p_{x,t}, p_{y,t}) = \log(\mathcal{C}_H) + F(p_{x,t}, p_{y,t}) \tag{55}$$

Function F depends only on variables that are exogenous from the viewpoint of the consumer and therefore, because of the separability, it drops out the portfolio choice.

Hence, the optimization problem of consumer is equivalent to the single-good consumption-investment problem, with consumption expenditure $\mathcal{C}_H$ replacing the consumption. Importantly, it implies that the prices of individual goods $p_{x,t}, p_{y,t}$ do not pose a risk that the consumer desires to hedge.

With log preference, consumers have constant consumption-to-wealth ratio. Thus, the pareto weights λ_t is equal to the consumption expenditure ratio, which in turn is equal to the wealth ratio between two countries $\lambda_t = \frac{W_{F,t}}{W_{H,t}}$. Substituting the demand functions in the budge constraints, we get the allocations (56)-(59).

$$x_t^H = \frac{\alpha}{\alpha + (1-\alpha)\lambda_t} X_t \tag{56}$$

$$x_t^F = \frac{(1-\alpha)\lambda_t}{\alpha + (1-\alpha)\lambda_t} X_t \tag{57}$$

$$y_t^H = \frac{1-\alpha}{1-\alpha+\alpha\lambda_t} Y_t \tag{58}$$

$$y_t^F = \frac{\alpha\lambda_t}{1-\alpha+\alpha\lambda_t} Y_t. \tag{59}$$

A.3 SDFs and Asset Prices

SDF. Let $\mathcal{N}_{t+1}^c$ denote the set of all indices of agents in country c who receives worthless ideas at time $t+1$. In what follows, we will focus on the exposition on the Home consumer. By definition,

$$\frac{M_{t+1}^H}{M_t^H} = \beta \mathrm{E}\left(\frac{c_{t+1}^{i,H}}{c_t^{i,H}}\right)^{-1} = \beta\left(\frac{\int_{i\in\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H}}{\int_{i\in\mathcal{N}_{t+1}^H} dC_t^{i,H}}\right)^{-1} \tag{60}$$

where the first equation follows from the consumer's Euler equation and the second equation follows from the probability of receiving a profitable firm being zero. As a result, households' anticipated consumption growth coincides with the consumption growth of the cohort $\mathcal{N}_{t+1}^H$. Market clearing implies:

$$C_{t+1}^H = \int_{i\in\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H} + \int_{i\notin\mathcal{N}_{t+1}^H} dC_{t+1}^{i,H} \tag{61}$$

Note that $\mathbf{1}_{i\notin\mathcal{N}^H_{t+1}} \times \mathbf{1}_{i\notin\mathcal{N}^H_t} = 0$ almost surely, so

$$\int_{i\in\mathcal{N}^H_{t+1}} dC^{i,H}_t = C^H_t \tag{62}$$

Combining (60)-(62) along with the allocation rules (56)-(59) we have that

$$\frac{M^H_{t+1}}{M^H_t} = \beta(\frac{X_{t+1}}{X_t})^{-\alpha}(\frac{Y_{t+1}}{Y_t})^{\alpha-1}\left(\frac{\alpha+(1-\alpha)\lambda_t}{\alpha+(1-\alpha)\lambda_{t+1}}\right)^{-\alpha}\left(\frac{\alpha\lambda_t+1-\alpha}{\alpha\lambda_{t+1}+1-\alpha}\right)^{\alpha-1}\left(1-\frac{\int_{i\notin\mathcal{N}^H_{t+1}} dC^{i,H}_{t+1}}{\int_{i\in[0,1]} dC^{i,H}_{t+1}}\right)^{-1} \tag{63}$$

Note that with log preference, consumption bundles is proportional to consumption expenditure, which in turn is proportional to wealth. Therefore the last term can be written as

$$b_{H,t+1} = \frac{\int_{i\in\mathcal{N}^H_{t+1}} w^{i,H}_{t+1}}{\int_{i\in[0,1]} w^{i,H}_{t+1}} = \frac{\int_{i\in\mathcal{N}^H_{t+1}} d\mathcal{C}^{i,H}_{t+1}}{\int_{i\in[0,1]} d\mathcal{C}^{i,H}_{t+1}} = 1 - \frac{\int_{i\notin\mathcal{N}^H_{t+1}} dC^{i,H}_{t+1}}{\int_{i\in[0,1]} dC^{i,H}_{t+1}} \tag{64}$$

Substituting back we obtain (24). Similarly, we can derive the SDF for foreign consumers.

$$\frac{M^F_{t+1}}{M^F_t} = \beta\left(\frac{X_{t+1}}{X_t}\right)^{\alpha-1}\left(\frac{Y_{t+1}}{Y_t}\right)^{-\alpha}\frac{1}{b_{F,t+1}}\frac{\lambda_t}{\lambda_{t+1}}\left(\frac{\alpha+(1-\alpha)\lambda_t}{\alpha+(1-\alpha)\lambda_{t+1}}\right)^{\alpha-1}\left(\frac{\alpha\lambda_t+1-\alpha}{\alpha\lambda_{t+1}+1-\alpha}\right)^{-\alpha} \tag{65}$$

Asset Prices. Let us first focus on the stock market in Home country. The SDF can be used to price the risky stocks by no arbitrage:

$$S^H_t = p_{x,t}X_t + \mathrm{E}_t[\sum_{t+1}^{T} M^H_s p_{x,s} X_s e^{-\sum_{t+1}^{s} u^H_j}] \tag{66}$$

Note that M^H_t is the SDF using consumption bundles of the home country, if we define ζ^H_t as the SDF using local goods of home country, then we have

$$M^H_t p_{x,t} = \zeta^H_t$$

Note that the first-order condition of X-good for consumers gives:

$$\zeta^H_s = \beta^{s-t}\frac{\alpha}{c^H_{x,s}} \tag{67}$$

where $c_{x,s}^H$ is the total consumption of X goods by the households who have not received any profitable firms between $t+1$ and s, which has a probability of one. Therefore,

$$c_{x,s}^H = \frac{\alpha}{\alpha + (1-\alpha)\lambda_s} X_s \Pi_{t+1}^s b_{H,s} \tag{68}$$

Substituting (67) and (68) into (66), we have

$$S_t^H = p_{x,t} X_t \mathrm{E}_t [\sum_{t+1}^T \beta^{s-t} \frac{\Pi_{t+1}^s \frac{1}{b_{H,s}} (\alpha + (1-\alpha)\lambda_s)}{\frac{1}{b_{H,t}} (\alpha + (1-\alpha)\lambda_t)} e^{-\sum_{t+1}^s u_j^H}] + p_{x,t} X_t \tag{69}$$

The derivation for foreign country's stock market is similar.

A.4 The Change of Wealth Distribution

In A.2 we show that the optimization of consumer is equivalent to the single-good consumption -investment problem, with consumption expenditure $\mathcal{C}$ replacing the consumption. Moreover, the consumers do not hedge the prices of individual goods $p_{x,t}, p_{y,t}$.

This implies that the consumers in home and foreign are solving the same portfolio-choice problem. As a result, their optimal portfolios and wealth growth are the same across different states. Hence, the wealth ratio at $t+1$ is given by

$$\lambda_{t+1} = \frac{\int_{i\in[0,1]} w_{t+1}^{i,F}}{\int_{i\in[0,1]} w_{t+1}^{i,H}} = \frac{\int_{i\in\mathcal{N}_{t+1}^F} w_{t+1}^{i,F} + \int_{i\notin\mathcal{N}_{t+1}^F} w_{t+1}^{i,F}}{\int_{i\in\mathcal{N}_{t+1}^H} w_{t+1}^{i,H} + \int_{i\notin\mathcal{N}_{t+1}^H} w_{t+1}^{i,H}} \tag{70}$$

Note that the total value of profitable firms at $t+1$ is related to the displacement shocks u_{t+1}^H, u_{t+1}^F. From 8 and 9 it follows that the total value of new firms are:

$$\int_{i\notin\mathcal{N}_{t+1}^H} w_{t+1}^{i,H} = S_{t+1}^H (1 - e^{-u_{t+1}^H}) \tag{71}$$

$$\int_{i\notin\mathcal{N}_{t+1}^F} w_{t+1}^{i,F} = S_{t+1}^F (1 - e^{-u_{t+1}^F}) \tag{72}$$

And the total value of old firms is

$$\int_{i\in\mathcal{N}_{t+1}^F} w_{t+1}^{i,F} + \int_{i\in\mathcal{N}_{t+1}^H} w_{t+1}^{i,H} = S_{t+1}^H e^{-u_{t+1}^H} + S_{t+1}^F e^{-u_{t+1}^F} \tag{73}$$

Because the consumers in home and foreign hold the same portfolio, the wealth ratio for the households that do not receive new firms are the same at t and $t+1$. Hence,

$$\lambda_t = \frac{\int_{i\in[0,1]} w_t^{i,F}}{\int_{i\in[0,1]} w_t^{i,H}} = \frac{\int_{i\in\mathcal{N}_{t+1}^f} w_t^{i,F}}{\int_{i\in\mathcal{N}_{t+1}^h} w_t^{i,H}} \tag{74}$$

Combining (70)-(74) we obtain

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}\left(S_{t+1}^H e^{-u_{t+1}^F} + S_{t+1}^F e^{-u_{t+1}^F}\right) + \frac{1}{\lambda_t} S_{t+1}^F (1 - e^{-u_{t+1}^F})}{\frac{1}{1+\lambda_t}\left(S_{t+1}^H e^{-u_{t+1}^F} + S_{t+1}^F e^{-u_{t+1}^F}\right) + S_{t+1}^H (1 - e^{-u_{t+1}^H})} \tag{75}$$

A.5 Approximation

We now derive the approximate analytical solutions near the long-term steady state. That is, when $\lambda_t = 1$ and when u_{t+1}^H, u_{t+1}^F are small.

By symmetry, when $\lambda_t = 1$ the price-dividend ratio of the stock markets are the same. Let us denote this ratio as C_{pd}, i.e.,

$$\left(\frac{S_t^H}{p_{x,t} X_t}\right)_{\lambda_t=1} = \left(\frac{S_t^F}{p_{y,t} Y_t}\right)_{\lambda_t=1} = C_{pd} \tag{76}$$

Using the price ratio relation given by (21), we can rewrite (75) as

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}\left(e^{-u_{t+1}^H} pd_{t+1}^H + \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} e^{-u_{t+1}^F} pd_{t+1}^F\right) + \frac{1}{\lambda_t}\frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}}(1 - e^{-u_{t+1}^F})}{\frac{1}{1+\lambda_t}\left(e^{-u_{t+1}^H} pd_{t+1}^H + \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} e^{-u_{t+1}^F} pd_{t+1}^F\right) + (1 - e^{-u_{t+1}^H}) pd_{t+1}^H} \tag{77}$$

where pd_{t+1}^c is the price-dividend ratio of the stock market in country $c \in \{H, F\}$ at $t+1$. To further simplify, we use the fact that u_{t+1}^H, u_{t+1}^F are small so that $pd_{t+1}^c \approx C_{pd}$ for $c \in \{H, F\}$. Denote the total wealth of stock market as $\bar{W} = W_H + W_F$, we make the following observation:

$$\bar{W} = S_{t+1}^H + S_{t+1}^F \tag{78}$$

$$\frac{S_{t+1}^F}{S_{t+1}^H} \approx \frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} \tag{79}$$

The second equation is because $pd_{t+1}^c \approx C_{pd}$. It follows that

$$S_{t+1}^H = \frac{\alpha+(1-\alpha)\lambda_{t+1}}{1+\lambda_{t+1}}\bar{W}, \qquad S_{t+1}^F = \frac{1-\alpha+\alpha\lambda_{t+1}}{1+\lambda_{t+1}}\bar{W} \tag{80}$$

The dynamics of wealth distribution can thus be written as

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) + \frac{1}{\lambda_t}(1-\alpha+\alpha\lambda_{t+1})(1-e^{-u^F_{t+1}})}{\frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) + (\alpha+(1-\alpha)\lambda_{t+1})(1-e^{-u^H_{t+1}})} \tag{81}$$

Denote the common terms in both the numerator and denominator as

$$B = \left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) \tag{82}$$

Some algebra gives

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\frac{1}{1+\lambda_t}B + \frac{1-\alpha}{\lambda_t}(1-e^{-u^F_{t+1}})}{\frac{1}{1+\lambda_t}B + \alpha(e^{-u^F_{t+1}} - e^{-u^H_{t+1}}) + (1-\alpha)\lambda_{t+1}(1-e^{-u^H_{t+1}})} \tag{83}$$

To progress further, we use the result from A.2 that consumers in both countries have the same portfolios and therefore the same wealth growth. At $t+1$, the wealth of households in both countries who do not receive profitable firms is

$$\int_{i\in N^h_{t+1}} w^{i,H}_{t+1} + \int_{i\in N^f_{t+1}} w^{i,F}_{t+1} = \left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right) \tag{84}$$

$$\frac{1}{1+\lambda_{t+1}}\bar{W} \tag{85}$$

Because consumers hold the same portfolio, we have

$$\frac{\int_{i\in N^H_{t+1}} w^{i,H}_{t+1}}{\int_{i\in N^F_{t+1}} w^{i,F}_{t+1}} = \frac{\int_{i\in[0,1]} w^{i,H}_t}{\int_{i\in[0,1] w^{i,F}_{t+1}}} = \frac{1}{\lambda_t} \tag{86}$$

Therefore

$$\int_{i\in N^H_{t+1}} w^{i,H}_{t+1} = \frac{1}{1+\lambda_t}\left(e^{-u^H_{t+1}}(\alpha+(1-\alpha)\lambda_{t+1}) + (1-\alpha+\alpha\lambda_{t+1})e^{-u^F_{t+1}}\right)\frac{1}{1+\lambda_{t+1}}\bar{W} \tag{87}$$

On the other hand, by definition

$$\int_{i\in[0,1]} w^{i,H}_{t+1} = \int_{i\in N^H_{t+1}} w^{i,H}_{t+1} + \int_{i\notin N^H_{t+1}} w^{i,H}_{t+1} \tag{88}$$

so that

$$\int_{i\in N_{t+1}^H} w_{t+1}^{i,H} = \int_{i\in[0,1]} w_{t+1}^{i,H} - \int_{i\notin N_{t+1}^H} w_{t+1}^{i,H}$$
$$= \frac{1}{1+\lambda_{t+1}}\bar{W} - (1-e^{-u_{t+1}^H})\frac{\alpha+(1-\alpha)\lambda_{t+1}}{1+\lambda_{t+1}}\bar{W} \tag{89}$$

Substituting (87) and (89) into (83), after some algebra we get

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1-\alpha+\alpha e^{-u_{t+1}^H}+(1-\alpha)(1-e^{-u_{t+1}^F})\frac{1}{\lambda_t}}{1-\alpha+\alpha e^{-u_{t+1}^F}+(1-\alpha)\lambda_t(1-e^{-u_{t+1}^H})} \tag{90}$$

Using the fact that $e^x \approx 1+x$ and $\lambda_t = 1$, we have

$$\Delta\log\lambda_{t+1} = \log\frac{1-\alpha+\alpha e^{-u_{t+1}^H}+(1-\alpha)(1-e^{-u_{t+1}^F})\frac{1}{\lambda_t}}{1-\alpha+\alpha e^{-u_{t+1}^F}+(1-\alpha)\lambda_t(1-e^{-u_{t+1}^H})}$$
$$\approx u_{t+1}^F - u_{t+1}^F \tag{91}$$

To get the approximate expression for log growth of consumption ratio, substituting (56)-(59) into (14), we have

$$\Delta c_{t+1}^H - \Delta c_{t+1}^F = (2\alpha-1)[\Delta\log X_{t+1} - \Delta\log Y_{t+1} + \Delta\log\frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}}] - \Delta\log\lambda_{t+1}$$

note that $\lambda_{t+1} \approx 1+\Delta\log\lambda_{t+1}$, so we have

$$\Delta\log\frac{1-\alpha+\alpha\lambda_{t+1}}{\alpha+(1-\alpha)\lambda_{t+1}} \approx (2\alpha-1)\Delta\log\lambda_{t+1} \tag{92}$$

substituting back we get (31). The derivation for log growth of output ratio is straightforward from definitions.

B Model with Epstein-Zin preference

With Epstein-Zin preference, we can not invoke the result in Cuoco and He (2001) because the preference is not time-additive. But the representative agent constructed previously still exists in each country because the aggregation property only depends on the homotheticity of the preference. So in this case, the representative agent constructed above behaves the same as an individual agent in a country but scaled up to the country-level wealth.

B.1 Dynamics of the Consumption Ratio

Denote $W_t^c = W(\hat{C}_t^c, U_{t+1}^c)$ as the utility of the representative agent of country c. Denote the partial derivatives with respect to composite consumption and continuation utility as $W_{1,t}^c, W_{2,t}^c$, we have

$$\frac{\partial W_t^c}{\partial \bar{C}_t^c} = \frac{\partial W_t^c}{\partial \hat{C}_t^c}\frac{\partial \hat{C}_t^c}{\partial \bar{C}_t^c} = W_{1,t}^c(\bar{C}_t^c)^{h-1}$$

$$\frac{\partial W_t^c}{\partial U_{t+1}^c} = W_{2,t}^c$$

The intertemporal marginal rate of substitution of representative agent of country c is

$$M_{t,t+1}^c = \frac{\frac{\partial W}{\partial U}^c_{t+1}\frac{\partial W}{\partial \bar{C}}^c_{t+1}}{\frac{\partial W}{\partial \bar{C}}^c_{t}} = \frac{W_{2,t}^c W_{1,t+1}^c}{W_{1,t}^c}(\frac{\bar{C}_{t+1}^c}{\bar{C}_t^c})^{h-1} \tag{93}$$

International trade of X good implies that the marginal utilities of good X for $t = 1, 2, ...$ in each possible state is

$$\left(\prod_{j=0}^{t-1} W_{2,j}^H\right) W_{1,t}^H \bar{C}_t^H \frac{\alpha}{x_t^H}(\bar{C}_t^H)^{h-1} = (\bar{C}_t^F)^{h-1}\frac{1-\alpha}{x_t^F}\bar{C}_t^F W_{1,t}^F \left(\prod_{j=0}^{t-1} W_{2,j}^f\right) \tag{94}$$

Define the date t Pareto weights as

$$\Lambda_t^c = \Lambda_0^c \left(\prod_{j=0}^{t-1} W_{2,j}^c\right) W_{1,t}^c \bar{C}_t^c (\bar{C}_t)^{h-1}$$

$$= \Lambda_{t-1}^c W_{2,t-1}^c \frac{W_{1,t}^c}{W_{1,t-1}^c}(\frac{\bar{C}_t^c}{\bar{C}_{t-1}^c})^{h-1}\frac{\bar{C}_t^c}{\bar{C}_{t-1}^c} = S_{t-1}^c M_{t-1,t}^c \exp(\Delta c_t^c)$$

Since the economy starts with a symmetric setup $\Lambda_0^H = \Lambda_0^F$. We can rewrite (94) as

$$\Lambda_t^H \frac{\alpha}{x_t^H} = \frac{1-\alpha}{x_t^F}\Lambda_t^F$$

Denote $\lambda_t = \frac{\Lambda_t^F}{\Lambda_t^F}$ as the ratio of Pareto weights. The optimality condition can be written as

$$\lambda_t = \frac{\alpha x_t^F}{(1-\alpha)x_t^H} \tag{95}$$

Similar to the log case, note that with Cobb-Douglas preference over different goods, households consumption expenditure share for each good is fixed. That is, foreign households spend $1-\alpha$ on X-good and home households spend α on X-good. Therefore, (95) shows that λ_t is also the consumption expenditure between foreign and home. That is, $\lambda_t = \frac{p_F C_F}{p_H C_H} = \frac{\mathcal{C}_{F,t}}{\mathcal{C}_{H,t}}$. Also, we have that

$$\lambda_{t+1} = \lambda_t \frac{M_{t,t+1}^F e^{\Delta c_{t+1}^F}}{M_{t,t+1}^H e^{\Delta c_{t+1}^H}} \tag{96}$$

B.2 Allocations and Exchange Rate

Similar to the log case, since the ratio of consumption expenditure is $\lambda_t = \frac{\mathcal{C}_{F,t}}{\mathcal{C}_{H,t}}$, we have

$$x_t^H = \frac{\alpha \mathcal{C}_{H,t}}{p_{x,t}}, y_t^H = \frac{(1-\alpha)\mathcal{C}_{H,t}}{p_{y,t}}, x_t^F = \frac{(1-\alpha)\mathcal{C}_{F,t}}{p_{x,t}}, y_t^F = \frac{\alpha \mathcal{C}_{F,t}}{p_{y,t}}$$

substituting these demands into resource constraints, we get the allocations (56), (57), (58), (59). Given these allocations, we can calculate the consumption bundles:

$$\bar{C}_{H,t} = (x_t^H)^\alpha (y_t^H)^{1-\alpha} \tag{97}$$

$$\bar{C}_{F,t} = (x_t^F)^{1-\alpha} (y_t^F)^\alpha \tag{98}$$

We can also compute the price of consumption bundles in home and foreign countries:

$$p_t^H = \frac{p_{x,t} x_t^H + p_{y,t} y_t^H}{C_{H,t}} \tag{99}$$

$$p_t^F = \frac{p_{x,t} x_t^F + p_{y,t} y_t^F}{C_{F,t}} \tag{100}$$

Note that the relative price of good Y in terms of good X is

$$p_t = \frac{X_t}{Y_t} \frac{1-\alpha+\alpha\lambda_t}{\alpha+(1-\alpha)\lambda_t} \tag{101}$$

By definition, the exchange rate is the ratio of price of consumption bundles:

$$E_t = \frac{p_t^H}{p_t^F} = \frac{\bar{C}_{F,t}}{\bar{C}_{H,t}} \frac{1}{\lambda_t} \tag{102}$$

The exchange rate growth is

$$\frac{E_{t+1}}{E_t} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{C}_{F,t+1}/\bar{C}_{F,t}}{\bar{C}_{H,t+1}/\bar{C}_{H,t}} \tag{103}$$

Note that (96) and (103) shows that in our model exchange rate growth is equal to the growth of SDF, as the model has an integrated financial markets.

B.3 SDF

Let us focus on the home country. The derivation for foreign country is similar. Since preference is homothetic, consumption is proportional to wealth. To calculate the SDF of the representative agent, we need to consider two groups of population: the population that receive the new firms in the current period (with measure π, denote as N); and the population that does not receive the new firms in the current period (with measure $1-\pi$, denote as O).

To this end, first note that $b_{i,t+1}$ is the fraction of wealth account for by the cohort that does not receive profitable projects from period t to $t+1$ in country i. The wealth shares of these two groups within the home country are

$$b_{H,t}(1-\pi), b_{H,t}\pi + 1 - b_{H,t}$$

The consumption growth and relative consumption growth for group O are $\frac{\bar{C}_{t+1}}{\bar{C}_t} b_{H,t+1}$ and $b_{H,t+1}$. And the consumption growth and relative consumption growth for group O are $\frac{b_{H,t+1}\pi+1-b_{H,t+1}}{\pi}\frac{\bar{C}_{t+1}}{\bar{C}}$ and $\frac{b_{H,t+1}\pi+1-b_{H,t+1}}{\pi}$. Therefore, the growth in the composite consumption for two groups {O, N} are (we omit the country index H from now on)

$$\begin{aligned}
\frac{\hat{C}_{t+1}}{\hat{C}_t}_O &= \left(\frac{\bar{C}_{t+1} b_{t+1}}{\bar{C}_t}\right)^h (b_{t+1})^{1-h} = \frac{\bar{C}^h_{t+1}}{\bar{C}^h_t} b_{t+1} \\
\frac{\hat{C}_{t+1}}{\hat{C}_t}_N &= \left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi}\right)^h \left(\frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi}\right)^{1-h} \\
&= \left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^h \frac{b_{t+1}\pi + 1 - b_{t+1}}{\pi}
\end{aligned}$$

Similarly, we can derive the growth in continuation utility for these two groups. Since the consumption to utility ratio are equalized across two groups, we have

$$\frac{\left(\frac{U_{O,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)}{\left(\frac{U_{N,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)} = \frac{b_{t+1}}{\frac{b_{t+1}\pi+1-b_{t+1}}{\pi}} \tag{104}$$

The SDF of these two groups can be written as

$$M_{O,t,t+1} = \beta(\frac{\hat{C}_{t+1}}{\hat{C}_t})_O^{-\frac{1}{\psi}}(\frac{\bar{C}_{t+1}}{\bar{C}_t})^{h-1}\left(\frac{U_{O,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

$$M_{N,t,t+1} = \beta(\frac{\hat{C}_{t+1}}{\hat{C}_t})_N^{-\frac{1}{\psi}}(\frac{\bar{C}_{t+1}}{\bar{C}_t})^{h-1}\left(\frac{U_{N,t+1}^{1-\gamma}}{\mathrm{E}_t(U_{t+1}^{1-\gamma})}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

In this economy, each investor's own inter-temporal marginal rate of substitution is a valid SDF. Hence the cross-sectional average of investors' inter-temporal marginal rates of substitution is a valid stochastic discount factor. That is,

$$\begin{aligned} M_{t,t+1} &= (1-\pi)M_{O,t,t+1} + \pi M_{N,t,t+1} \\ &= \beta\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\frac{h}{\psi}+h-1}\left(\pi\left(\frac{b_{t+1}\pi+1-b_{t+1}}{\pi}\right)^{-\frac{1}{\psi}}\left(\frac{U_{N,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}} + (1-\pi)b_{t+1}^{-\frac{1}{\psi}}\left(\frac{U_{O,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}\right) \end{aligned}$$

Combining with (104), we have (43).

B.4 Wealth ratio

First, note that the marginal utility of consumption of the representative agent in each country is

$$\frac{\partial \tilde{U}}{\partial C} = (1-\tilde{\beta})\tilde{U}^{\frac{1}{\psi}}\hat{C}^{-\frac{1}{\psi}}\bar{C}^{h-1}$$

we can compute the wealth of households who didn't receive projects at t, in units of local consumption bundles:

$$\begin{aligned} \hat{W}_H &= \frac{\tilde{U}}{\frac{\partial \tilde{U}}{\partial C_H}} \\ &= \frac{1}{1-\beta}(\tilde{U})^{1-1/\psi}(\hat{C}_{H,t})^{\frac{1}{\psi}}\bar{C}_{H,t}^{1-h} \end{aligned}$$

$$= \frac{1}{1-\beta}(\frac{\tilde{U}_{H,}t}{\hat{C}_{H,t}})^{1-1/\psi}\hat{C}_{H,t}\bar{C}_{H,t}^{1-h}$$
$$= \frac{1}{1-\beta}(\frac{\tilde{U}_{H,}t}{\hat{C}_{H,t}})^{1-1/\psi}\bar{C}_{H,t}$$

Similarly we can derive the wealth for foreign country,

$$\hat{W}_F = \frac{1}{1-\beta}\tilde{U}_{F,t}^{1-1/\psi}\hat{C}_{F,t}^{1/\psi}\bar{C}_{F,t}^{1-h} = \frac{1}{1-\beta}(\frac{\tilde{U}_{F,}t}{\hat{C}_{F,t}})^{1-1/\psi}\bar{C}_{F,t} \tag{105}$$

Note that the wealth above are calculated in the units of local consumption bundles, so the ratio of two countries' wealth should be adjusted by the price of their respective consumption bundles

$$\frac{W_F}{W_H} = \frac{\hat{W}_F}{\hat{W}_H}\frac{p_F}{p_H} = \left(\frac{\frac{U_{F,t}}{\hat{C}_{F,t}}}{\frac{U_{H,t}}{\hat{C}_{H,t}}}\right)^{1-1/\psi}\lambda_t \tag{106}$$

The second equation comes from the fact that $\lambda = \frac{p_F\bar{C}_F}{p_H C_H}$ (Recall (95)).

B.5 Asset Prices

Similar to the log case, we have

$$\begin{aligned}
S_t^H &= p_{x,t}X_t + \mathrm{E}_t[M_{t,t+1}^H S_{t+1}^H] \\
pd_t^H &= \mathrm{E}_t[M_{t+1}^H\frac{p_{x,t+1}X_{t+1}}{p_{x,t}X_t}(1+pd_{t+1}^H)e^{-u_{t+1}^H}] \\
S_t^F &= p_{y,t}Y_t + \mathrm{E}_t[M_{t,t+1}^F S_{t+1}^F] \\
pd_t^F &= \mathrm{E}_t[M_{t+1}^H\frac{p_{y,t+1}Y_{t+1}}{p_{y,t}Y_t}(1+pd_{t+1}^F)e^{-u_{t+1}^F}]
\end{aligned}$$

B.6 Trade and Capital Flows

The net export as a fraction of total output is

$$\frac{NX_t^H}{X_t} = \frac{p_{x,t}X_t - p_{x,t}x_t^H - p_{y,t}y_t^H}{p_{x,t}X_t} = 1 - \frac{1}{\alpha+(1-\alpha)\lambda_t} \tag{107}$$
$$\frac{NX_t^F}{Y_t} = \frac{p_{y,t}Y_t - p_{y,t}Y_t^F - p_{y,t}x_t^H}{p_{y,t}Y_t} = 1 - \frac{\lambda_t}{1-\alpha+\alpha\lambda_t} \tag{108}$$

The net international investment position scaled by country's wealth, is

$$\frac{A_t^H}{p_{x,t}X_t} = \frac{W_t^H - S_t^H}{W_t^H} \tag{109}$$

$$\frac{A_t^F}{p_{y,t}Y_t} = \frac{W_t^F - S_t^F}{W_t^F} \tag{110}$$

C Numerical Procedure

Here, we briefly describe the numerical procedure for solving the model.

C.1 All equations to solve

The equilibrium is obtained by jointly solving the set of non-linear equations that describe the equilibrium conditions: (11), (12),(14),(15),(21), (25), (33), (34), (56),(57),(58),(59), (106), (43). We put all the equations here, as below:

On the aggregate level, we have

$$d\log X = \mu + \delta u_H + \varepsilon_H$$

$$d\log Y = \mu + \delta u_F + \varepsilon_F$$

For each country's allocation we have (56)-(59).

$$x_t^H = \frac{\alpha}{\alpha + (1-\alpha)\lambda_t} X_t \tag{111}$$

$$x_t^F = \frac{(1-\alpha)\lambda_t}{\alpha + (1-\alpha)\lambda_t} X_t \tag{112}$$

$$y_t^H = \frac{1-\alpha}{1-\alpha+\alpha\lambda_t} Y_t \tag{113}$$

$$y_t^F = \frac{\alpha\lambda_t}{1-\alpha+\alpha\lambda_t} Y_t. \tag{114}$$

The displacement effect

$$b_{H,t+1} = 1 - \frac{(1+pd_{H,t+1})(1-e^{-u_{H,t+1}})}{\left(1+pd_{H,t+1} + (1+pd_{F,t+1})\frac{p_{y,t+1}Y_{t+1}}{p_{x,t+1}X_{t+1}}\right)\frac{1}{1+w_{t+1}}} \tag{115}$$

$$b_{F,t+1} = 1 - \frac{(1+pd_{F,t+1})(1-e^{-u_{F,t+1}})}{\left((1+pd_{H,t+1})\frac{p_{x,t+1}X_{t+1}}{p_{y,t+1}Y_{t+1}} + (1+pd_{F,t+1})\right)\frac{w_{t+1}}{1+w_{t+1}}} \tag{116}$$

where the dividend ratio is

$$\frac{p_{y,t}Y_t}{p_{x,t}X_t} = \frac{1-\alpha+\alpha\lambda_t}{\alpha+(1-\alpha)\lambda_t}$$

Post-Dividend price-dividend ratio are given by

$$pd_t^H = \mathrm{E}_t[M_{t,t+1}^H \frac{p_{x,t+1}X_{t+1}}{p_{x,t}X_t}(1+pd_{t+1}^H)e^{-u_{t+1}^H}] \tag{117}$$

$$pd_t^F = \mathrm{E}_t[M_{t,t+1}^F \frac{p_{y,t+1}Y_{t+1}}{p_{y,t}Y_t}(1+pd_{t+1}^F)e^{-u_{t+1}^F}] \tag{118}$$

Aggregate consumption is given by

$$C_t^H = (x_t^H)^{\alpha}(y_t^H)^{1-\alpha} \tag{119}$$

$$C_t^F = (x_t^F)^{1-\alpha}(y_t^F)^{\alpha} \tag{120}$$

The two SDFs are given by (43),

$$\frac{M_{t+1}^c}{M_t^c}_o = \beta\left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}}\right)^{-\frac{h}{\psi}+h-1} b_{o,c,t+1}^{-\frac{1}{\psi}}\left(\frac{U_{o,c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{121}$$

$$\frac{M_{t+1}^c}{M_t^c}_n = \beta\left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}}\right)^{-\frac{h}{\psi}+h-1} b_{n,c,t+1}^{-\frac{1}{\psi}}\left(\frac{U_{n,c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}} \tag{122}$$

where

$$b_n = \frac{b\pi+1-b}{\pi}$$

$$b_o = b$$

$$\frac{U_{n,t+1}}{C_t} = \frac{U_{n,t+1}}{C_{n,t+1}}\frac{\bar{C}_{t+1}}{\bar{C}_t}(\frac{b\pi+1-b}{\pi})$$

$$\frac{U_{o,t+1}}{C_t} = \frac{U_{o,t+1}}{C_{o,t+1}}\frac{\bar{C}_{t+1}}{\bar{C}_t}b$$

We use cross-sectional average as the aggregate SDF. I.e.,

$$\frac{M_{t+1}}{M_t} = \beta\left(\frac{\bar{C}_{c,t+1}}{\bar{C}_{c,t}}\right)^{-\frac{h}{\psi}+h-1}\left(\pi\left(\frac{b_{c,t+1}\pi+1-b_{c,t+1}}{\pi}\right)^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}} + (1-\pi)b_{c,t+1}^{-\frac{1}{\psi}+\frac{1/\psi-\gamma}{1-\gamma}}\right)\left(\frac{\bar{U}_{c,t+1}^{1-\gamma}}{\mathrm{E}_t[U_{c,t+1}^{1-\gamma}]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

and wealth ratio is given by (106) and the lambda ratio is given by (96). Recursively definition of continuation utility are given by (123) - (124)..

These are all the equations.

Specifically, we need to numerically solving four functions for any given state: Price-Dividend ratio of H/F and the expected continuation Utility of H/F. The price-dividend ratios are recursively defined above. Next we derive the recursive definition for expected continuation utility:

We focus on the case for the home country and omit the country index. Consider a household with wealth share ω_i, his continuation utility is

$$V_{i,t+1} = [(1-\beta)\hat{C}_{i,t+1}^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}[V_{i,t+2}^{1-\gamma}]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}$$

where

$$\hat{C}_{i,t} = (\omega_{i,t+1}\bar{C}_{i,t+1})^h (\frac{\omega_{i,t+1}\bar{C}_{i,t+1}}{\bar{C}_{i,t+1}})^{1-h} = \omega_{i,t+1}\bar{C}_{t+1}^h$$

So we have the utility as a function of wealth share

$$V_{i,t+1}(\omega_i) = [(1-\beta)(\omega_{i,t+1}\bar{C}_{t+1}^h)^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}[V_{i,t+2}(\omega_{i,t+2}|\omega_{i,t+1})^{1-\gamma}]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\psi}}}$$

Normalize it we have

$$\begin{aligned}
\frac{V_{i,t+1}(\omega_{i,t+1})}{\bar{C}_{t+1}^h} &= \left[(1-\beta)(\omega_{i,t+1})^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}\left[\left(\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+1}^h}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \\
&= \left[(1-\beta)(\omega_{i,t+1})^{1-\frac{1}{\psi}} + \beta \mathrm{E}_{t+1}\left[\left(\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+2}^h}\frac{\bar{C}_{t+2}^h}{\bar{C}_{t+1}^h}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned}$$

Due to homotheticity, we know that $\frac{V_{i,t+1}(\omega_{i,t+1})}{\bar{C}_{t+1}^h}$ is linear in $\omega_{i,t+1}$, also it implies that $\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+2}^h}$ is linear in $\omega_{i,t+2}$. Dividing both sides by $\omega_{i,t+1}$

$$\frac{V_{i,t+1}(\omega_{i,t+1})}{\bar{C}_{t+1}^h\omega_{i,t+1}} = \left[(1-\beta) + \beta \mathrm{E}_{t+1}\left[\left(\frac{V_{i,t+2}(\omega_{i,t+2})}{\bar{C}_{t+2}^h\omega_{i,t+2}}\frac{\bar{C}_{t+2}^h\omega_{i,t+2}}{\bar{C}_{t+1}^h\omega_{i,t+1}}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

So we can write the utility-consumption ratio as

$$
\begin{aligned}
UC_{i,t+1}(\lambda_{t+1}) &= \left[(1-\beta) + \beta \mathrm{E}_{t+1}\left[\left(UC_{i,t+2}(\lambda_{t+2})\frac{\bar{C}^h_{t+2}\omega_{i,t+2}}{\bar{C}^h_{t+1}\omega_{i,t+1}}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \\
&= \left[(1-\beta) + \beta \underbrace{\mathrm{E}_{t+1}\left[\left(UC_{i,t+2}(\lambda_{t+2})\frac{\hat{C}_{t+2}}{\hat{C}_{t+1}}\right)^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}_{Q(\lambda_{t+1})^{\frac{1-1/\psi}{1-\gamma}}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \qquad (123)
\end{aligned}
$$

And the expected continuation utility, normalized by current consumption, is

$$
\begin{aligned}
Q_t &= \mathrm{E}_t\left[\left(UC_{i,t+1}(\lambda_{t+1})\frac{\hat{C}_{t+1}}{\hat{C}_t}\right)^{1-\gamma}\right] \\
&= \mathrm{E}_t\left((1-\pi)\left(UC_{i,t+1}(\lambda_{t+1})(\frac{\bar{C}_{t+1}}{\bar{C}_t})^h b\right)^{1-\gamma} + \pi\left(UC_{i,t+1}(\lambda_{t+1})(\frac{\bar{C}_{t+1}}{\bar{C}_t})^h \frac{b_H\pi + 1 - b_{H,t}}{\pi}\right)^{1-\gamma}\right) \qquad (124)
\end{aligned}
$$

For continuation utility, we have recursively defination given by (123) - (124).

C.2 Solving on the Grids

Grids. In order to solve the model numerically, we need to set up the grids for state and shocks. The neutral shock has two grid points $[-\sigma_e, \sigma_e]$ where σ_e is its standard deviation. The displacement shock has three grid points $[u_1, u_2, u_3]$. The transition matrix of displacement shock is parametrized by two parameters p, q. The log λ_t is discretized on 45 grid points. We set the bounds for $\log\lambda$ at -5.0 and 5.0.

Algorithm. We solve the equilibrium using policy iteration. This algorithm is based on the fact that value function is the solution of a fixed point problem generated by a contraction mapping.

To initiate the process, we need to start with an initial guess of price-dividend ratio and utility-consumption ratio. We use the static steady state values for the initial guess.

For any point on the grid, we need to solve a set of non-linear equations. Specifically, at time t, given a combination of shocks $u^H_t, u^F_t, \varepsilon^H_t, \varepsilon^F_t$, we need to solve λ_t, w_t. To do so, we first need to guess a value $\hat{\lambda}_t$ and then interpolate the price-dividend ratio and utility-consumption ratio using $\hat{\lambda}_t, u_t^H, u_t^F$. Then, using these imputed values, we solve $\tilde{\lambda}_t$, $\tilde{w}_t$ from the set of non-linear equations.

The difficulty is the fact that for some guessed values, the solution for the set of non-linear equations does not exist. So we try different random guesses starting with the state λ_{t-1}. In particular, we search for guess with the following form:

$$\lambda_{t-1} + \hat{\varepsilon} r |\lambda_{t-1}|$$

where $\hat{\varepsilon} \in N(0,1)$ is a random normal variable. $|\lambda_{t-1}|$ is the abosolute value of λ_{t-1}. r is the variable that starts with 0.05 and it increases by 0.15 for every 3000 attempts. And once it increase by 0.15, the threshold of attempts also raises by 1500. That is, after r becomes 0.2, it will need additional 4500 attempts to be raised again to 0.35, and so on.

If the solution from the above guess is not equal to our initial guess, i.e., $\tilde{\lambda}_t \neq \hat{\lambda}_t$, we update our guess according to $\hat{\lambda}' = (1 - w_u)\lambda_{t-1} + w_u \tilde{\lambda}_t$ and repeat previous step. The weight on the new solution starts with $w_u = \frac{1}{2}$ and get updated every 5 iterations. In essence, we also make w_u random so that it helps the convergence. We iterate the previous step until $\hat{\lambda}_t$ and $\tilde{\lambda}_t$ converges.

Finally, we do not directly solve on the grid of 45 points for $\log \lambda_t$. Instead, we first solve for the log-linearised version of the model, on a subset of the state space. Then, we extrapolate the log-linearized solution on a grid of 5 points with a larger subset of state space and use it as an initial guess and solve for the solution on the grid. Next, we extrapolate the solution of previous step (5 points) on a 7 points grid and use that as an initial guess and solve for the solution on the 7-points grids, and so on.

In summary, we do it iteratively. Gradually, we obtain the solution of the 45 points grid on the full state space. Doing so means that we only update the solution marginally at each step. In theory, for each adjacent steps, the solutions are very close in the function space. As a result, this practice does not only increase the probability of solving the model at each step, but also speeds up the process significantly.

D Estimation

D.1 Data Source

The FX data is from World Development indicators (WDI) from World Bank. Sample period is 1971-2019.

The consumption, GDP and net export data also come from the World Bank. We use households final consumption expenditure for consumption series, and the difference between the indices of export of goods and imports of goods and services as our net export series. Both consumption and GDP are real, PPP-adjusted.

Inflation rates are calculated using Consumer Price Index (CPI) from world bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country.

Real interest rates are constructed using three-month T-bills yields from the Global Financial Data, adjusting for realized inflation using annual changes in CPI. The interest rates series for New Zealand and Switzerland starts from 1978 and 1980, respectively. For the rest, the sample period is 1971-2019.

Data on equity index returns (MSCI series) is obtained from Datastream. Equity returns data for New Zealand starts from 1980, the rest is 1971-2019. Data on top 1% percentage income share is from World Inequality Database [8].

D.2 Identification

In Figure (9)–(11) we plot the slope of the model's implied moments $X(\theta)$ to small changes in parameters around the optimum $\hat{\theta}$. Specifically, we report

$$E_{i,j} = \frac{dX^j(\theta)}{d\theta^i} \tag{125}$$

Which is the numerical gradient computed using a five-point stencil around $\hat{\theta}$.

In addition, in Figure (6)–(8) we plot the Gentzkow and Shapiro (2014) measure of sensitivity of parameters to moments. We report the measure in elasticity form

$$\hat{\lambda}_{i,j} = \lambda_{i,j} \frac{X^j(\theta)}{\theta^i}$$

where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j. The matrix Λ is computed as

$$\Lambda = -(G'WG)^{-1}G'W$$

Where G is the numerical gradient of the sample moments $g(\theta) = X - \mathcal{X}$ and W is the weighting matrix.

In what follows, we summarize the main patterns in these Figures.

- The parameter δ is identified by (not well identified)

[8] https://wid.world/

- The mean and volatility of risk-free rate, and the volatility of exchange rates. The volatility of the exchange rate is mostly determined by the effective displacement effect in the market. When δ is small, the growth impact of u shock is small relative to its displacement effect. As a result, a lower level of δ leads to a higher impact of displacement effect and therefore more volatile pricing kernels. When δ increases, it increase the growth rate and therefore the mean of the risk-free rate.
- The correlation between exchange rate and net export. The displacement shock leads to not only currency appreciation, but also an increase in output growth. All else equal, an increase in δ strengthens the correlation between exchange rate and net export.

• The parameter μ is identified by (relatively well identified)

- The mean of consumption growth and output growth. The higher the μ, the higher the consumption and the output growth.

• The parameter h is identified by (not well identified)

- The mean of the excess returns. As h increases, the effective size of u shock diminishes. Since the equity risk premium is mostly driven by the displacement shock, its magnitude decreases with h.
- The correlation between exchange rate and consumption growth. The higher the h, the less of the households perceived impact of u shocks, as they put less weight on the effect of declined wealth share. Since the u (ε) shock contributes to the countery-cyclicality (pro-cyclicality) of net export, the correlation between exchange rate and consumption helps determine the level of relative consumption.

• The parameter β and ξ are identified by (Relatively well identified)

- The mean of risk-free rate and the level of income inequality. The risk-free rate in the model is directly linked to the effective discount rate, which itself is a product of discount rate and the survival rate (one minus death rate). These two parameters play a similar role in the model, as they both contribute to the effective discount rate. But they affect inequality in a slightly different way: the higher the discount rate, the higher the price-dividend ratio and therefore a lower share of top income – as the accumulated wealthy people earn less dividend on their wealth. On the other hand, as the death rate increases, it lowers wealth inequality – as people have less expected "time" to accumulate their wealth before they die.

Consequently, the level of inequality and the risk-free rate help determine these two parameters.

- The parameter α is identified by (relatively well identified)
  - The volatility of exchange rate. The higher the home-bias, the larger the effective size of u shock at its own country whose risks can not be diversified away. Hence it weakens the effect of trade and therefore increases the volatility of SDF.
  - The mean and volatility of stock returns. The higher level of home-bias increases the volatility of pricing kernels and therefore the volatility of stock markets.
- The parameter γ is identified by (not well identified)
  - The mean and volatility of excess returns. This parameter is not well identified – has relatively large standard errors.
- The parameters ψ is identified by (not well identified)
  - The volatility of risk-free rate. A higher willingness for inter-temporal substitute leads to lower variability in risk-free rate.
- The parameter p is identified by (relatively well identified)
  - The coefficient of consumption in the bi-variate regression with consumption and inequality. All else equal, a more persistent low state weakens the linkage between consumption and u shock, and therefore lowers the coefficient of consumption in the bi-variate regression with consumption and inequality.
- The parameter q is identified by (relatively well identified)
  - The volatility of risk-free rate and consumption growth. The higher persistence leads to a higher variability in the long-term consumption growth. The volatility of consumption growth. All else equal, a more persistent high-state of u shock increase the volatility of u shock. Given that u shock is less correlated than the ε shock, it lowers the correlation between consumption growth.
  - The size of excess returns. With a recursive preferences, a more persistent shock leads a higher compensation for risks in equilibrium.
  - The coefficients of bi-variate regression with consumption and stock markets, as it effectively controls the size of (large) displacement shock.

- The parameter u_1, u_2 are identified by (relatively well identified)
  - The volatility of risk-free rate and stock market. The difference between u_1 and u_2 determines the volatility of u shock, which in turn determines the volatility of most of the dynamics in the model. All else equal, a volatile stock market and exchange rate can be interpreted as evidence of more dispersed u_1 and u_2. That is, a small u_1 and a large u_2.
  - The level of top income share. The displacement shock determines the level of inequality. All else equal, a larger magnitude of displacement shocks lead to a higher level of top income share.
  - A larger u_2 also increases the magnitude of coefficients in the bi-variate regression with consumption and inequality, and the coefficients in the bi-variate regression with consumption and stock returns.
- The parameter σ_e is identified by (relatively well identified)
  - The volatility of consumption growth and output growth. Given that the estimated δ is relatively small, the volatility of aggregate output and aggregate consumption is mostly driven by the neutral shock.
- The parameter ρ_e is identified by (relatively well identified)
  - The correlation between aggregate bilateral consumption and bilateral output. As mentioned above, the volatility of aggregate consumption and output are determined by the neutral shock. Consequently, the correlation between aggregate consumption and aggregate output are determined by the correlation of neutral shocks.
  - The correlation between exchange rate and cnosumption/output growth. Recall that the neutral shock contributes to the counter-cyclicality of exchange rate while displacement shock contributes to the pro-cyclicality of exchange rate. A more correlated neutral shock weakens the effective size of neutral shock on the exchange rate and therefore strengthens the pro-cyclicality.
- The parameter ρ_c is identified by (relatively well identified)
  - The volatility of stock markets and exchange rate. As ρ_c increase, u shocks are becoming less correlated. This leads to a more volatile exchange rate.

- The correlation between the stock markets. Recall in figure 3 that stock market volatility is mostly driven by u shock and that u shock contributes to the negative correlation in the stock market. Therefore, the positive correlation between the stock market in the data is informative about the amount of technological spillover – that is, the positive correlation between u shocks in the model.

- The parameter π is identified by (not well identified)
  - The size of excess returns. A higher π lowers the effectiveness of displacement shocks. As the excess returns are mostly driven by the displacement shock, the size of it helps determine the size of population that is affected.
  - The coefficient of inequality in the bi-variate regression. A higher level of π lowers the concentration of capital income and therefore weakens the relation between u shocks and top income share. As a result, a higher π lowers the correlation between top income share and exchange rate.
  - The level of income inequality. The displacement effects in wealth shares decreases in π.

D.3 Estimation Methodology

The model has a total of 16 parameters. We put two restrictions on the dynamics of u shocks to reduce the number of parameters. First, we assume that $u_1 = u_2$. Hence, a transition from u_1 to u_2 only affects the future distribution of u (as the transition probabilities change) rather than the current level of displacement. Second, we assume that the matrix T corresponds to transition matrix of a discretized AR(1) process, so that it could be parameterized by only two parameters—the corresponding autocorrelation parameter p and q. Specifically, we assume that the transition matrix has the following form

$$T = \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-q) & pq + (1-p)(1-q) & q(1-p) \\ (1-q)^2 & 2q(1-q) & q^2 \end{bmatrix} \tag{126}$$

Where p^2 is the probability of staying in the lowest state once already there and q^2 is the probability of staying in the highest state once there. We estimate the remaining parameters of the model using a simulated minimum distance method Lee and Ingram (1991). Specifically, given a vector of X of target statistics in the data, we obtain parameter estimates by

$$\hat{p} = \arg\min_{p \in \mathcal{P}} \left(X - \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p) \right)' W \left(X - \frac{1}{S} \sum_{i=1}^{S} \hat{X}_i(p) \right) \tag{127}$$

Where $\hat{X}_i(p)$ is the vector of statistics computed in one simulation of the model. Our choice of weighting matrix $W = diag(XX')^{-1} I_W$ penalizes proportional deviations of the model statistics from their empirical counterparts. I_W is a diagonal matrix that adjusts for the relative importance of the statistics in our estimation. We apply a factor of 10 on the equity risk premium and the volatility of exchange rate. The rest elements on the diagonal of I_W are normalized to one.

We use different weights on the diagonal of I_W to reflect the relative importance of the following moments: equity risk premium and the volatility of exchange rate. We do this because the magnitude of these moments are relatively well documented in the literature, and also speaks directly to the model's mechanism. For instance, the level of income inequality is directly linked to the size of u shock that drives most of the dynamics in the model.

Our estimation targets are reported in the first column of Table 6. They include a combination of first and second moments of aggregate quantities, asset prices and exchange rates. In additional to these standard international moments in the literature, we also target a set of correlations. The neutral shock and displacement shock have different implications for the cyclicality of the exchange rates. Thus, the set of correlation between exchanges rates and consumption, output and stock market, together with the set of bilateral correlations, are informative about the relative magnitude of these two shocks.

In addition, we target the average top 1% income inequality of the United States and the estimated coefficients of bi-variate regressions (38). In the model, we consider the stock market as a levered claim of domestic consumption goods by a factor of 2.

We simulate the model at annual frequency. For each simulation, we first simulate 100 years data as burn-in, to remove the samples' dependencies on the initial condition. Then, we simulate the data for 50 years – the same length as our empirical sample. The simulation starts with the symmetric steady state where the displacement shocks are at the middle state and $\lambda = 1$. In each iteration we simulate 10000 samples, and simulate pseudo-random variables using the same seed in each iteration.

We compute standard errors for the vector of parameter estimates $\hat{p}$ as

$$V(\hat{p}) = (1 + \frac{1}{S}) \left(\frac{\partial}{\partial p} \mathcal{X}(p)' W \frac{\partial}{\partial p} \mathcal{X}(p) \right)^{-1} \frac{\partial}{\partial p} \mathcal{X}(p)' W' V_X(\hat{p}) W \frac{\partial}{\partial p} \mathcal{X}(p) \left(\frac{\partial}{\partial p} \mathcal{X}(p)' W \frac{\partial}{\partial p} \mathcal{X}(p) \right)^{-1} \tag{128}$$

where

$$V_X(\hat{p}) = \frac{1}{S}\sum_{i=1}^{S}(\hat{X}_i(p) - \mathcal{X}(\hat{p}))(\hat{X}_i(\hat{p}) - \mathcal{X}(\hat{p}))'$$

is the estimate of the sampling variation of the statistics in X computed across simulations.

The standard errors calculated in (127) are computed using the sampling variation of the target statistics across simulations (128).

Solving each iteration of the model is costly, and thus computing the minimum (127) using standard methods is infeasible. We therefore use the Radial Basis Function (RBF) algorithm in Bjrkman and Holmstrm (2000). The Bjrkman and Holmstrm (2000) algorithm first fits a response surface to data by evaluating the objective function at a few points. Then, it searches for a minimum by balancing between local and global search in an iterative fashion. We use a commercial implementation of the RBF algorithm that is available through the TOMLAB optimization package.

D.4 Construction of Estimation Targets

Consumption, output and net export. Output is gross domestic product. Consumption is households and NPISHs final consumption expenditure (private consumption). Net export is the exports of goods and services minus the imports of good and services.

Standard deviation of aggregate quantities. We first calculate the standard deviation for each US-foreign country pair, and then we take the average and use that as our target.

Correlations between aggregate quantities. Similar to the standard deviation, we first calculate the correlation for each US-foreign country pair, and then we take the average and use that as our target.

Real exchange rate. Inflation rates are calculated using Consumer Price Index (CPI) from world bank. The real exchange rate are calculated by adjusting nominal exchange rates by the relative CPI index of the corresponding country.

Risk free rate and Stock market returns. Risk free rate is constructed using three-month T-bills yield, adjusting for realized inflation using annual changes in CPI. Stock market returns are obtained using MSCI indexes from Datastream.

UIP coefficient. for each US-foreign country pair, we regress the exchange rate growth from t to $t+1$ on the interest rate differentials at t:

$$\Delta e_{US,F,t,t+1} = \alpha_F + \beta_{UIP,F}(r_{F,t} - r_{US,t}) + \varepsilon_{F,t}$$

Then we take an average of the estimated $\beta_{UIP,F}$ across all countries F in our sample.

Inequality and the Bi-variate regression. Data is from World Inequality Database, the top 1% income share including capital income. For the bi-variate regression, we use the estimated coefficients of the panel regression of (38) and (3) with country fixed effects. In these regressions, independent variables are standardized using unconditional moments.

Figure 6: We report the Gentzkow and Shapiro (2014) sensitivity measure of the estimated parameters to moments. We report the measure in elasticity form, $\lambda_{i,j}\frac{X^j}{\theta^i}$ where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j.

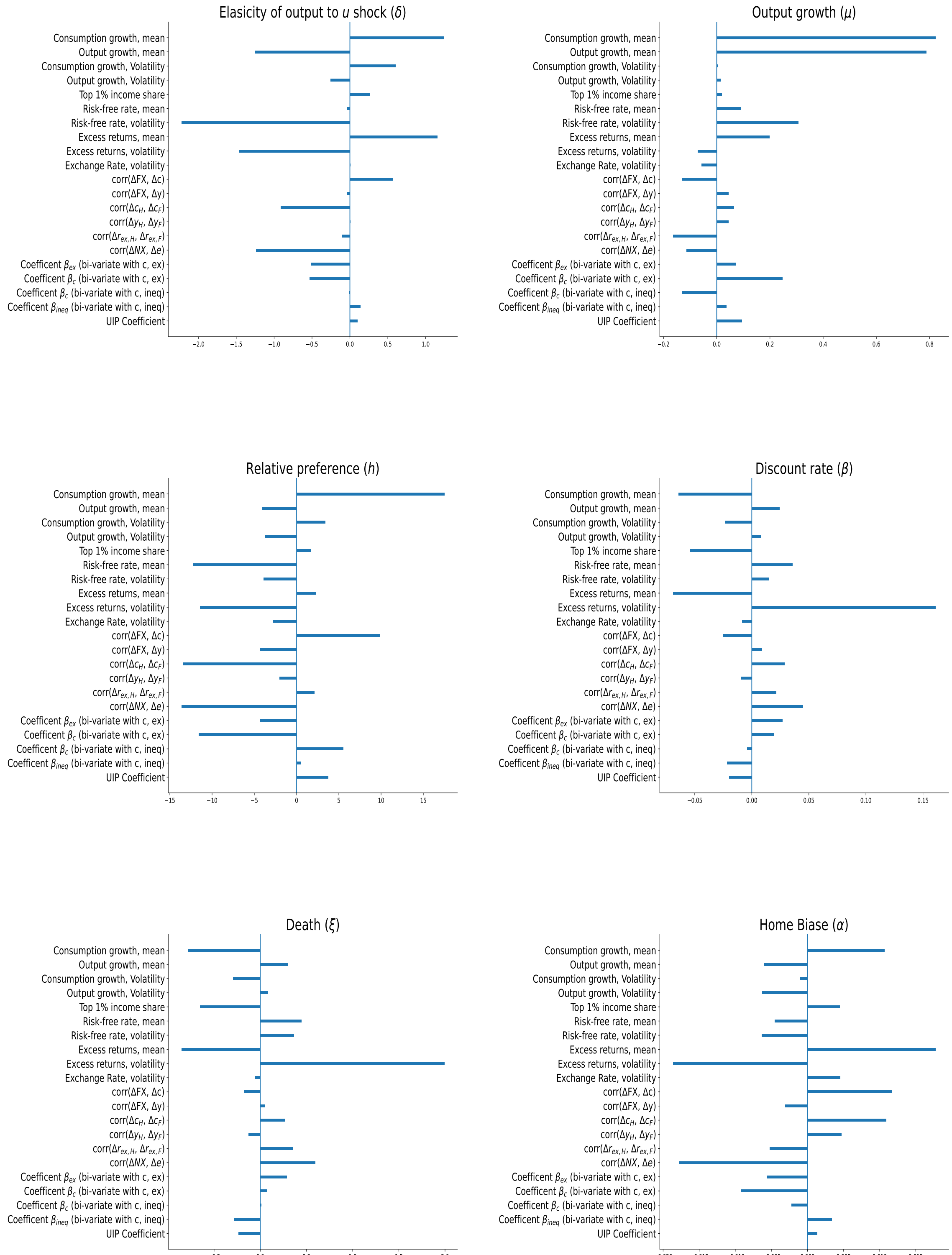


Figure 7: We report the Gentzkow and Shapiro (2014) sensitivity measure of the estimated parameters to moments. We report the measure in elasticity form, $\lambda_{i,j}\frac{X^j}{\theta^i}$ where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j.

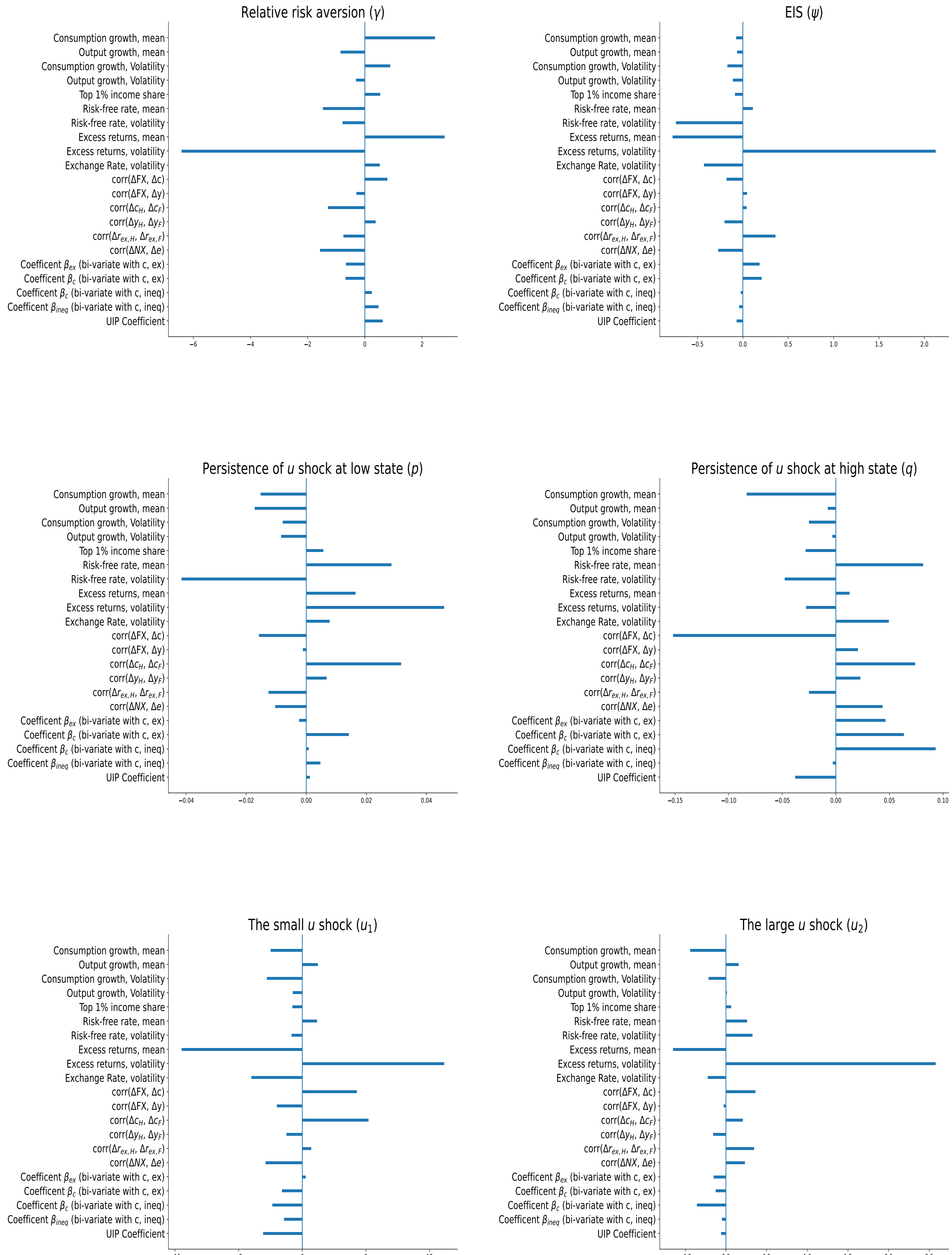


Figure 8: We report the Gentzkow and Shapiro (2014) sensitivity measure of the estimated parameters to moments. We report the measure in elasticity form, $\lambda_{i,j}\frac{X^j}{\theta^i}$ where $\lambda_{i,j}$ is the element of the sensitivity matrix Λ that corresponds to parameter i and moment j.

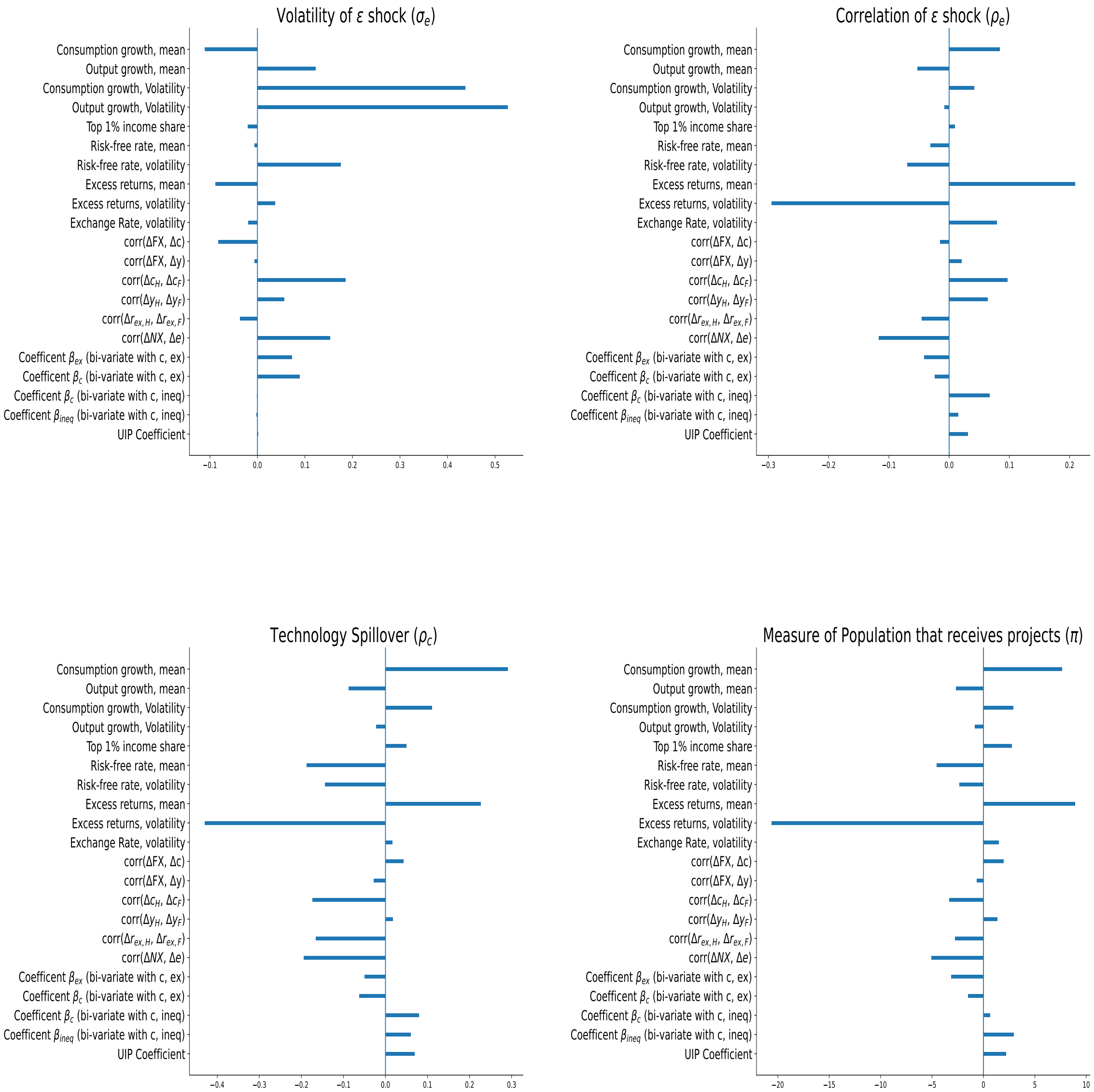


Figure 9: We report the sensitivity of moments estimate $\mathcal{X}(\theta)$ to parameter θ. Specifically, we report the numerical derivative $\frac{dX^j(\theta)}{d\theta^i}$– computed using a 5-point stencile – of moments j to parameter i.

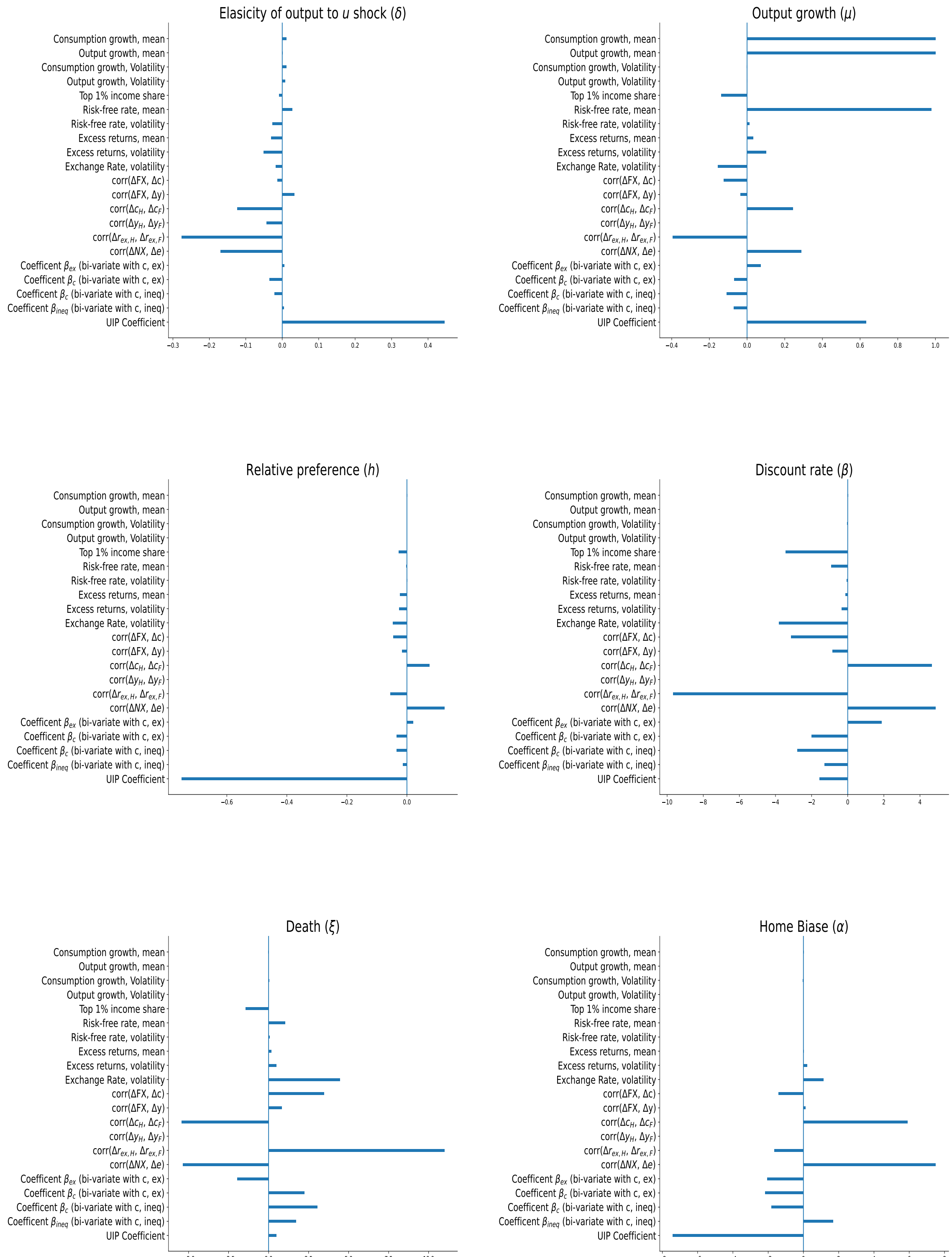


Figure 10: We report the sensitivity of moments estimate $\mathcal{X}(\theta)$ to parameter θ. Specifically, we report the numerical derivative $\frac{dX^j(\theta)}{d\theta^i}$– computed using a 5-point stencile – of moments j to parameter i.

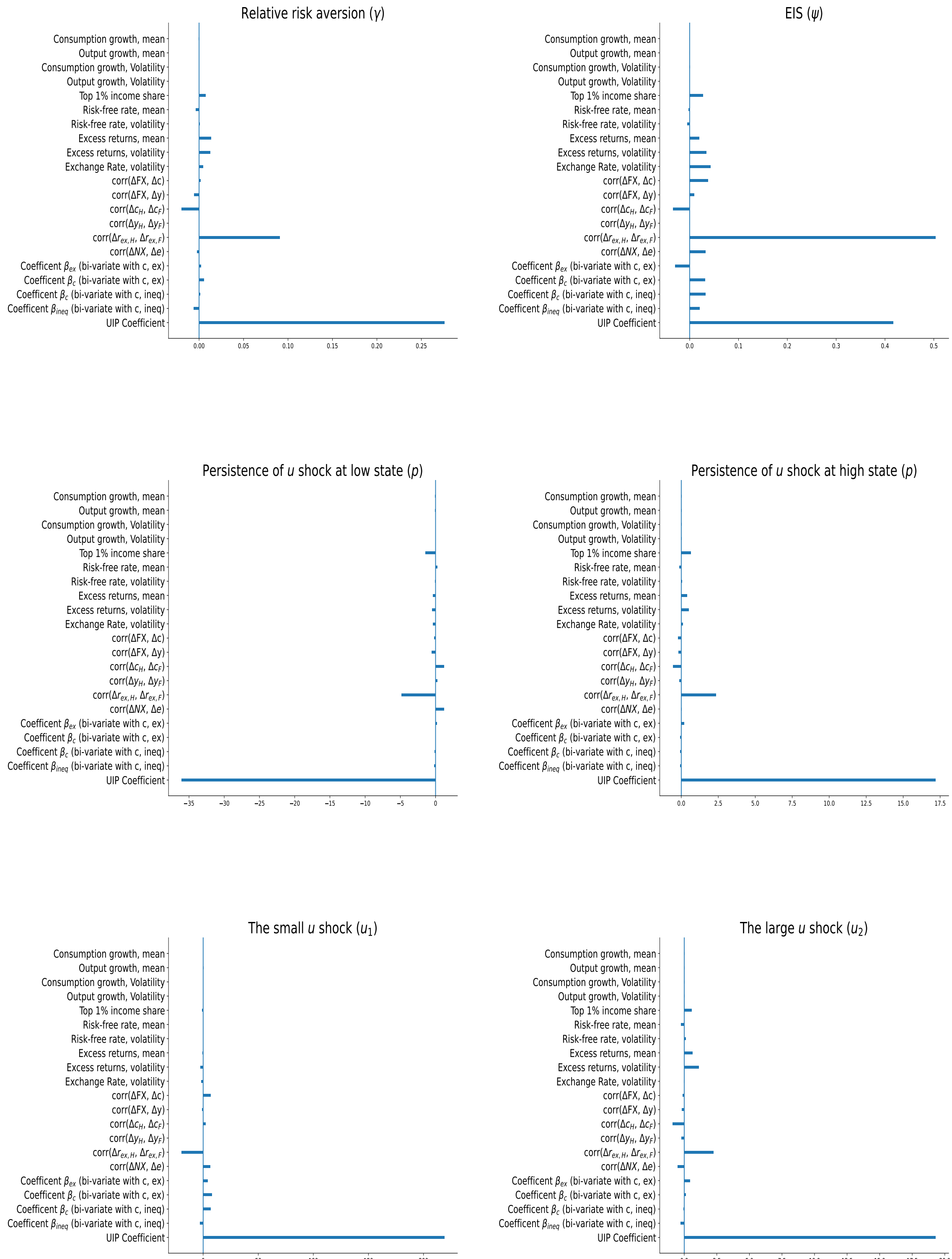
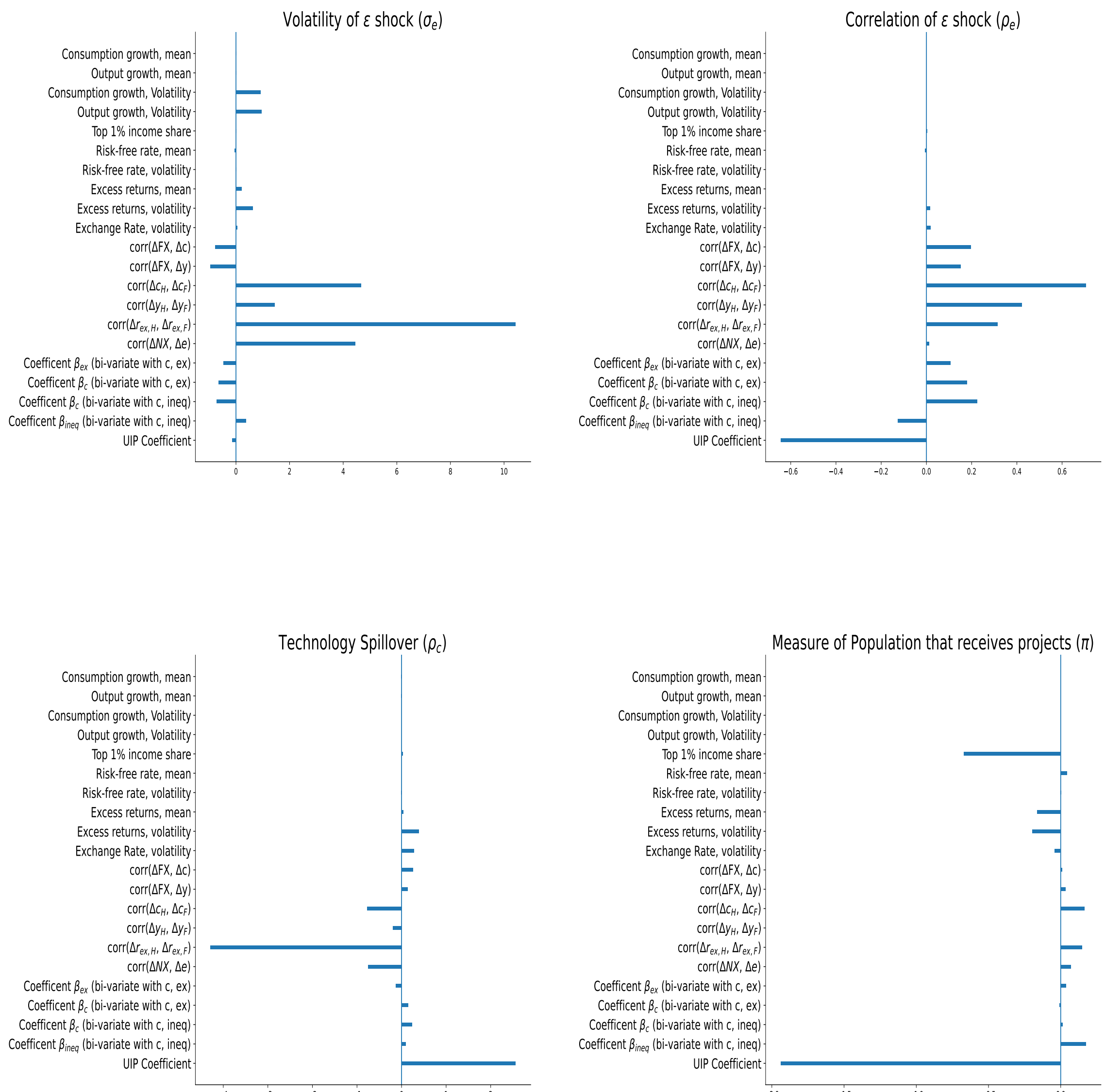


Figure 11: We report the sensitivity of moments estimate $\mathcal{X}(\theta)$ to parameter θ. Specifically, we report the numerical derivative $\frac{dX^j(\theta)}{d\theta^i}$– computed using a 5-point stencile – of moments j to parameter i.



Appendix Figures and Tables

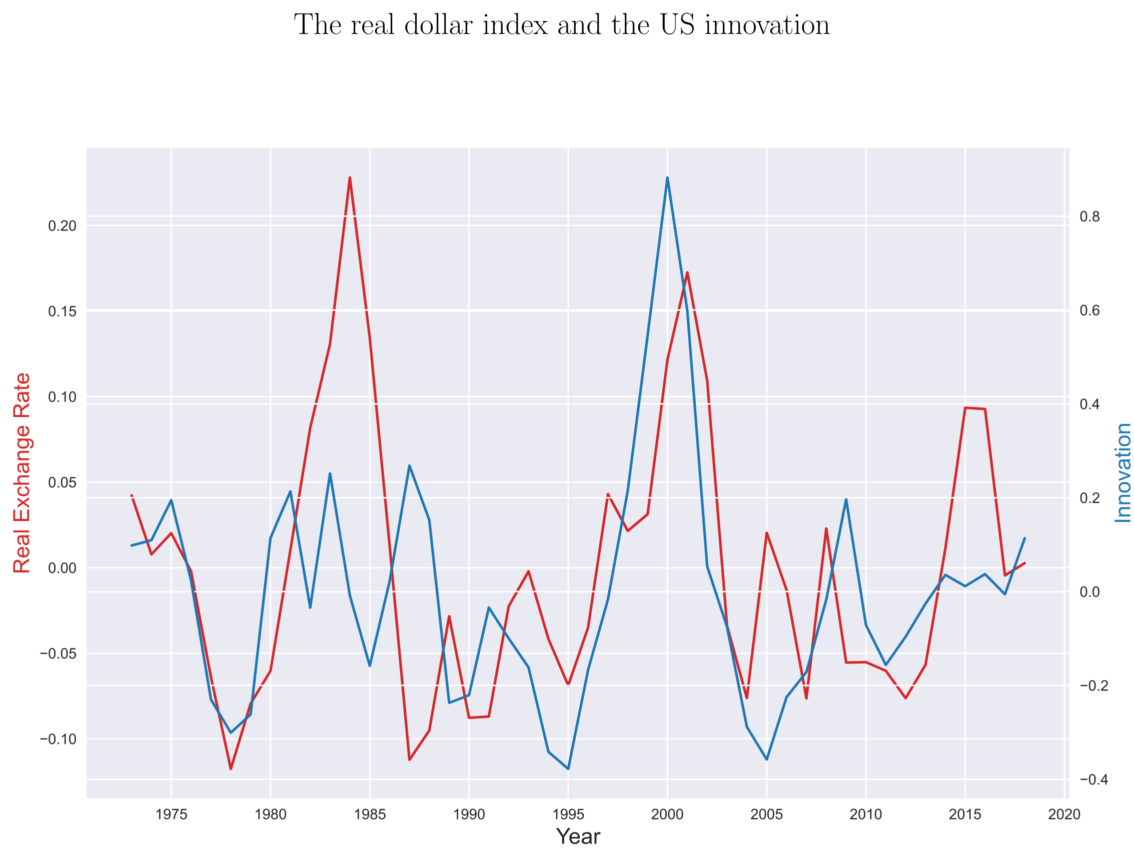


Figure A.1: This figure plots the real dollar index and the US innovation. The dollar index in red is the traded-weighted real advanced foreign economy dollar (AFE) indexes, calculated by the Fed. The US innovation series in blue plots the sum of the value of patents granted in year t scaled by aggregate output, using methodology in Kogan et al. (2017).

Table A.1: Output growth, stock market returns and exchange rate growth

	GDP growth	Stock return	$R^2(\%)$	Observations
Panel	0.010**	-0.015**	17.15	458
	(0.004)	(0.007)		
Australia	0.010	-0.011	31.16	49
	(0.013)	(0.015)		
Canada	0.015	-0.005	26.98	49
	(0.009)	(0.009)		
France	0.008	-0.008	40.50	28
	(0.019)	(0.03)		
Germany	0.002	-0.031	40.34	28
	(0.018)	(0.019)		
Italy	0.001	-0.014	26.08	28
	(0.018)	(0.021)		
Japan	-0.010	-0.003	21.06	49
	(0.015)	(0.02)		
New Zealand	0.040**	-0.022	41.32	41
	(0.018)	(0.019)		
Norway	0.008	0.011	16.84	49
	(0.015)	(0.016)		
Sweden	0.010	-0.038**	17.16	49
	(0.016)	(0.016)		
Switzerland	0.013	-0.048***	41.38	39
	(0.019)	(0.016)		
United Kingdom	0.023	-0.020	27.50	49
	(0.018)	(0.022)		

Notes: The table reports regression results of the growth of log exchange rate on both 1-year log output growth (first column) and growth of log top 1% income share ratio (second column)

$$\log e_{t+1} - \log e_t = \beta_1 \Delta \log C_{t,t+1} + \beta_2 \Delta r_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. Stock market returns (MSCI Indexes) data are from Datastream. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table A.2: Exchange rate growth and inequality growth

	Inequality growth	$R^2(\%)$	Observations
Panel	0.012**	14.26	406
	(0.005)		
Australia	0.023	15.52	49
	(0.015)		
Canada	-0.007	10.04	49
	(0.007)		
France	0.016	18.42	28
	(0.022)		
Germany	0.030	19.48	18
	(0.028)		
Italy	0.065**	36.11	18
	(0.024)		
Japan	0.005	7.28	39
	(0.013)		
New Zealand	0.019	16.01	49
	(0.014)		
Norway	-0.012	14.05	39
	(0.016)		
Sweden	0.015	13.24	39
	(0.021)		
Switzerland	0.014	14.50	39
	(0.011)		
United Kingdom	0.029*	29.29	39
	(0.015)		

Notes: The table reports regression results of the growth of log exchange rate on log income inequality growth ratio.

$$\log e_{t+1} - \log e_t = \beta \Delta \log I_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table A.3: Consumption growth, inequality growth and exchange rate growth

	Consumption growth	Inequality growth	$R^2(\%)$	Observations
Panel	0.014**	0.017**	16.87	406
	(0.005)	(0.006)		
Australia	-0.000	0.051***	43.15	49
	(0.009)	(0.012)		
Canada	0.028*	-0.013	22.45	49
	(0.014)	(0.01)		
France	0.008	0.013	40.92	28
	(0.023)	(0.013)		
Germany	0.004	-0.033	46.79	18
	(0.025)	(0.026)		
Italy	0.006	0.026	46.60	18
	(0.024)	(0.034)		
Japan	-0.039	0.024	16.76	39
	(0.027)	(0.02)		
New Zealand	0.015	0.015	19.25	49
	(0.017)	(0.016)		
Norway	0.038*	0.003	22.67	39
	(0.021)	(0.021)		
Sweden	0.008	0.032	20.76	39
	(0.027)	(0.022)		
Switzerland	0.014	0.009	22.09	39
	(0.015)	(0.013)		
United Kingdom	0.027*	0.036*	41.19	39
	(0.016)	(0.018)		

Notes: The table reports regression results of the growth of log exchange rate on both 1-year log consumption growth (first column) and growth of log top 0.1% income share ratio (second column)

$$\log e_{t+1} - \log e_t = \beta_1 \Delta \log C_{t,t+1} + \beta_2 \Delta I_{t,t+1} + \gamma \log e_t + \varepsilon_{t+1}$$

The sample period is 1971-2019. The unbalanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, France and Italy. Independent variables are standardized to unit standard deviation using unconditional moments. In individual country regressions, standard errors (in parentheses) are obtained using Newey-West with one period lag. The Panel regressions include country fixed effects, and standard errors in parentheses are obtained by clustering at the country level. Income inequality data is from World Inequality Database. Exchange rate, consumption and GDP data are from the World Bank and the IMF. Stock market returns (MSCI Indexes) data are from Datastream. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.