# Incentives for Traders: Ideal and Heuristic Contracts

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**Abstract**

Separation of information gathering and execution is a common assumption in models of delegated portfolio management. This assumption is, however, inappropriate for contracting with traders, for whom information gathering and action are inseparable, as in Demski and Sappington (1987)'s delegated expertise. Because the optimal contract is not solvable in this setting, we look instead at heuristic contracts based on what is readily observed, and we compare their efficiency to using the unavailable ideal contract (as in Dybvig, Farnsworth, and Carpenter (2010)). We find that the heuristic contract captures most of the ideal contract's efficiency gain. A convex quadratic reward for a large portfolio position is most useful for improving a linear contract. A negative term proportional to realized variance of the underlying is also useful, because it makes it more costly to miss directional signals. If the incentive contract is convex quadratic in return, it is a disaster because it creates a bad incentive to take on arbitrarily large risk.

Keywords: Contract theory, information acquisition, trading execution, quadratic incentive terms, futures trading.

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# 1 Introduction

When an investment bank hires a trader, what is the best contract? We build models of compensation of futures desk traders who exert costly effort to obtain information and execute trades based on the information. In this paper, we propose heuristic contracts that incentivize effort and beneficial use of the information. We find that a quadratic reward on risky positions can significantly incentivize effort while a negative term on realized variance of the underlying can provide additional marginal incentives for effort. However, a convex quadratic reward in return is not useful, because it creates a bad incentive to take on arbitrarily large risk.

A trader's problem is different from most existing models, because a trader's information-gathering and execution of trades cannot be separated. On the one hand, information may be hard to communicate, possibly due to the necessarity of a deep knowledge base, or due to the time limit of the transitory trading opportunity. On the other hand, it is also hard for the principal to back out the trader's information *ex post*. This makes it inappropriate to use models, such as those of idealized incentive contracting, which assume that trading and information-gathering are separable.[1]

Our analysis can be viewed as delegated expertise modeled by Demski and Sappington (1987). Like them, we assume that information acquisition and execution are inseparable and that both are delegated to the agent. We use a doctor's diagnosis to illustrate delegated expertise. The doctor obtains a lot of observations, for example, the patient's description of symptoms and other behavior, sound of the heart, blood and urine test results, chest X-ray etc., and combines the information with previous experience and knowledge to rule out possiblities. The information is high-dimensional and is costly and complicated to communicate. Even if the doctor can communicate all the information, the patient wouldn't know the diagnosis, because it's necessary to use the doctor's experience and medical training to process the information. It would be prohibitively costly (and for experience, impossible) to communicate the doctor's observations and the doctor's knowledge base. The same applies to traders who have expertise and access to private information that is useful in making a trading decision. If anything, trading is a purer case: while it is efficient for the doctor to communicate some processed information to the patient, not communicating anything seems reasonable for a trader.

[1] "Separable" can mean separate people or that contracting is separable. Separable contracting means the structure of the contract makes it as if separate agents are making the choices.

Timing is another consideration that makes it hard, if not impossible, for traders to communicate useful information. Traders learn information in the process of trading and tend to have profit opportunities that will evaporate if they don't hit the quotes immediately. For example, in an OTC market, traders call others to bargain on what price is available and build optimal trading patterns based on heuristics from the past that may be hard to communicate. The bargaining process is also part of information gathering. If information gathering and trade execution were conducted separately, by the time the information gatherer reported information to the actual trader, the trading opportunity would be gone. This inseparability also applies to electronic trading, especially the high speed trading in markets that vary from millisecond to millisecond.

An alternative interpretation of observability of the information is that it may not be reported, but instead is inferred from trades or actions of the traders. In Dybvig, Farnsworth, and Carpenter (2010), there is a frictionless market with a single price, and everyone knows the stock price and the trading opportunities, at least ex post. With precise information ex post, we could punish the agent for not doing what the principal would have the agent do. However, in reality the complex process of trading makes it almost impossible to back out the exact information the trader used in the trading process. This is true especially for OTC securities but realisitically for almost everything, only the trader knows what trading opportunities are available, even ex post. We can think of this as a matter of dimensionality; we cannot infer high-dimensional information from low-dimensional trades. This makes contracting in practice different from the benchmark model. In other words, backing out everything we would like to know is much easier in our tractable models with few enough variables to be solved than it is in practice.

Dybvig, Farnsworth, and Carpenter (2010) have two choices that should be optimal: the effort level and the portfolio choice. In the first-best, both the effort and portfolio choice (and thus the signal based on which the portfolio choice is made)[2] can be verified. In the second-best, the principal can only observe the signal, but not the agent's effort. In the third-best, neither is observable to the principal. The agent reports the signal but needs incentives to make reporting truthful. In this paper, we will use the word "idealized optimal contracting"[3] for the third-best in Dybvig, Farnsworth, and Carpenter (2010).

[2]The first-best solution in Dybvig, Farnsworth, and Carpenter (2010) has a one-to-one correspondence between the reported signal and portfolio choice.

[3]Justification of "idealized optimal contracting" is the revelation principle, which says that optimal contracting can be obtained by a "direct mechanism," in which all the agent's information is reported truthfully.

By extension, we can view delegated expertise in Demski and Sappington (1987) as the fourth-best of the problem: while both effort and signal are not observable, there is also no opportunity to report the signal between receipt of the signal and choice of portfolio. Given that the information is high-dimensional and hard to quantify, and there might not be a time between receipt of signal and outcomes for communication or recording of information to take place. In our paper, we have a heuristic formulation for this fourth-best, and it is compared with the "idealized optimal contracting" benchmark of the third-best.

As has been a traditional starting from Ross (1973), we formulate the problem from the principal's perspective: The principal selects the contract for the agent and plans the agent's choices, subject to the constraint that the agent is willing to accept the contract and act as the principal plans. The principal and agent face two information-asymmetry problems: the principal does not observe either the agent's effort (hidden action) or the agent's information (hidden information). Therefore both moral hazard and adverse selection exist in the problem. It is important for the principal to design a contract that incentivizes the agent to pick the optimal effort to obtain a signal and to execute trading based on the contract.

We propose heuristic contracts based on the ideal solution in the constrained case, adding restrictions of the signal reporting or trading constraint using revelation principle. We analyze the constrained problem using revelation principle as in Hölmstrom (1977)[4] and Myerson (1979), and compute a numerical solution[5]. The ideal contract rewards the agent for taking active positions and imposes something like a quadratic bet on the signal being correct. The rewards incentivize the agent to take risks, and the quadratic bet encourages effort to obtain precise information and penalizes imprecise signal. The observation of the ideal (numerical) solution informed our selection of heuristic contracts to consider.

The effective heuristic contracts we find use quadratic incentive terms. Admati and Pfleiderer (1997) shows that compensation linear in returns does not provide incentive for effort because the impact of the slope can be undone by changing the portfolio weight. This typically leads to underprovision of effort because effort that is optimal for the

[4]The revelation principle is on page 7 of Hölmstrom (1977), and is proven in footnote 7 to Chapter I.

[5]Actually, we solve the problem analytically in the first-best where neither type of information asymmetry is a problem, and in second-best if only effort is unobservable. See both solutions in Appendix D. For the third-best case where neither the effort nor the signal is observable to the principle, we can only obtain a numerical solution, as presented in Section 5.

single-agent problem ignores the benefit of effort for the principal. It might seem natural to have compensation convex and quadratic in returns, to make the agent more willing to take on risk and therefore have more incentive to expend effort. However, we prove that such contracts are terrible because they give the agent an incentive to take on an arbitrarily large amount of risk. Given that compensation quadratic in returns doesn't work, we consider instead adding quadratic terms in the portfolio weight and the realized variance of the underlying contract. These terms allow compensation to mimic the payoff of the unavailable ideal benchmark, and capture most of the gains of using the unavailable ideal contract.

Heuristic contracts that we proposed for the trader's problem are effective in balancing incentive for effort and efficiency of risk sharing. We find that allowing for a negative term on variance only works through incentivizing the effort, and cannot change the risk sharing between the agent and principal. However, adding a quadratic term on risky position improves both the incentive for effort and allows for better risk sharing. Intuitively, adding a non-negative reward increases the expected payoff of the agent, and if the agent's risk tolerance is thus higher, it makes the agent's optimal share in this portfolio increases. The agent is thus more incentivized to make effort to obtain and use more precise information[6].

One thing to notice is that our heuristic rules have a requirement that the trader's positions on quadratic terms need to be prespecified in the contracts, in order to prevent the agent from placing bets on volatility of the market, by trading options, to undo the incentives as in Admati and Pfleiderer (1997). In practice, it should be reasonable for the investment banks to implement similar requirement to prevent traders from holding private positions to undo their incentive contracts.

## 1.1 Connection to the literature

As we know from Admati and Pfleiderer (1997), a linear contract cannot incentivize effort, nonlinear contracts with various incentive terms has been studied in the literature. For example, Stoughton (1993) studies quadratic contracts proposed in Bhattacharya and Pfleiderer (1985) and shows that they help to overcome the underinvestment problem of linear contracts. Huang, Qiu, and Yang (2020) shows that adding a quadratic

[6]Pay attention that higher expected payoff does not always lead to a higher risk tolerance in general, as discussed in Ross (1973)

trading cost[7] helps to prevent the manager from undoing the contract completely, but it is not necessarily the correct amount of curvature for optimal contracting. Li and Tiwari (2009) proposed contracts with an asset-based linear part (as in Stoughton (1993)) and a benchmark-adjusted linear part (as in Admati and Pfleiderer (1997)) plus an option-like bonus. They find that with the given structure, the option-like term plays the key role to incentivize the effort. In this paper, we compare various quadratic terms in a general and comprehensive setting, and find that a convex quadratic reward for a large portfolio position is most useful for improving a linear contract, while a negtive term on realized variance of the underlying is also helpful.

Although benchmarking might be important in delegated portfolio management, it is not an issue in our problem. Or equivalently, we can consider the benchmarking position to be zero in the futures' trading problem. For a futures' desk trader whose job is to conduct informed trading, the obtained signal should inform on trading direction, i.e., whether to go long or short in the futures contracts. With neutral information, the optimal trading position should be zero, and there is no superior risk premium to collect. As a result, the trading positions built by traders can been considered as "Active Share" in Cremers and Petajisto (2009), who find that agents' active deviations from benchmark are usually informative and can predict performance. Further, Hunter, Kandel, Kandel, and Wermers (2014) develop "active peer benchmark" to enhance effective fund selection, while in this paper we look at the agent's problem in isolation without considering the contests among traders. We can interpret the traders in our model as liquidity providers in commodity futures market who trade on transitory profit opportunities. See Cheng and Xiong (2014) for a review on financialization of the commodity futures market and how financial investors affect risk sharing and information discovery in commodity markets.

In a dynamic setting where both the principal and agent learn about the agent's ability in the delegated investment process, Pegoraro (forthcoming) finds that the optimal contract should have the incentive pay and delegated capital being convex in past performance. While good past performance indicates better ability of the agent in that paper, a larger risky position taken by the agent indicates more effort and risk-taking of the agent in our model. Consistently, a convex term on risky positions is the most useful incentive term in our optimal heuristic contracts, and it incentivizes more effort and promotes better risk sharing. In addition, we show that adding a negative (concave) term on realized variance can incentivize effort as well. In a continuous model, Sung (2020)

[7]Such a market friction is similar to price pressure .

models the agency problem with mean-volatility joint ambiguity uncertainties and finds that the optimal contract consists of two sharing rules: one for realized outcome and one for realized volatility. The volatility sharing rule complements the other rule by aligning the agent's worst prior with that of the principal. Interestingly, our effective heuristic contracts have a negative term on the realized variance that aligns the interest of the agent with that of the principal. More specifically, such a sharing rule on the second moment of return can effectively incentivize the trader to make effort to obtain better information and take risk.

In this paper, we find that the key role of optimal contracts is to incentivize the agent to take active positions, and makes it unattractive to choose low effort and play it safe by taking little risk. This is consistent with the literature, for example, Stoughton (1993), Admati and Pfleiderer (1997), Li and Tiwari (2009) and Dybvig, Farnsworth, and Carpenter (2010). However, we find that the model's implication that we need to give the trader more incentive to take on risk a bit paradoxical in the face of an emphasis by practitioners on limiting the risk taken on by traders; and we discuss several alternative assumptions without resolving the paradox. In the literature, San Martín (2018) investigates the delegated trading problem from a risk management perspective, and finds that risk limits to the agent is an important component of the optimal contract that helps overcome agency frictions. Moreover, the model predicts that risk limits will be relaxed as the cost of accessing an expert second opinion increases, but tightened as the trader's cost of providing accurate research increases.

The paper is structured as follows: We first lay out the basic setting of the agency problem in Section 2. The details on assumptions and results of the heuristic model is given in Section 4, which is developed based on the benchmark solution in Section 5. Potential problems of quadratic contracts in return are discussed in Section 4.2. Section 6 concludes.

# 2 General Setting

We first lay out the general setting of the trader's problem, which fits both the constrained model and heuristic model. The problem is formulated from the principal's perspective as in Ross (1973).

**Initial Wealth and Market Return.** We assume that the market is complete and market states are distinguished by security prices. Let $x$ denote the market state and let $f(x)$ be the price density on the futures return. It is a single period model of commodity futures trading and payoffs will be realized at the end of the period. Traders are small in the futures market, i.e. their trades do not affect futures price.[8] The investor's initial wealth is assumed to be $W_0$, and the trader has no initial wealth in our model.

**Information Generation.** We assume that through making effort, the agent has the ability to generate information about futures' return. We model the information generation process using mixture model as in Rogerson (1985) that the signal obtained by the agent is a mixture of two distributions, the "informed" distribution and the "uninformed" distribution. Both the agent and principal believe that the agent's effort could increase the informativeness of the signal, which is represented by the "informed" part of the signal denoted $f^I(x,s)$, and that the other distribution should be "uninformed", denoted $f^U(x,s)$.[9]

Given effort $e \in [0,1]$, the joint density of return $x$ and signal $s$ is

$$f(x,s;e) = \pi(e) f^I(x,s) + (1-\pi(e)) f^U(x,s), \tag{2.1}$$

where

$$f^I(x,s) \sim N\left(0, \begin{pmatrix} \gamma^2+\sigma^2 & \gamma^2 \\ \gamma^2 & \gamma^2 \end{pmatrix}\right) \qquad \text{“informed”},$$
$$f^U(x,s) \sim N\left(0, \begin{pmatrix} \gamma^2+\sigma^2 & 0 \\ 0 & \gamma^2 \end{pmatrix}\right) \qquad \text{“uninformed”}.$$

Here, $\pi(\cdot)$ is a function describing how the effort determines the informativeness of the signal. In the informed distribution, $x$ and $s$ has a correlation $\rho = 1/\sqrt{1+\sigma^2/\gamma^2} > 0$, while $\rho = 0$ in the uninformed distributions. Thus in the mixture model, signal $s$ conveys information about the true return "$x$" through the informed channel whose weight can be affected by the effort. However, through uninformed distributions, signal cannot convey information of market state "$x$".

If $\pi(e)$ is a monotonically increasing function, it means that the agent's effort increases

[8]A possible extension is to consider when traders have market impact. Then the trader's strategic behavior in building the positions may make the contracting problem even more complicated.

[9]It is possible to allow for the agent and principal holding different beliefs or priors on asset prices and the associated risk, however this does not change the main conclusion.

the informed part of return density, and thus improves the understanding of futures' return. We assume that $s$ and $x$ are independent in the uninformed distributions, and we have expressions in products of the marginal distributions:

$$f^I(x,s) = f(x)f^I(s|x),$$
$$f^U(x,s) = f(x)f^U(s).$$

**Optimal Contracting.** In the setting, the principal designs the incentive scheme and proposes the contract to the agent, and lets the agent decide whether to accept or to reject it. Once the contract is accepted, the agent will choose the optimal effort to exert under the scheme of this contract. As a result of the effort, the agent receives the trading signal, based on which the agent constructs the portfolio. At the end, portfolio return is realized and payoffs are made to the agent and the principal according to the agreed contract.

We define the utilities for the agent and the principal as $u^A$ and $u^P$ respectively. They can be expressed as functions of $(x,s)$ in the constrained problem, which means that the level of utility of the agent and the principal depends on the asset return $x$ and trading signal $s$, rather than a constant value of choice. The effort is costly exerted, we assume the cost for the agent to expend the effort is $c(e)$. Following Grossman and Hart (1983), we use utility levels rather than consumption level as the choice variables. The agency problem is formulated as maximizing the principal's utility subjecting to a budget constraint and a participation constraint for the trader to accept the contract.

## 2.1 The Trader's Problem

As stated in the general setting, we formulate the trader's problem from the principal's perspective. In order to ensure the trader exert optimal effort and optimally use the perceived signal in the trading portfolio, we need to impose incentive compatible constraints such that the optimal choice for the trader would be exactly expected by the principal in the contract design.

The principal will maximize his utility

$$\max_{u^A,u^P,e^*} \int\int u^P(x,s)\left(\pi(e^*)f^I(x,s) + (1-\pi(e^*))f^U(x,s)\right)dxds, \tag{2.2}$$

subject to the budget constraint

$$(\forall s \in S) \int \left(W_A(x,s) + W_P(x,s)\right) f(x) dx = W_0, \tag{2.3}$$

the participation constraint[10]

$$\int\int u^A(x,s) \left(\pi(e^*) f^I(x,s) + (1-\pi(e^*)) f^U(x,s)\right) dx ds - c(e^*) \geqslant U_0, \tag{2.4}$$

and a constraint for simultaneous incentive compatibility of effort and truthful signal reporting

$$\{e^*, s\} = \underset{e,\theta(s)}{\arg\max} \int\int u^A(x,\theta(s)) \left(\pi(e) f^I(x,s) + (1-\pi(e)) f^U(x,s)\right) dx ds - c(e), \tag{2.5}$$

where $W_0$ is the initial wealth of the principal; $U_0$ is the reservation utility of the agent, which means the agent will only accept the contract if the agent's utility of accepting the contract is at least $U_0$; $e^*$ is the optimal effort, and $c(\cdot)$ is the cost function of effort. $\theta(s)$ is the signal that agent chooses to use, or to report in the benchmark model, and it is a function of the true signal $s$. When $\theta(s) = s$, the agent chooses to honestly use or report the obtained signal.

**Preferences.** With further assumption of constant absolute risk aversion (CARA) for both the agent and principal, we have the payoff functions for the agent and principal as

$$W_A(x,s) = -\frac{\ln\left(-u^A(x,s)\right)}{A},$$
$$W_P(x,s) = -\frac{\ln\left(-u^P(x,s)\right)}{P},$$

where $A$ and $P$ are the levels of risk aversion of the agent and principal, respectively.

# 3 Idealized Benchmark: Overview

In this section, we give a brief introduction to solution of the constrained problem. Although what we are interested to investigate in this paper is the agency problem that

[10] Note here $e^*$ indicates the cost of efforts the agent makes to achieve the trading signal. Without loss of generality, we can replace the monotonic function of cost $c(e)$ with $e$.

the agent does not have the opportunity to report the obtained signal. The nice thing about using a constrained setting as in Dybvig, Farnsworth, and Carpenter (2010) is that they use the revelation principle (Hölmstrom (1977)) which allows to solve the problem directly. This constrained problem makes sense for agents who actually need to report the obtained signal, or who can only choose from a menu of given portfolios. Based on the numerical solution to the constrained problem, we have proposed heuristic contracts that are effective without this additional signal reporting stage in Section 4.

Although our constrained problem provides a modified version of our target problem imposing a constraint as in Dybvig, Farnsworth, and Carpenter (2010), it is still very different from the other paper in several aspects: First, we solve the constrained problem with a different utility function: they use log utility and we use CARA throughout our paper. Second, the approach might be similar to solve the constrained problem, however the calculation is totally different when changing the utility function. Third, our target in this paper is not about solving a different version of the constrained agency problem, however it is to use the constrained solution as a starting point to solve the original trader's problem, and to further discuss the properties of heuristic solutions and of quadratic contracts that are commonly used in the literature and in practice.

There is no analytical solution for the payoff of the principal and agent. We solve for the form of principal's payoff as a function of budget share and pricing density, and obtain the numerical solution with reasonable parameters.

**Proposition 3.1.** *The solution of the expected utility conditional on $s$ for the principal in the principal's problem can be expressed as*

$$u^P(x,s) = -\exp(-PB^P(s) + \int f(x)\ln\left[\pi(e)f^I(s|x) + (1-\pi(e))f^U(s)\right]dx \\ - \ln\left[\pi(e)f^I(s|x) + (1-\pi(e))f^U(s)\right]) \quad (3.6)$$

*where*

$$B^P(s) = -\frac{1}{P}\int \ln\left(-u^P(x,s)\right) f(x)dx$$

*is the principal's budget share.*

*Furthermore, the Lagrange multiplier for budget constraint (2.3) is*

$$\lambda_B(s) = P\exp\left(-PB^P(s) + \int f(x)\ln\left[\pi(e)f^I(s|x) + (1-\pi(e))f^U(s)\right]dx\right). \quad (3.7)$$

*Proof.* See in Appendix B □

**Numerical Solution** Because analytical solutions are not available, we switch to numerical methods to obtain the solutions numerically. Let $n(\cdot;\cdot,\cdot)$ be the normal density parameterized by its mean and variance. The parameter values used are $\sigma = 0.1$, $\gamma = 0.2$, $W_0 = 100$, $U_0 = -0.9$, and the risk-free rate $r = 0$. In general, traders usually have higher risk aversion than partners in an investment bank, thus we consider possible pairs of risk aversion of $(A, P)$ as $(2, 1)$, $(2, 0.2)$ or $(10, 1)$. The optimal effort is set at $e = 0.4$.

We discretize $f^U(s)$, $f(x)$, and $f(x|s)$ in $N_x$ market states and $N_s$ signal states. The benchmark model is based on Dybvig et al. (2010) except with different assumptions about preferences (CARA instead of CRRA) and the distribution of payoffs (normality of the futures price with zero mean futures return). These assumptions value short and long positions symmetric, which is a good assumption for futures traders, who are not in the business of collecting the general market risk premium.

Figure 3.1 plots the numerical solution of the expected utility for agent in the constrained problem, where the axes are signal $s$ and market state $x$. We can draw a few observations from the ideal contract: First, when signal and market are both high (or both low), the agent is rewarded the most. We can observe from the figure that the expected utility is higher on the diagonal where $x$ matches with $s$. Especially, it reaches the peaks in the two corners, indicating the highest rewards for the correctly predicted extreme signals.

Second, for any given signal $s$, the expected utility exhibits a concave shape with respect to market state $x$. The observed concave reward on $x$ may come close to a quadratic bet on the signal being correct, which provides high utility if the signal predicts the market state, and low utility if the signal is imprecise.

However, reality differs in multiple directions from the contracting model setting. Once observed the signal, the profits are not immediate but need to be executed by the trader himself. The restriction on reporting the signal to an accountant for execution may be a too strong assumption to consider the case of a trader. Thus, we propose heuristic contracts that are more realistic and easy to implement in reality based on the above observations from the numerical solution.

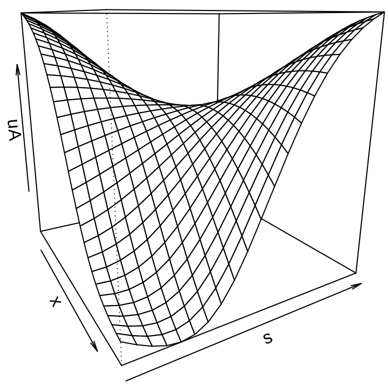


Figure 3.1: The ideal contract in the constrained problem, effort = 0.4

# 4 Heuristic Contracts

One of our main contribution of this paper is developing heuristic contracts that are effective and simple to implement in practice. The effectiveness of using heuristic rules comes close to the ideal solution in the benchmark model in Section 5, which is however not directly applicable for a trader's problem. The heuristic contracts fit the trader's problem setting with simple compensation rules motivated by the benchmark solution, and they explicitly present the economic driving forces of the effective contracts.

## 4.1 Heuristic Model

Based on observations of constrained optimum presented in Section 5, we propose heuristic contracts consist of a linear sharing plus several possible quadratic incentive terms. For example, adding a quadratic reward on risky positions to incentivize using extreme signals, and adding a negative term on the realized variance to reward efforts in obtaining a more precise signal. As a result, we build several possible heuristic contracts presented in the formulas (4.8) and (4.9).

The trader's problem is presented as in formulas (2.2)-(2.5). More specifically, we write down the payoff functions of proposed heuristic contracts for the agent and principal respectively, which nest several possible heuristic contracts.

$$W^A = \phi_0 + \phi_1 \theta(s) X + \phi_\theta \left(\theta(s)\right)^2 + v_a \left(X^2 - P_x\right), \tag{4.8}$$

$$W^P = W_0 + \theta(s) X + v_p \left(X^2 - P_x\right) - W^A, \tag{4.9}$$

where $W_0$ is the initial wealth of the principal, $\phi_0$ is a constant, representing a fixed payment from principal to agent; $\phi_1$ is the coefficient of linear sharing, and $\theta(s)X$ is the portfolio P&L from futures trading, where $X$ is the realized futures return, $\theta(s)$ is the agent's choice of futures position depending on the signal $s$; $\phi_\theta$ is a constant, representing the amount of a quadratic reward on risky position $\theta(s)$, which is paid from the principal to the agent; $v_a$ and $v_p$ are the agent and principal's exposures to the realized variance of the futures return, where $X^2$ is the derivative contract on the realized variance of futures return, and $P_x$ is the price of such a derivative contract.

The heuristic contract in (4.8) and (4.9) nests many possibilities. For example, if only $(\phi_0, \phi_1)$ are nonzero, the heuristic contract degenerates to a linear contract which cannot effectively incentivize effort as shown in Admati and Pfleiderer (1997). When $\phi_\theta$ is nonzero, a quadratic reward on trading position is added in the contract to incentivize the agent. When $v_a$ is nonzero, the agent is exposed to the realized variance of futures return, which is paid by the principal to the agent. If $v_p = v_a$, the principle hedges the variance risk on the market; and if $v_p - v_a$ is nonzero, the principal chooses to actively expose himself to the realized variance risk, betting the signal that the agent chooses to use being correct.

We impose the heuristic rules to the optimization problem in (2.2)-(2.5), and solve for the optimal contract in each heuristic case. In Tables 4.1 - 4.2, we present the results of the optimal heuristic contracts. Each table presents the optimal solutions for one pair of risk aversion parameters $(A, P)$, respectively. Each column in the tables represents the optimal solution of coefficients $(\phi_1, \phi_\theta, v_a, v_p)$ in one heuristic rule, and a coefficient is omitted when the corresponding incentive term is not included in the heuristic contract. We do not present $\phi_0$ in the tables, as it is a residual term that can be calculated based on the values of other choice variables[11]. The optimal solutions of $(\phi_0, \phi_1, \phi_\theta, v_a, v_p)$ should be implemented as constants specified in the contract proposed to the agent.

In each table, the optimal heuristic rules are compared with the autarky solution (*"Autarky"*) in the first column and the idealized optimal contract (*"Ideal"*) in the last column. We set the expected utility of principal (*"EUP"*) and its certain equivalent (*"CE"*) of the idealized optimum as 100% and that of the autarky without hiring as 0%, and calculate the EUP and CE of each heuristic solution. Optimal effort $(e^*)$ in each case is presented in the last row of the table.

From the tables, we observe that linear contracts (*"Linear"*) in general can achieve only less than half of the effectiveness of the idealized optimum. However, adding a quadratic reward on risky positions can significantly increase the effectiveness of a heuristic contract, while adding a negative term on the realized variance can provide marginal incentives for effort. As a result, if both of these two incentive terms are imposed, the heuristic contracts can reach close to or even more than 90% of the effectiveness of the idealized optimum.

To better understand the incentive contribution of the realized variance term, we can think of several cases: when the agent accepts a contract with this negative incentive

[11] For simplicity and optimization purpose, we rewrite and use $\phi_0$ as a function of the other variables in our optimization routine, based on the participation constraint (Eq. (2.4)). Imposing the participation constraint binding, we can solve for $\phi_0$ as a function of the other choice variables.

term and chooses not to trade (i.e. $\theta(s) = 0$), he is thus exposed to additional variance risk. Irrespective of what the signal is, the variance from the commodity futures market will lead to a loss to the agent. However, when the agent chooses to trade based on a non-zero signal, this negative term of realized variance actually tilts more of the payoff towards the signal being correct, and hedges the second order risk of this bet. Thus, if the agent's signal is correct, the contract will lead to a higher payoff; and if the signal is imprecise, it will lead to a lower payoff.

As a result, the negative term on realized variance can both incentivize the agent to make effort to obtain and use more precise information, and encourage using extreme signals. In addition, when the principal chooses to actively expose to this risk (i.e. $v_p - v_a \neq 0$), the principal is leveraging up his payoff betting on the signal received and used by the agent being correct.

We further investigate the classic trade off on linear sharing ($\phi_1$) and effort ($e^*$) under these heuristic rules. For linear contracts, the agent adjusts the exposure to the same portfolio independent of $\phi_1$, which neutralizes any possible impact of $\phi_1$ on effort (Admati and Pfleiderer (1997)). As a result, $\phi_1$ in the linear case is set for optimal risk sharing given effort. For the first three columns of heuristic contracts with only negative term(s) for realized variance, the optimal effort $e^*$ increases when the terms on realized variance get more flexible, however the linear sharing $\phi_1$ is almost unchanged. Thus, the quadratic terms on the realized variance indeed incentivize effort in our model, but it does no have much impact on the risk sharing.

In contrast, for the heuristic contracts adding a reward for risk positions, both the effort and linear share of the agent increase. Thus, the reward on risky positions works not only on the effort but also through risk sharing. Intuitively, adding a non-negative reward increases the expected payoff of the agent, and if the agent's risk tolerance is thus higher, it makes the agent's optimal share in this portfolio increases. As a result, the agent has more incentives to make effort to obtain and use a more precise signal in our model. Pay attention that higher expected payoff does not always lead to a higher risk tolerance in general, as discussed in Ross (1973).

| | Autarky | Linear | Heuristic | | | | | | | | Ideal |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ | | 0.0909 | 0.0910 | 0.0875 | 0.0873 | 0.1893 | 0.1916 | 0.1718 | 0.1690 | |
| $v_p$ | | | -0.1117 | | -0.7138 | | -0.4955 | | -0.8703 | |
| $v_a$ | | | | -0.3734 | -0.4389 | | | -0.2266 | -0.3059 | |
| $\phi_\theta$ | | | | | | 5.0920 | 5.2541 | 4.8266 | 4.9950 | |
| EUP | -1 | -0.9937 | -0.9937 | -0.9903 | -0.9894 | -0.9788 | -0.9783 | -0.9776 | -0.9762 | -0.9744 |
| CE | 0 | 0.0063 | 0.0063 | 0.0097 | 0.0107 | 0.0214 | 0.0219 | 0.0227 | 0.0241 | 0.0259 |
| %EUP | 0 | 24.61 | 24.61 | 37.89 | 41.41 | 82.81 | 84.77 | 87.50 | 92.97 | 100 |
| %CE | 0 | 24.32 | 24.32 | 37.45 | 41.31 | 82.63 | 84.56 | 87.64 | 93.05 | 100 |
| $e^*$ | 0 | 0.2742 | 0.2742 | 0.4080 | 0.4315 | 0.5763 | 0.5792 | 0.6005 | 0.6142 | 0.6304 |

Table 4.1: This table presents the optimal contracts under different schemes of contracts, including the autarky case ("Autarky") where the principal does the trading himself without hiring the agent, a linear contract ("Linear"), a group of heuristic contracts ("Heuristic"), and the idealized optimal contract in the constrained problem ("Ideal"). It compares the expected utility of the principal ("EUP"), its certain equivalent ("CE"), and the achieved percentage of them in each contract if we take these of the idealized optimal contract as 100% and these of the autarky as 0%. The last row presents the optimal effort $e^*$ chosen by the agent under each contract. Parameter values $W_0 = 0$, $U_0 = -1$, $\sigma = 0.2$, $\gamma = 0.1$, cost function is $c(e) = h * e^2/(1-e)$, where $h = 0.05$. Risk aversion $A = 10$, $P = 1$.

| | Autarky | Linear | Heuristic | | | | | | | Ideal |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ | | 0.3333 | 0.3335 | 0.3320 | 0.3319 | 0.4373 | 0.4391 | 0.4335 | 0.4317 | |
| $v_p$ | | | -0.1117 | | -0.5890 | | -0.2957 | | -0.7048 | |
| $v_a$ | | | | -0.2487 | -0.4504 | | | -0.1526 | -0.3908 | |
| $\phi_\theta$ | | | | | | 3.4426 | 3.5000 | 3.4086 | 3.4905 | |
| EUP | -1 | -0.9933 | -0.9932 | -0.9929 | -0.9924 | -0.9868 | -0.9867 | -0.9867 | -0.9860 | -0.9842 |
| CE | 0 | 0.0067 | 0.0068 | 0.0071 | 0.0076 | 0.0133 | 0.0134 | 0.0134 | 0.0141 | 0.0159 |
| %EUP | 0 | 42.41 | 43.04 | 44.94 | 48.10 | 83.54 | 84.18 | 84.18 | 88.61 | 100 |
| %CE | 0 | 42.14 | 42.77 | 44.65 | 47.80 | 83.65 | 84.28 | 84.28 | 88.68 | 100 |
| $e^*$ | 0 | 0.2742 | 0.2742 | 0.2918 | 0.3061 | 0.4499 | 0.4516 | 0.4543 | 0.4650 | 0.4707 |

Table 4.2: This table presents the optimal contracts under different schemes of contracts, including the autarky case ("Autarky") where the principal does the trading himself without hiring the agent, a linear contract ("Linear"), a group of heuristic contracts ("Heuristic"), and the idealized optimal contract in the constrained problem ("Ideal"). It compares the expected utility of the principal ("EUP"), its certain equivalent ("CE"), and the achieved percentage of them in each contract if we take these of the idealized optimal contract as 100% and these of the autarky as 0%. The last row presents the optimal effort $e^*$ chosen by the agent under each contract. Parameter values $W_0 = 0$, $U_0 = -1$, $\sigma = 0.2$, $\gamma = 0.1$, cost function is $c(e) = h * e^2/(1-e)$, where $h = 0.05$. Risk aversion $A = 2$, $P = 1$.

### 4.1.1 Regression Verification of Heuristic Rules

We further verify the heuristic rules using weighted nonlinear regression of the constrained optima. We regress the constrained optimal payoffs of the agent and principal jointly on the market signal $x$. The regression equations are presented in (4.10)-(4.11), with weights equal to the distribution in the mixture model (2.1).[12]

$$y_1 = a_0 + \phi_1 \theta_s x + \phi_\theta \theta_s^2 + v_a x^2, \tag{4.10}$$

$$y_2 = b_0 + \theta_s x + v_p x^2, \tag{4.11}$$

where we use the market signal $x$ as the independent variable, the payoff for agent in the constrained optimum as the dependent variable $y_1$ and the total payoff for the principal and agent as $y_2$ to estimate the parameter set of $(\phi_1, \phi_\theta, v_a, v_p, \theta_s, a_0, b_0)$. We present the following results for the risk aversion parameter set of $(A, P) = (10, 1)$ and $(2, 1)$ as in Table 4.1-4.2 individually:

| | A=10, P=1 | A=2, P=1 |
|---|---|---|
| $\phi_1$ | 0.1582 | 0.4306 |
| $v_p$ | -0.9808 | -0.8118 |
| $v_a$ | -0.2774 | -0.4408 |
| $\phi_\theta$ | 0.0029 | 0.0016 |
| $R^2$ | 98.44% | 96.98% |
| $e^*$ | 0.6304 | 0.4707 |

Table 4.3: Weighted Nonlinear Regression

We observe that the coefficients of $\phi_1$, $v_a$, and $v_p$ come close to the solutions to heuristic contracts in Table 4.1-4.2. The sign of $\phi_\theta$ is positive which is consistent with that in the heuristic rules but the magnitude is smaller. In general, using the proposed heuristic rules can generate a very high R square in this regression.

## 4.2 Why Not Contracts Quadratic in Return?

Admati and Pfleiderer (1997) find that linear contracts cannot effectively incentivize effort as the agent can undo the slope by taking less portfolio risk exposure. Thus, a quadratic

[12]We also test nonlinear regression without weight, and the regression results of coefficients and $R^2$ come close to these of the weighted regression. The conclusion does not change.

term in return which may potentially incentivize the effort or better risk-taking would be of great interest to discuss. However, we find that contracts quadratic in return have problems too.

In this section, we discuss on potential problems of using contracts or utility functions in a quadratic form of portfolio return.[13] Assuming a contract or utility function in a quadratic form is generally unrealistic as the payoff of such contract or the utility obtained is actually unbounded if choice variables can be arbitrarily large. For example, in the problem to incentivize a trader, if the principal chooses a quadratic contract (with positive quadratic term), the agent can receive unlimited payoff by choosing an arbitrarily large portfolio $\theta(s)$. We look at the case when the agent is given a contract with positive quadratic term to incentivize effort. When the quadratic term has a zero coefficient, this quadratic contract degenerates to a linear contract as in Admati and Pfleiderer (1997) that the agent can effectively undo any incentive provided by a linear contract.

We assume that the agent's utility function is defined on both positive and negative payoffs, and the utility is strictly increasing on the payoff. Let $x$ be the return on asset, and $prob(x = 0) = 0$. We show that with such a quadratic contract, the agent can achieve arbitrarily large certain equivalent (CE), and thus the CE is unbounded.

**Theorem 4.1.** *Assume that the agent is given a quadratic contract with the payoff expressed as*

$$W_A(X;\theta) = \phi_0 + \phi_1(\theta X) + \phi_2(\theta X)^2 \tag{4.12}$$

*where $X$ is the futures return, $\phi_0$, $\phi_1$ and $\phi_2 > 0$ are the constant coefficients of the quadratic contract, and $\theta$ is the position on futures contract chosen by the agent. The expected utility of agent for a given $\theta$ is*

$$\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] = \int_{x=-\infty}^{\infty} u\left(\phi_0 + \phi_1(\theta x) + \phi_2(\theta x)^2\right) dF(x) \tag{4.13}$$

*where the utility function of agent, $u : [c_{\min}, \infty) \to \mathbb{R}$, is strictly increasing, and $c_{\min} \equiv \min_{\theta X}\{\phi_0 + \phi_1(\theta X) + \phi_2(\theta X)^2\} = \phi_0 - \frac{\phi_1^2}{4\phi_2}$. The futures return $X$ follows a distribution on $(-\infty, \infty)$, whose cumulative distribution function (CDF) is $F$, and whose probability density function (PDF) exists everywhere.*[14] *Then, the agent has no optimum.*

[13] For payoffs of general polynomial functions, things may become more subtle.

[14] Here, $x$ is not a constant, otherwise the problem of choosing $\theta$ to maximize $\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right]$ does not have a solution.

*Proof.* A general proof is in Appendix. Here we present a simplified version when $prob(X = 0) = 0$ or $X$ has no mass point.

As assumed the agent has a strictly increasing utility function and the contract payoff is $W_A = \phi_0 + \phi_1 X\theta + \phi_2 X^2\theta^2$ with $\phi_2 > 0$. For the positive quadratic payoff with full support, there exists a minimal value of certain equivalent $c_{\min}$. We show that $\forall \bar{c} \in \mathbb{R}$, there exist an $\theta$ such that

$$\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] > u\left(\bar{c}\right). \tag{4.14}$$

1. If $\bar{c} < c_{\min}$, then $u$ is strictly increasing implies that

$$\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] \geqslant u\left(c_{\min}\right) > u(\bar{c})$$

2. If $\bar{c} \geqslant c_{\min}$: choose a large enough $c^* > \bar{c}$, let $\pi = \theta X$ be the portfolio P&L, there exist $\bar{\pi} > 0$ and $\underline{\pi} < 0$, such that $W_A\left(\underline{\pi}\right) = W_A\left(\bar{\pi}\right) = c^*$, and $W_A\left(\pi\right) > c^*$ for $\pi$ outside $[\underline{\pi}, \bar{\pi}]$. Thus

$$\begin{aligned}
\mathbb{E}\left[u\left(W_A\left(X;\theta\right)\right)\right] &= \int_{x=-\infty}^{\infty} u\left(\phi_0 + \phi_1\left(\theta x\right) + \phi_2\left(\theta x\right)^2\right) dF(x) \\
&= \int_{x=-\infty}^{\underline{\pi}/\theta} u\left(\phi_0 + \phi_1\left(\theta x\right) + \phi_2\left(\theta x\right)^2\right) dF(x) \\
&\quad + \int_{x=\underline{\pi}/\theta}^{\bar{\pi}/\theta} u\left(\phi_0 + \phi_1\left(\theta x\right) + \phi_2\left(\theta x\right)^2\right) dF(x) \\
&\quad + \int_{x=\bar{\pi}/\theta}^{\infty} u\left(\phi_0 + \phi_1\left(\theta x\right) + \phi_2\left(\theta x\right)^2\right) dF(x) \\
&> u(c^*)F(\underline{\pi}/\theta) + u(c_{\min})\left(F(\bar{\pi}/\theta) - F(\underline{\pi}/\theta)\right) + u(c^*)\left(1 - F(\bar{\pi}/\theta)\right) \\
&= u(c^*)\left(1 - \left(F(\bar{\pi}/\theta) - F(\underline{\pi}/\theta)\right)\right) + u(c_{\min})\left(F(\bar{\pi}/\theta) - F(\underline{\pi}/\theta)\right) \\
&\xrightarrow[\theta\uparrow\infty]{} u(c^*) > u(\bar{c})
\end{aligned}$$

□

We further look at the expected utility for given $\theta$. If the utility for given $\theta$, which does not depend on signal $s$, has no maximum, then we won't have a solution to maximizing the expected utility by choosing $\theta(s)$ depending on $s$. As a result, for increasing $u(c)$, $X \in \mathcal{L}^2$ not a constant, the problem of choosing $\theta$ to maximize $\mathbb{E}\left[u\left(W_A\left(X;\theta\right)\right)\right]$ does not have a solution.

**Lemma 4.2.** *Assume that futures return $X$ is in the $\mathcal{L}^2$-space, i.e. it is square integrable. If the payoff function $W_A$ is quadratic with $\phi_2 > 0$ and utility function $u$ is concave, then*

$$\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] < \infty \tag{4.15}$$

*Proof.* For any $c_0$ in the support of $u(\cdot)$, we have $u(c) \leqslant u(c_0) + m(c - c_0)$ for any $m \in \nabla u(c_0)$. Since $\phi_2 > 0 \Rightarrow c_{\min} \in \mathbb{R} \Rightarrow \mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] \geqslant u\left(c_{\min}\right)$.

$$\begin{aligned}\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] &\leqslant \mathbb{E}\left[u\left(c_0\right) + m\left(c - c_0\right)\right] \\ &= u\left(c_0\right) + m\mathbb{E}\left[\phi_0 + \phi_1\left(\theta X\right) + \phi_2\left(\theta X\right)^2 - c_0\right] \\ &= u\left(c_0\right) + m\left(\phi_0 - c_0\right) + m\phi_1\theta\mathbb{E}\left[X\right] + m\phi_2\theta^2\mathbb{E}\left[X^2\right].\end{aligned}$$

which is finite, since $X \in \mathcal{L}^2$ is equivalent to $\mathbb{E}[X] < \infty$ and $\mathbb{E}[X^2] < \infty$. As a result, $\mathbb{E}\left[u\left(W_A(x;\theta)\right)\right]$ is bounded and thus smaller than infinity. □

**Theorem 4.3.** *Assume that futures return $X$ is in the $\mathcal{L}^2$-space, the payoff function $W_A$ is quadratic, utility function $u$ is increasing and concave. Let $\bar{u} \equiv \sup_c \# u(c)$ which could be infinity. Then for any $\theta$,*

$$\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] < u_{\max} \equiv (1 - \Delta F(0))\bar{u} + \Delta F(0)u(W_A(0)) \tag{4.16}$$

*and $u_{\max}$ could be infinity as well.*

*Proof.* There exist two cases:

1. If $u$ is bounded, then $\bar{u} \neq \infty$. Since $X = 0 \Rightarrow \theta X = 0 \Rightarrow W_A = W_A(0)$, we have $u_{\max} = \left(1 - prob(X = 0)\right)\bar{u} + prob(X = 0)u\left(W_A(0)\right)$.

$$\begin{aligned}\mathbb{E}\left[u\left(W_A\left(x;\theta\right)\right)\right] &= \int_{x=-\infty}^{\infty} u\left(W_A\left(x;\theta\right)\right)f(x)dx \\ &< \int_{x=-\infty}^{0-} \bar{u}f(x)dx + u(W_A(0))prob(X=0) + \int_{x=0+}^{\infty} \bar{u}f(x)dx \\ &= \bar{u}[1 - prob(X=0)] + u(W_A(0))prob(X=0) = u_{\max}\end{aligned}$$

If $prob(X = 0) \neq 0$, then $u_{\max} \neq \bar{u}$.

2. If $u$ is unbounded, thus $\bar{u} = \infty$ and $u_{\max} = \infty$.
As shown in Lemma 4.2, $\mathbb{E}\left[u\left(W_A(X;\theta)\right)\right] < \infty = u_{\max}$

□

**Theorem 4.4.** *Under the assumptions of Theorem 4.3, if the payoff function $W_A$ is strictly convex, then there exist an increasing sequence of $\{\theta_i\}$ with $\lim_{i\uparrow\infty}\theta_i = \infty$ such that*

$$\lim_{i\uparrow\infty}\mathbb{E}\left[u\left(W_A(X;\theta_i)\right)\right] = u_{\max} \tag{4.17}$$

*Proof.* As assumed $W_A$ is strictly convex, thus there exists a minimal value of certain equivalent $c_{\min}$; and for large enough $\bar{c} > c_{\min}$, there exist $\bar{\pi}$ and $\underline{\pi}$, $\bar{\pi} > 0 > \underline{\pi}$, such that $W_A(\underline{\pi}) = W_A(\bar{\pi}) = \bar{c}$, and $W_A(\pi) > \bar{c}$ for $\pi$ outside $[\underline{\pi}, \bar{\pi}]$. For any $\theta_i > 0$ in $\{\theta_i\}$,

$$\begin{aligned}
\mathbb{E}\left[u\left(W_A\left(X;\theta_i\right)\right)\right] &= \int_{x=-\infty}^{\infty} u\left(W_A(\theta_i X)\right) dF(x) \\
&= \int_{x=-\infty}^{\underline{\pi}/\theta_i} u\left(W_A(\theta_i X)\right) dF(x) + \int_{x=(\underline{\pi}/\theta_i)+}^{0-} u\left(W_A(\theta_i X)\right) dF(x) \\
&\quad + u\left(W_A(0)\right)\Delta F(0) \\
&\quad + \int_{x=0+}^{\bar{\pi}/\theta_i} u\left(W_A(\theta_i X)\right) dF(x) + \int_{x=(\bar{\pi}/\theta_i)+}^{\infty} u\left(W_A(\theta_i X)\right) dF(x) \\
&\geqslant u(\bar{c})F(\underline{\pi}/\theta_i) + u(c_{\min})\left(F(0-) - F\left((\underline{\pi}/\theta_i)+\right)\right) + u(W_A(0))\Delta F(0) \\
&\quad + u(c_{\min})\left(F(\bar{\pi}/\theta_i) - F(0)\right) + u(\bar{c})\left(1 - F((\bar{\pi}/\theta_i)+)\right) \\
&\xrightarrow[i\uparrow\infty]{} u(\bar{c})F(0-) + u(W_A(0))\Delta F(0) + u(\bar{c})(1 - F(0)) \\
&= u(\bar{c})[1 - prob(X=0)] + u(W_A(0))prob(X=0) \\
&\xrightarrow[\bar{c}\uparrow]{} u_{\max} \\
\mathbb{E}\left[u\left(W_A\left(x;\theta_i\right)\right)\right] &= \int_{x=-\infty}^{\infty} u\left(\phi_0 + \phi_1\left(\theta_i x\right) + \phi_2\left(\theta_i x\right)^2\right) f(x)dx \\
&\leqslant \int_{x=-\infty}^{0-} \bar{u}f(x)dx + u(W_A(0))prob(X=0) + \int_{x=0+}^{\infty} \bar{u}f(x)dx \\
&= \bar{u}[1 - prob(X=0)] + u(W_A(0))prob(X=0) = u_{\max}
\end{aligned}$$

As a result, $\lim_{i\uparrow\infty}\mathbb{E}\left[u\left(W_A\left(x;\theta_i\right)\right)\right] = u_{\max}$. □

To summerize, convex quadratic contracts in returns give the agent an arbitrage (in the limit) at the expense of the principal.

# 5 Idealized Benchmark: Solution

In this section, we present the details of the constrained problem and its solution. As stated before, the nice thing about the constrained problem is that using the revelation principle (Hölmstrom (1977)) allows us to solve the problem directly. This constrained problem makes sense for agents who actually need to report the obtained signal, or who can only choose from a menu of given portfolios. In this paper, it serves as a starting point to search for heuristic contracts in the trader's problem where the agent does not have the opportunity to report the obtained signal.

We formulate the constrained problem from the principal's perspective as in Ross (1973). Grossman and Hart (1983) states that in a convex programming problem there exists an optimal way to implement an action by the agent, if the agent's preferences over income lotteries are independent of action. Under the convexity condition identified in Rogerson (1985), we can adopt the first-order approach to the principal-agent problem.

## 5.1 The Constrained Problem

Based on the general setting in Section 2, we formulate the constrained problem with an additional signal reporting stage.

In the constrained problem, both the effort and the signal are private information to the agent which are unobservable to the principal. Thus, both moral hazard and adverse selection arise in the problem. [15] In order to ensure the trader exert optimal effort and optimally use the perceived signal in the trading portfolio, we need to impose incentive compatible constraints such that the optimal choice for the trader would be exactly expected by the principal in the contract design. According to the revelation principle, we can design an incentive compatible contract that includes all the possible equilibrium choice functions for the agent and the principal.

[15] We can consider three cases: The first-best case is when both the agent's effort and the obtained trading signal are observable to the principal, thus they can be fully contracted in the incentive scheme. The second-best case is when the effort is private but the signal is observable to the principal, so moral hazard problem arises on the hidden action. The solution to the first- and second-best problems are given in Appendix D. What presented in the main text is the third-best case where both the effort and signal are private information to the agent, and they are unobservable to the principal. Thus both moral hazard and adverse selection arise in the third-best case.

The principal will maximize his utility

$$\max_{u^A,u^P,e^*} \int\int u^P(x,s)\left(\pi(e^*)f^I(x,s)+(1-\pi(e^*))f^U(x,s)\right)dxds, \tag{5.18}$$

subject to the budget constraint

$$(\forall s\in S)\int\left(W_A(x,s)+W_P(x,s)\right)f(x)dx=W_0, \tag{5.19}$$

the participation constraint[16]

$$\int\int u^A(x,s)\left(\pi(e^*)f^I(x,s)+(1-\pi(e^*))f^U(x,s)\right)dxds-c(e^*)=U_0, \tag{5.20}$$

and the incentive compatible constraint

$$\{e^*,s\}=\arg\max_{e,\rho(s)}\int\int u^A(x,\rho(s))\left(\pi(e)f^I(x,s)+(1-\pi(e))f^U(x,s)\right)dxds-c(e), \tag{5.21}$$

where $W_0$ is the initial wealth of the principal; $U_0$ is the reservation utility of the agent, which means the agent will only accept the contract if the agent's utility of accepting the contract is at least $U_0$; and $e^*$ is the optimal effort, $c(\cdot)$ is the cost function of effort.

For the constrained problem, we do not impose any prespecified form for the payoff of the principal and agent. Their levels of utility are directly solved from the above problem by using the revelation priciple.

In order to solve the problem, we need to maximize the principal's expected utility which is computed from the principal's utility level in each contingency $(x,s)$, and the joint distribution of $x$ and $s$ given effort level $e^*$. Choice variables are the utility levels for the agent and the principal, and optimal effort $e^*$.

The budget constraint provides separate specifications for each signal realization $s\in S$. $W_A$ and $W_P$ are the consumption levels computed from the utility and they are valued with pricing kernel $f(x)$ in the budget constraint. Participation constraint ensures that the agent can earn more than the reservation utility level $U_0$ if accepting this contract.

We can solve directly for the form of principal's payoff as a function of budget share and pricing density. It is because we want to give the principal the optimal amount given

[16]Note here $e^*$ indicates the cost of efforts the agent makes to achieve the trading signal. Without loss of generality, we can replace the monotonic function of cost $c(e)$ with $e$.

the budget share and pricing of payoffs, and once the contract is in place the principal does not have any choices that have to be incentivised.

**Proposition 5.1.** *The solution of the expected utility conditional on $s$ for the principal in the principal's problem can be expressed as*

$$u^P(x,s) = -\exp(-PB^P(s) + \int f(x) \ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right] dx \\ - \ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right]) \quad (5.22)$$

*where*

$$B^P(s) = -\frac{1}{P}\int \ln\left(-u^P(x,s)\right) f(x)dx$$

*is the principal's budget share.*

*Furthermore, the Lagrange multiplier for budget constraint (2.3) is*

$$\lambda_B(s) = P\exp\left(-PB^P(s) + \int f(x)\ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right] dx\right). \quad (5.23)$$

*Proof.* See in Appendix B □

Hereafter we denote

$$\ln^U(x,s;e) := \ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right]$$

for simplicity. We can understand this formula as the logarithm of the conditional distribution of the principal and the agent's beliefs given $x$.

Thus $u^P(x,s)$ and $\lambda_B(s)$ can be written as

$$u^P(x,s) = -\exp\left(-PB^P(s) + \int f(x)\ln^U dx - \ln^U\right) \quad (5.24)$$

$$\lambda_B(s) = P\exp\left(-PB^P(s) + \int f(x)\ln^U dx\right). \quad (5.25)$$

*Remark* 5.2. Formula (5.24) consists of two parts: the first part is the principal's utility from the budget share that principal receives; and the second part of $\left(\int f(x)\ln^U dx - \ln^U\right)$ is a random term with mean zero, which represents the risk or uncertainty based on the principal's belief of the signal.

**Proposition 5.3.** *Given CARA utility for the agent and the principal, interest rate* $r = 0$ *and market is complete,* $B^P(s) = \int \Phi_P(x,s)f(x)dx$ *by definition can be seen as the market value of principal's payoff in state* $s$*. Because the actual payoff received by the principal is* $\Phi(x,s)$*, we can define the principal's the profit and loss (P&L) in state* $s$ *as*

$$\Phi_P(x,s) - B^P(s) = \frac{\ln^C - \int f(x) \ln^C dx}{P}. \tag{5.26}$$

*The risk-neutral expected P&L with respect to market price is zero,*

$$\int (\Phi_P(x,s) - B^P(s))f(x)dx = \frac{\int f(x) \ln^C dx - \int f(x) \ln^C dx}{P} = 0.$$

*which confirms that given* $s$*, the market price of* $\Phi(x,s)$ *is exactly* $B^P(s)$*.*

*The actual expected P&L with respect to the conditional distribution of* $f(x|s)$ *however should be positive.*

$$\int \left(\Phi_P(x,s) - B^P(s)\right) f(x|s)dx > 0 \tag{5.27}$$

*where* $f(x|s) \sim N(s, \sigma^2)$ *and* $f(x) \sim N(0, \sigma^2 + \gamma^2)$*. Here it is feasible for the agents to receive a riskless return with zero P&L, so risk aversion implies they must receive a positive expected return to accept a risky payoff.*

*If* $\pi(e) = 0$*, no effort or effort is useless, and thus the trader does not observe any signal. Without information, no action is taken by the trader. We refer to this as the benchmark case, and the P&L is*

$$\Phi_P(x,s) - B^P(s) = \frac{\ln f^C(s) - \int f(x) \ln f^C(s)dx}{P} = 0;$$

*If* $\pi(e) = 1$*, which means that the trader makes the maximum effort and the P&L is*

$$\Phi_P(x,s) - B^P(s) = \frac{-\frac{\gamma^2}{\gamma^2+\sigma^2}x^2 + 2xs + \gamma^2}{2\sigma^2 P}, \tag{5.28}$$

*which increases with the precision of the signal, that is cov(x,s) getting higher, and decreases with agent trading actively.*

*Proof.* See in Appendix E □

## 5.2 Solution to the Constrained Problem

When both the effort and signal are unobservable, we need incentive compatible constraint for agent to use the true signal to build the portfolio. We can apply the first-order approach and replace the incentive compatibility constraint (5.21) with the agent's first-order conditions for portfolio choice, evaluated at $\rho(s) = s$:

$$(\forall s \in S) \int \frac{\partial u^A(x,s)}{\partial s} \left(\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right) f(x) dx = 0 \tag{5.29}$$

With this substitution, the investor's first-order condition for $u^A$ is

$$\begin{aligned} -\frac{\lambda_B(s)}{Au^A(x,s)} &= (\lambda_P + \lambda_s'(s)) \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right] + \lambda_e \pi'(e) \left[f^I(s|x) - f^U(s)\right] \\ &+ \lambda_s(s) \left[\pi(e) \frac{\partial f^I(s|x)}{\partial s} + (1-\pi(e)) \frac{\partial f^U(s)}{\partial s}\right] := \lambda_P G(x,s;e) \end{aligned} \tag{5.30}$$

where $\lambda_s$ is the Lagrangian multiplier on the truthful reporting constraint. We define the right hand side of the above equation as $\lambda_P G(x,s;e)$. Let $\ln^G(x,s;e) := \ln(G(x,s;e))$ and $\Pi(x,s;e) := \exp(\ln^\Pi(x,s;e))$, we have

$$\ln^G = \ln \left[\Pi + \frac{\partial}{\partial s} \left\{\lambda_s(s) \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right]\right\}\right].$$

Substitute $\lambda_B$ in (5.30), take logarithm of both side and multiply them by $f(x)$, and then integrate with respect to $x$, we have

$$\int f(x) \ln^U dx + \ln \frac{P}{A} + AB^A(s) - PB^P(s) = \ln \lambda_P + \int \ln^G f(x) dx$$

Because $B^A(s) + B^P(s) = W_0$, we solve the budget share of principal as

$$B^P(s) = \frac{A}{A+P} W_0 - \frac{1}{A+P} \left[\ln \frac{A\lambda_P}{P} + \int \left[\ln^G - \ln^U\right] f(x) dx\right]. \tag{5.31}$$

*Remark* 5.4. In (5.31), $\ln^G - \ln^U$ can be seen as the "belief arbitrage", though we don't have an analytical expression of $\ln^G$.

**Numerical Solution** Because analytical solutions are not available, we switch to numerical methods to obtain the solutions numerically. We assume the joint distribution of $(x, s)$ as described before, and $cov(x, s) > 0$ under the "informed" distribution, and $cov(x, s) = 0$ under the "uninformed" distributions. This is a model of market timing, and $x$ represents the excess return of futures contract.

Let $n(\cdot; \cdot, \cdot)$ be the normal density parameterized by its mean and variance. Then $f^U(s) = n(s; 0, \gamma^2)$ is the density of $s$ in both the "informed" and "uninformed" cases, $f(x) = n(x; 0, \sigma^2 + \gamma^2)$ is the density of $x$ in the "uninformed" cases, and $f(x|s) = n(x; s, \sigma^2)$ is the conditional density of $s$ given $x$ in the "informed" case. In this setting, a signal tells maximally a fraction $\rho^2 = \frac{\gamma^2}{\sigma^2+\gamma^2}$ of the futures' excess return. With maximum effort, i.e. $e = 1$, observing the signal is like fully observing this fraction of the market, and its excess return is up by $s$ and uncertainty (or variance) is reduced by $\gamma^2$.

The parameter values used are $\sigma = 0.1$, $\gamma = 0.2$, $W_0 = 100$, $U_0 = -0.9$, and the risk-free rate $r = 0$. In general, traders although wealthy too usually have higher risk aversion than partners in an investment bank, thus we consider possible pairs of risk aversion of $(A, P)$ as $(2, 1)$, $(2, 0.2)$ or $(10, 1)$. The cost function is assumed to be $c(e) = h * e^2/(1-e)$, where $h$ is a constant coefficient. The optimal effort is set at $e = 0.4$.

We discretize $f^U(s)$, $f(x)$, and $f(x|s)$ in $N_x$ market states and $N_s$ signal states. In order to circumvent the difficulty imposed by the presence of $\lambda'_s(s)$ in this first-order condition of the constrained problem, we work with a discrete version in which the incentive constraint for truthfully using signal in futures trading is replaced by two sets of constraints. The first set imposes the restriction that using the signal just higher than the true signal is not optimal and the other does the same for using the signal just lower than the true signal. Together this makes $2(N_s - 1)$ constraints. As the discretization becomes very fine, this problem approximates the continuous state case.

The benchmark model is based on Dybvig, Farnsworth, and Carpenter (2010) except with different assumptions about preferences (CARA instead of CRRA) and the distribution of payoffs (normality of the futures price with zero mean futures return). These assumptions value short and long positions symmetric, which is a good assumption for futures traders, who are not in the business of collecting the general market risk premium. The contracting problem is formulated from the principal's perspective to maximize principal's expected utility, constrained by the budget constraint and the participation constraint that the agent accepts the contract. By revelation principle, the agent can only report the signal to an accountant to execute or choose portfolio from a menu based on the obtained signal in the contracting problem. When both effort and signal are unobservable, the agent chooses the effort and signal as principal plans under the optimal contract. Figure 5.2 plots the numerical solution of the expected utility for agent in the constrained problem, where the axes are signal $s$ and market state $x$.

We can draw a few observations from the ideal contract: First, when signal and market are both high (or both low), the agent is rewarded the most. We can observe from the figure that the expected utility is higher on the diagonal where $x$ matches with $s$. Especially, it reaches the peaks in the two corners, indicating the highest rewards for the correctly predicted extreme signals.

Second, for any given signal $s$, the expected utility exhibits a concave shape with respect to market state $x$. The observed concave reward on $x$ may come close to a quadratic bet on the signal being correct, which provides high utility if the signal predicts the market state, and low utility if the signal is imprecise. Both observations are used in proposing our heuristic contracts for the trader's problem.

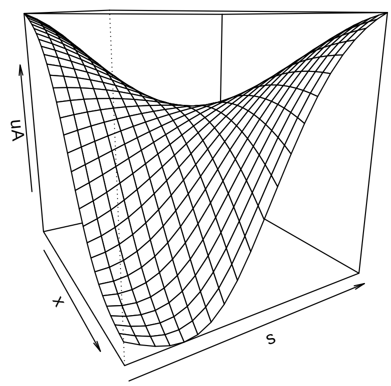


Figure 5.2: The ideal contract in the constrained problem, effort = 0.4

# 6 Conclusion

In this paper, we develop and show that heuristic contracts with linear sharing and quadratic terms can create incentives for traders (agent) that execute trades for the investment bank (principal). The primary incentive for effort comes from compensation for taking an active position, and this makes it unattractive to choose low effort and play it safe by taking little risk. Such a term also promotes better risk-sharing between the principal and agent. Additionally, adding a negative term on realized variance also helps to incentivize effort. However, contracts quadratic in return create a bad incentive for traders to take on arbitarily large risk.

We solve for optimal contracts in the constrained problem using revelation principle, based on which we have proposed above-mentioned heuristic contracts. The proposed heuristic contracts can recover approximately 90% the effectiveness of the optimal contract. The proposed heuristic rules give incentives for effort and seem easier to implement. In practice, this quadratic reward can be implemented as explicit terms written in the contract or in some implicit form of compensation, for example as a formula of calculating the end of year bonus.

In conclusion, we find that to incentivize effort and risk taking are still the key questions to address in our agency problem with CARA utility and the assumption that the agent is more risk averse than the principal. We find that a quadratic reward on risky position can effectively incentivize the effort, and a negative term on realized variance provides additional incentives for effort. In total, linear sharing plus these two incentive terms can reach close to or even more than 90% of the effectiveness of constrained optimum. We solve our problem for traders who executes trades themselves without reporting the obtained signal as in delegated expertise problem. Meanwhile, it might also be interesting to notice the potential risk management problem for traders who are less risk averse or even risk chasing, where the question of how to control risk might be of key concern to the principal, see San Martín (2018).

# A An Illustrative Example

We first illustrate the question in a simple case with two market states (bullish and bearish) and two possible signals (positive and negative) on futures return. The example shows us how a reward on risky position in the contract can create incentives for effort. The reward on risky position is also the key incentive in our proposed heuristic contracts in the general setting.

We assume a mixture model in which the probability matrix can be a mixture of uninformed $\pi^U$ and informed $\pi^I$, where

$$\pi^I = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}, \quad \text{and} \quad \pi^U = \begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}.$$

where each item $\pi_{i,j}$ in row $i$ and column $j$ denotes the probability of having futures return equals to $r_j$, and signal being $s_i$. With effort $e$, the trader's signal and return have joint distribution $\pi(e) = e\pi^I + (1-e)\pi^U$, which is

$$\pi(e) = \frac{1}{4}\begin{bmatrix} 1+e & 1-e \\ 1-e & 1+e \end{bmatrix}.$$

As a result, through making effort the agent can increase the informativeness of obtained signal. For simplicity, we assume that possible signals are $s = [-1, 1]^\top$, and futures return in two market states is $r = [-\Delta, \Delta]^\top$, where $\Delta$ is a positive constant. The agent will choose the optimal position $\theta(s)$ on futures contract for each signal $s$. As a result, the portfolio's profit and loss (P&L) matrix is $R = \theta r^\top$, and the expected profit equals to $\theta^\top \pi(e) r$.

Consider a incentive contract offering the agent with a payoff/wealth function:

$$W^A = W_0 + aR + b|\theta| + d, \tag{A.32}$$

where $W_0$, $a$, $b$ and $d$ are constants. $W_0$ is the initial wealth of the agent, $a$ is the linear sharing coefficient between the agent and the principal; $b$ is the coefficient on a reward from the principal to agent on taking risky position $\theta$; and $d$ represents a constant payment from principal to agent.

Suppose the agent has a log utility, and thus the expected utility of agent is

$$\begin{aligned}\mathbb{E}\left[U^A\right] =& \mathbb{E}\left[\log\left(W_0 + aR + b|\theta(s)| + d\right)\right] - c(e)\\ =& \mathbb{E}\left[\log\left(W_0 + a\theta(s)r + b|\theta(s)| + d\right)\right] - c(e).\end{aligned} \tag{A.33}$$

where $c(e)$ is the utility cost of effort. The agent maximizes the expected utility by choosing $\theta(s)$ and $e$. Without loss of generality, we can assume $\theta(s) = \theta^* s$, where $\theta^* > 0$.[17] Consider the first term in (A.33):

$$\begin{aligned}&\mathbb{E}\left[\log\left(W_0 + a\theta^* sr + b\theta^* + d\right)\right]\\ =&\pi_s(e)U\left(W_0 + a\theta^*\Delta + b\theta^* + d\right) + \pi_d(e)U\left(W_0 - a\theta^*\Delta + b\theta^* + d\right),\end{aligned}$$

where $\pi_s$ is the probability that signal matches with the payoff, and $\pi_d := 1 - \pi_s$ is the probability that signal does not match with the payoff.

Take the first order condition with respect to $\theta^*$,

$$\frac{\partial}{\partial\theta^*}\mathbb{E}\left[U^A\right] = 0,$$

which is

$$\pi_s(e)\left(a\Delta + b\right)U'\left(W_0 + a\theta^*\Delta + b\theta^* + d\right) = \pi_d(e)\left(a\Delta - b\right)U'\left(W_0 - a\theta^*\Delta + b\theta^* + d\right).$$

Since conditional probabilities $\pi_s(e) = (1+e)/2$ and $\pi_d(e) = (1-e)/2$, we thus have

$$\frac{(1+e)(a\Delta + b)}{W_0 + a\theta^*\Delta + b\theta^* + d} = \frac{(1-e)(a\Delta - b)}{W_0 - a\theta^*\Delta + b\theta^* + d}.$$

Simplifying the above equation, we obtain the optimal solution as follows:

$$\theta^* = \frac{(a\Delta e + b)(W_0 + d)}{a^2\Delta^2 - b^2}. \tag{A.34}$$

[17] One can show that in the two-state model, setting $\theta(-1) = -\theta(1)$ makes $\frac{\partial U^A}{\partial\theta(-1)} = -\frac{\partial U^A}{\partial\theta(1)}$.

Plug in (A.34) into agent's utility (A.33), we have

$$\begin{aligned}\mathbb{E}\left[U^A\right] =& \frac{1+e}{2}\log\left(\frac{a\Delta(1+e)(W_0+d)}{a\Delta-b}\right)+\frac{1-e}{2}\log\left(\frac{a\Delta(1-e)(W_0+d)}{a\Delta+b}\right)-c(e)\\ =& \frac{1+e}{2}\log(1+e)+\frac{1-e}{2}\log(1-e)+\frac{e}{2}\log\left(\frac{a\Delta+b}{a\Delta-b}\right)-c(e)+\log a\Delta\\ &+\log(W_0+d)-\frac{1}{2}\log\left(a^2\Delta^2-b^2\right). \end{aligned} \tag{A.35}$$

Let the cost function be

$$c(e)=h\left((1-e)\log(1-e)+e\right), \tag{A.36}$$

where the cost coefficient $h>0$. The cost function is defined on $[0,1)$. It is an increasing and convex function with $c(0)=c'(0)=0$, $c'(e)=-h\log(1-e)>0$, $c''(e)=\frac{h}{1-e}>0$.

Further, taking the first order condition of (A.35) with respect to effort $e$, we have

$$\begin{aligned}\frac{\partial}{\partial e}\mathbb{E}\left[U^A\right] &= \frac{1}{2}\log(1+e)-\frac{1}{2}\log(1-e)+\frac{1}{2}\log\left(\frac{a\Delta+b}{a\Delta-b}\right)-c'(e)\\ &= \frac{1}{2}\log(1+e)-\frac{1}{2}\log(1-e)+\frac{1}{2}\log\left(\frac{a\Delta+b}{a\Delta-b}\right)+h\log(1-e)\\ &= \frac{1}{2}\log\left(1+e\right)+(h-1/2)\log(1-e)+\frac{1}{2}\log\left(\frac{a\Delta+b}{a\Delta-b}\right) \end{aligned} \tag{A.37}$$

where $\Delta$, $h$ are positive constants as defined before; $a$ and $b$ are supposed to be positive to truly create a linear sharing with the agent, and a reward instead of a penalty to the agent, respectively; and $b<a\Delta$ is needed to make (A.37) well-defined.[18]

To obtain a solution to $\frac{\partial}{\partial e}\mathbb{E}\left[U^A\right]=0$, we consider the value of $h$. For any $h>1/2$,

[18]Consider that the part of $\mathbb{E}\left[U^A\right]$ that depends on $\theta$:

$$\begin{cases} a\theta(1)\Delta+b|\theta(1)|, & \text{when signal matches payoff}\\ -a\theta(1)\Delta+b|\theta(1)|, & \text{when signal does not match payoff}\end{cases}$$

When $b<a\Delta$ means that the reward is limited to the level of profit/loss from risk sharing. However when $b>a\Delta$, in the first case the agent is always better off taking higher risky position $\theta$, while in the second case, the agent always gets a reward larger than possible loss from the risk sharing. Thus there exists arbitrage opportunities when $b>a\Delta$.

(A.37) is positive at $e = 0$ and negative at $e = 1$. The second-order derivative:

$$\begin{aligned}\frac{\partial^2 U^A}{\partial e^2} &= \frac{1}{2}\left(\frac{1}{1+e} + \frac{1}{1-e}\right) - c''(e) \\ &= \frac{1}{2}\left(\frac{1}{1+e} + \frac{1}{1-e}\right) - \frac{h}{1-e},\end{aligned} \tag{A.38}$$

is a decreasing function of $e \in [0, 1)$. Thus there exists a unique solution to $\frac{\partial}{\partial e}\mathbb{E}\left[U^A\right] = 0$, and the optimal $e^*$ approaches to 1 when $b/a$ increases on $[0, \Delta)$. The results show that the incentive contract is effective, the contract terms $a$ and $b$ matter to the agent's choice of effort, and the agent cannot undo the incentive created by the principal.

Further when $h > 1$, (A.38) is a negative and decreasing function, thus there exists a unique solution to $\frac{\partial}{\partial e}\mathbb{E}\left[U^A\right] = 0$. The optimal $e^*$ can be any value in $[0, 1)$ and increases with $b/a \in [0, \Delta)$. As a result, the principal can effectively select any effort that the principal wants the agent to make by choosing $a$ and $b$ in the contract, and the agent cannot undo the incentive provided in the contract by the principal as in Admati and Pfleiderer (1997).

# B Proof of Proposition 3.1

*Proof.* Because $u^P$ only appears in (2.2) and (2.3), the optimal solution must solve sub-problem of (2.2) subject to (2.3).

$$\begin{aligned}\max_{u^P} \int\int & u^P(x,s)\left(\pi(e)f^I(x,s) + (1-\pi(e))f^C(x,s)\right)dxds \\ &- \int \lambda_B(s)\left[\int\left(-\frac{\ln\left(-u^A(x,s)\right)}{A} - \frac{\ln\left(-u^P(x,s)\right)}{P}\right)f(x)dx - W_0\right]ds\end{aligned} \tag{B.39}$$

where $\lambda_B$ is the Lagrange multiplier of the budget constraint. Take the first order condition with respect to $u^P$, and rearranging, gives

$$\pi(e)f^I(x,s) + (1-\pi(e))f^C(x,s) = -\lambda_B(s)\frac{f(x)}{Pu^P(x,s)}$$

As $f^I(x,s) = f(x)f^I(s|x)$ and $f^C(x,s) = f(x)f^C(s)$,

$$\pi(e)f^I(s|x) + (1-\pi(e))f^C(s) = -\lambda_B(s)\frac{1}{Pu^P(x,s)} \tag{B.40}$$

Budget constraint gives that

$$B^A(s) + B^P(s) = W_0$$

where $B^i(s) = \int -\frac{\ln(-u^i(x,s))}{i} f(x)dx$, $i = A$ or $P$.

Thus

$$\begin{aligned} B^P(s) &= W_0 - B^A(s) \\ &= W_0 + \frac{1}{A}\int \ln\left(-u^A(x,s)\right) f(x)dx \end{aligned}$$

From B.40, (take logarithm, multiply by $f(x)$, and integrate w/t $x$),

$$\lambda_B(s) = P\exp\left(-PB^P(s) + \int f(x)\ln\left[\pi(e)f^I(s|x) + (1-\pi(e))f^C(s)\right]dx\right) \tag{B.41}$$

Substitute $\lambda_B(s)$ back into the first order condition

$$\begin{aligned} u^P(x,s) &= -\frac{\lambda_B(s)}{P}\frac{1}{\pi(e)f^I(s|x) + (1-\pi(e))f^C(s)} \\ &= -\frac{\exp\left(-PB^P(s) + \int f(x)\ln\left[\pi(e)f^I(s|x) + (1-\pi(e))f^C(s)\right]dx\right)}{\pi(e)f^I(s|x) + (1-\pi(e))f^C(s)} \\ &= -\exp(-PB^P(s) + \int f(x)\ln\left[\pi(e)f^I(s|x) + (1-\pi(e))f^C(s)\right]dx \\ &\quad - \ln[\pi(e)f^I(s|x) + (1-\pi(e))f^C(s)]) \end{aligned}$$

□

# C Proof of Proposition 5.3

*Proof.* By CARA property, $u^P(\cdot)$ is a concave increasing function. Suppose $E\left[\Phi_P(x,s)|s\right] \leqslant B^P(s)$ and $\Phi_P(x,s)$ is not constant conditional on $s$. Thus

$$\begin{aligned} E\left[u^P(\Phi_P(x,s))|s\right] &< u^P\left(E\left[\Phi_P(x,s)|s\right]\right) && \text{by Jensen's inequality} \\ &\leqslant u^P\left(B^P(s)\right) && \text{by monotonity of } u^P \end{aligned} \tag{C.42}$$

which contradicts $\Phi_P$ is optimal for the principal.

If $\pi(e) = 1$:

$$\Phi_P(x,s) - B^P(s) = \frac{\ln f^I(s|x) - \int f(x) \ln f^I(s|x) dx}{P}. \tag{C.43}$$

where $f(x) = \frac{1}{\sqrt{2\pi(\gamma^2+\sigma^2)}} \exp -\frac{x^2}{2(\gamma^2+\sigma^2)}$ and $f^I(s|x) = \frac{1}{\sqrt{2\pi\gamma^2\sigma^2/(\gamma^2+\sigma^2)}} \exp\left[-\frac{\left(s-\frac{\gamma^2}{\gamma^2+\sigma^2}x\right)^2}{2\gamma^2\sigma^2/(\gamma^2+\sigma^2)}\right]$. Thus we have

$$\ln f^I(s|x) = -\frac{\left(s - \frac{\gamma^2}{\gamma^2+\sigma^2}x\right)^2}{2\gamma^2\sigma^2/(\gamma^2+\sigma^2)} - \frac{1}{2}\ln\left(2\pi\frac{\gamma^2\sigma^2}{\gamma^2+\sigma^2}\right), \tag{C.44}$$

where the constant term of $-\frac{1}{2}\ln\left(2\pi\frac{\gamma^2\sigma^2}{\gamma^2+\sigma^2}\right)$ will be canceled in $R^I$, so we only need to care about

$$F(x,s) := -\frac{\left(s - \frac{\gamma^2}{\gamma^2+\sigma^2}x\right)^2}{2\gamma^2\sigma^2/(\gamma^2+\sigma^2)}.$$

Substitute the above terms and simplify (C.43), we get

$$\begin{aligned} \Phi_P(x,s) - B^P(s) &= \frac{F(x,s) - \int F(x,s) f(x) dx}{P} \\ &= \frac{\int f(x)\left(\frac{\gamma^4}{(\gamma^2+\sigma^2)^2}x^2 - \frac{2s\gamma^2}{\gamma^2+\sigma^2}x\right)dx - \left(\frac{\gamma^4}{(\gamma^2+\sigma^2)^2}x^2 - \frac{2s\gamma^2}{\gamma^2+\sigma^2}x\right)}{P}\frac{(\gamma^2+\sigma^2)}{2\gamma^2\sigma^2} \\ &= \frac{\gamma^2 - \frac{\gamma^2}{\gamma^2+\sigma^2}x^2 + 2xs}{2P\sigma^2}. \end{aligned} \tag{C.45}$$

□

# D First-best and Second-best solution of the constrained problem

In the benchmark problem, we can consider three cases: The first-best case is when both the agent's effort and the obtained trading signal are observable to the principal, thus they can be fully contracted in the incentive scheme. The second-best case is when the effort is private but the signal is observable to the principal, so moral hazard problem arises on the hidden action. Finally, the third-best case is when both the effort and signal are private information to the agent, and they are unobservable to the principal. Thus both moral hazard and adverse selection arise in the third-best case.

In formulating the three problems, the difference occurs in the incentive compatible constraint. In the first-best case, given everything is observable to the principal and they can be fully contracted, we do not need the additional incentive compatible constraint at all. In the second-best case, only effort is unobservable to the principal, thus an incentive compatibility condition is needed to incentivise the trader to choose the optimal effort $e^*$. Equation (5.21) should be replaced by

$$e^* = \arg\max_e \int\int u^A(x,s)\left(\pi(e)f^I(x,s) + (1-\pi(e))f^U(x,s)\right)dxds - c(e). \quad \text{(D.46)}$$

Analytical solution exist for the first- and second-best case, however, there is only numerical solution to the third-best, when both effort and signal are unobservable to the principal.

## D.1 First-best

In the first-best contract, we have the payoff for the trader:

**Proposition D.1.** *Agent's fee in the first-best case is*

$$\Phi(x,s) = \frac{P}{A+P}W_0 + \frac{1}{A+P}\left(\ln\frac{A\lambda_P}{P} + \int f(x)\left(\ln^U - \ln^C\right)dx\right) + \frac{1}{A}\left(\ln^U - \int f(x)\ln^U dx\right), \quad \text{(D.47)}$$

*where $\lambda_P$ is the Lagrange multiplier on the participation constraint.* $\ln^U - \ln^C$ *can be seen* *as a "belief arbitrage", which comes from the difference of beliefs between the agent and the principal. The last term* $\frac{1}{A}(\ln^U - \int f(x)\ln^U dx)$ *is a random term with mean zero, which represents the risk or uncertainty based on the uninformed belief.*

*Proof.* See Appendix E □

*Remark* D.2. $\Phi_A$ can be written as $\Phi_A = B^A + \frac{1}{A}\left(\ln^U - \int f(x)\ln^U dx\right)$. Thus we define the P&L for the agent as $\Phi_A - B^A$, which is

$$\Phi_A(x,s) - B^A(s) = \frac{\ln^U - \int f(x)\ln^U dx}{A}. \tag{D.48}$$

The total portfolio P&L is

$$\text{portfolio P\&L} = \frac{\ln^C - \int f(x)\ln^C dx}{P} + \frac{\ln^U - \int f(x)\ln^U dx}{A}. \tag{D.49}$$

The portfolio has a positive expected P&L with respect to $f(x|s)$, and without superior information ($\pi(e) = 0$), the portfolio has zero P&L.

*Remark* D.3. If we switch off the difference in beliefs (when $k = 1$), then the budget share for the agent will be

$$B^A = \frac{P}{A+P}W_0 + \frac{1}{A+P}\ln\frac{A\lambda_P}{P}, \tag{D.50}$$

and the payoff is

$$\Phi_A(x,s) = \frac{P}{A+P}W_0 + \frac{1}{A+P}\ln\frac{A\lambda_P}{P} + \frac{1}{A}\left(\ln^U - \int f(x)\ln^U dx\right). \tag{D.51}$$

Thus the signal affects the payoff for the agent but not the budget sharing.

## D.2 Second-best

In the second-best, only effort is private to the agent. Thus, we need an incentive compatibility constraint of effort in the optimization problem:

$$e^* = \arg\max_e \int\int u^A(x,s)\left(\pi(e)f^I(x,s) + (1-\pi(e))f^U(x,s)\right) dxds - c(e). \tag{D.52}$$

Because the agent's utility is affine in effort and the cost of effort is convex, we can adopt the first-order approach and use the following sufficient first-order condition of the effort incentive compatibility in the optimization problem

$$\int\int u^A(x,s)\pi'(e)\left(f^I(s|x)-f^U(s)\right)f(x)dxds-c'(e)=0. \tag{D.53}$$

The first-order condition for $u^A$ is

$$-\frac{\lambda_B(s)}{Au^A(x,s)}=\lambda_P\left[\pi(e)f^I(s|x)+(1-\pi(e))f^U(s)\right]+\lambda_e\pi'(e)\left[f^I(s|x)-f^U(s)\right]$$

where $\lambda_e$ is the Lagrange multiplier on the incentive constraint of effort.

**Proposition D.4.** *The second-best contract gives the agent a payoff*

$$\Phi(x,s)=\frac{P}{A+P}W_0+\frac{1}{A+P}\left(\ln\frac{A\lambda_P}{P}+\int f(x)\left(\ln^{\Pi}-\ln^{C}\right)dx\right)$$
$$+\frac{1}{A}\left(\ln^{\Pi}-\int f(x)\ln^{\Pi}dx\right)$$

*where*

$$\ln^{\Pi}(x,s;e):=\ln\left[\pi(e)f^I(s|x)+(1-\pi(e))f^U(s)+\frac{\lambda_e}{\lambda_P}\pi'(e)\left(f^I(s|x)-f^U(s)\right)\right].$$

*The budget share for agent is*

$$B^A(s)=\frac{P}{A+P}W_0+\frac{1}{A+P}\left(\ln\frac{A\lambda_P}{P}+\int f(x)\left(\ln^{\Pi}-\ln^{C}\right)dx\right).$$

*Proof.* See in Appendix F □

*Remark* D.5. In the second-best, the "belief arbitrage" term is $\ln^{\Pi}-\ln^{C}$. Comparing with the first-best, the belief arbitrage term in the second-best rewards the agent when $f^I(s|x)-f^U(s)>0$ (by definition of $\ln^{\Pi}$), which gives incentives for the agent to make effort.

*Remark* D.6. In the second-best, the P&L for the agent is

$$\Phi_A(x,s)-B^A(s)=\frac{\ln^{\Pi}-\int f(x)\ln^{\Pi}dx}{A}. \tag{D.54}$$

The total portfolio P&L is

$$\text{portfolio P\&L} = \frac{\ln^C - \int f(x) \ln^C dx}{P} + \frac{\ln^\Pi - \int f(x) \ln^\Pi dx}{A}. \tag{D.55}$$

The portfolio has a positive expected P&L with respect to $f(x|s)$, and without superior information ($\pi(e) = 0$), the portfolio has zero P&L.

*Remark* D.7. If we switch off the difference in beliefs (when $k = 1$), then the budget share for the agent will be

$$B^A(s) = \frac{P}{A+P} W_0 + \frac{1}{A+P}\left(\ln \frac{A\lambda_P}{P} + \int f(x) \left(\ln^\Pi - \ln^U\right) dx\right). \tag{D.56}$$

Thus without belief difference, budget share in the second-best is a function of $s$ and the sharing rule incentivize the agent to make effort.

# E Proof of Proposition D.1

*Proof.* The first order condition for $u^A$

$$-\frac{\lambda_B(s)}{Au^A(x,s)} = \lambda_P \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right] \tag{E.57}$$

where $\lambda_P$ is the Lagrange multiplier on the participation constraint.

Substitute $\lambda_B(s)$ as in 5.25 gives

$$\frac{P \exp\left(-PB^P(s) + \int f(x) \ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^C(s)\right] dx\right)}{A\left(-u^A(x,s)\right)} \\ = \lambda_P \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right]$$

Take natural logarithm, then multiply by $f(x)$, and integrate with respect to $x$ gives

$$-PB^P(s) + \int f(x) \ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^C(s)\right] dx \\ + \ln P - \ln A - \int f(x) \ln\left(-u^A(x,s)\right) dx \\ = \ln \lambda_P + \int f(x) \ln\left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right] dx$$

$AB^A(s) = -\int f(x) \ln \left(-u^A(x,s)\right) dx$ gives

$$B^A(s) = \frac{P}{A} B^P(s) + \frac{1}{A} \ln \frac{A\lambda_P}{P} + \frac{1}{A} \int f(x) \left[\ln^U - \ln^C\right] dx$$

Since $B^A(s) + B^P(s) = W_0$,

$$\begin{aligned} B^P(s) &= \frac{1}{A+P} \left[ AW_0 - \ln \frac{A\lambda_P}{P} - \int f(x) \left[\ln^U - \ln^C\right] dx \right] \\ &= \frac{A}{A+P} W_0 - \frac{1}{A+P} \left( \ln \frac{A\lambda_P}{P} + \int f(x) \left[\ln^U - \ln^C\right] dx \right) \end{aligned} \tag{E.58}$$

$$B^A(s) = \frac{P}{A+P} W_0 + \frac{1}{A+P} \left( \ln \frac{A\lambda_P}{P} + \int f(x) \left[\ln^U - \ln^C\right] dx \right). \tag{E.59}$$

Thus

$$\begin{aligned} u^A(x,s) &= -\frac{P \exp\left[-PB^P(s) + \int f(x) \ln \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^C(s)\right] dx\right]}{A\lambda_P \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right]} \\ &= -\frac{P \exp\left(\frac{A}{A+P} \int f(x) \ln^C dx - \frac{P}{A+P} \left(AW_0 - \ln \frac{A\lambda_P}{P} - \int f(x) \ln^U dx\right)\right)}{A\lambda_P \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right]}. \end{aligned}$$

Take the inverse function of $u^A(\Phi)$, the function of the agent's fee is obtained. □

# F Proof of Proposition D.4

*Proof.* In the first-order condition for $u^A$

$$\begin{aligned} -\frac{\lambda_B(s)}{Au^A(x,s)} &= \lambda_P \left[\pi(e) f^I(s|x) + (1-\pi(e)) f^U(s)\right] + \lambda_e \pi'(e) \left[f^I(s|x) - f^U(s)\right] \\ &= \lambda_P f^U(s) + \left[\lambda_P \pi(e) + \lambda_e \pi'(e)\right] \left[f^I(s|x) - f^U(s)\right] \end{aligned} \tag{F.60}$$

Substitute $\lambda_B(s)$

$$\frac{P \exp\left(-PB^P(s) + \int f(x) \ln^C dx\right)}{A\left(-u^A(x,s)\right)} = \lambda_P f^U(s) + \left[\lambda_P \pi(e) + \lambda_e \pi'(e)\right] \left[f^I(s|x) - f^U(s)\right]$$

Take logarithm on both sides, multiply by $f(x)$ and integrate with respect to $x$:

$$
\begin{aligned}
&-PB^P(s) + \int f(x)\ln^C dx + \ln P - \ln A + AB^A(s) \\
&\qquad = \ln\lambda_P + \int \ln\left[f^U(s) + \left(\pi(e) + \frac{\lambda_e}{\lambda_P}\pi'(e)\right)\left(f^I(s|x) - f^U(s)\right)\right] f(x)dx,
\end{aligned}
$$

which equals to

$$
\begin{aligned}
B^A(s) = &\frac{P}{A}B^P(s) + \frac{1}{A}\ln\frac{A\lambda_P}{P} - \frac{1}{A}\int f(x)\ln^C dx \\
&+ \frac{1}{A}\int \ln\left[f^U(s) + \left(\pi(e) + \frac{\lambda_e}{\lambda_P}\pi'(e)\right)\left(f^I(s|x) - f^U(s)\right)\right] f(x)dx.
\end{aligned}
$$

We define

$$
\ln^\Pi := \ln\left[f^U(s) + \left(\pi(e) + \frac{\lambda_e}{\lambda_P}\pi'(e)\right)\left(f^I(s|x) - f^U(s)\right)\right]. \tag{F.61}
$$

Because $B^A(s) + B^P(s) = W_0$, we have

$$
B^P(s) = \frac{A}{A+P}W_0 + \frac{1}{A+P}\left[\ln\frac{P}{A\lambda_P} + \int\left(\ln^C - \ln^\Pi\right) f(x)dx\right] \tag{F.62}
$$

From F.60,

$$
u^A(x,s) = \frac{-P\exp\left(-PB^P(s) + \int f(x)\ln^C dx\right)}{A\left(\lambda_P f^U(s) + [\lambda_P\pi(e) + \lambda_e\pi'(e)]\left[f^I(s|x) - f^U(s)\right]\right)}
$$

Substitute $B^P(s)$ in (F.62), and $u^A = -\exp\left(-A\Phi(x,s)\right)$,

$$
\begin{aligned}
\Phi(x,s) =& \frac{P}{A}B^P(s) - \frac{1}{A}\int f(x)\ln^C dx - \frac{1}{A}\ln\frac{P}{A\lambda_P} + \frac{1}{A}\ln^\Pi \\
=& \frac{P}{A+P}W_0 + \frac{1}{A+P}\left(\ln\frac{A\lambda_P}{P} + \int f(x)\left(\ln^\Pi - \ln^C\right)dx\right) \\
&+ \frac{1}{A}\left(\ln^\Pi - \int f(x)\ln^\Pi dx\right).
\end{aligned}
$$

□

## G Heuristic contract

| | Benchmark | Linear | Heuristic | | | | | | | | Ideal |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ | | 0.0909 | 0.0910 | 0.0875 | 0.0873 | 0.1893 | 0.1916 | 0.1718 | 0.1690 | |
| $\bar{v}_p$ | | | -0.5583 | | -3.5690 | | -2.4774 | | -4.3513 | |
| $\bar{v}_a$ | | | | -1.8671 | -2.1945 | | | -1.1329 | -1.5295 | |
| $\phi_\theta$ | | | | | | 1.0184 | 1.0508 | 0.9653 | 0.9990 | |
| EUP | -1 | -0.9937 | -0.9937 | -0.9903 | -0.9894 | -0.9788 | -0.9783 | -0.9776 | -0.9762 | -0.9744 |
| CE | 0 | 0.0316 | 0.0316 | 0.0487 | 0.0533 | 0.1071 | 0.1097 | 0.1133 | 0.1204 | 0.1297 |
| %EUP | 0 | 24.61 | 24.61 | 37.89 | 41.41 | 82.81 | 84.77 | 87.50 | 92.97 | 100 |
| %CE | 0 | 24.36 | 24.36 | 37.55 | 41.09 | 82.58 | 84.58 | 87.36 | 92.83 | 100 |
| $e^*$ | 0 | 0.2742 | 0.2742 | 0.4080 | 0.4315 | 0.5763 | 0.5792 | 0.6005 | 0.6142 | 0.6304 |

Table G.4: This table lists the benchmark case (Benchmark) where the principal does the trading without hiring the agent, a linear contract (Linear), a group of heuristic contracts (Heuristic), and the third-best optimum in the constrained problem (Ideal). It compares expected utility of the principal (EUP), the certain equivalence (CE), and the achieved percentage of them in each contract if we take the third-best ideal case as 100% and the benchmark as 0%. The last row presents the optimal effort $e^*$ chosen by the agent under each contract. Parameter values $W_0 = 0$, $U_0 = -1$, $\sigma = 0.2$, $\gamma = 0.1$, $k = 1$, cost function is $c(e) = h * e^2/(1-e)$, where $h = 0.05$. Risk aversion $A = 2$, $P = .2$.

# H Numerical Solution of the Third-best

The optimal solution of the agent's utilities in the first-best is displayed in Figure H.3. With a higher signal, the portfolio position on the futures contracts is larger.

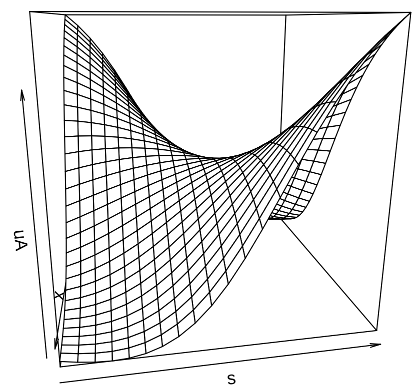


Figure H.3: Agent's utility in the first-best, effort = 0.4

The figures of agent's utility in the second- and third-best look very similar to that in the first-best. However, examining the incremental changes in the contract from first-best to second-best to third-best can provide us insightful observations. Figure H.4 displays the change of agent's utilities in the second-best minus that in the first-best. When signal "$s$" and market state "$x$" are both high (or both low), i.e. $f^I(x|s) > f^U(s)$, we observe that the agent is rewarded most in those states. Whereas in the other corners of the distribution, the agent has lower utility than in the first-best case which can be seen as a penalty for imprecise signal. The additional reward to correct signal and penalty for imprecise signal offered by the optimal contract in the second-best provides agent the incentive to exert effort.

Figure H.5 further displays the change of the agent's utilities in the third-best minus that in the second-best. The difference between these two contracts is that the third-best provides additional incentives to use the true signal. Fix $x$, we can observe that the plot is very convex with respect to $s$, indicating that the third-best contract provides extra rewards for reporting more extreme signals compared to the second-best.

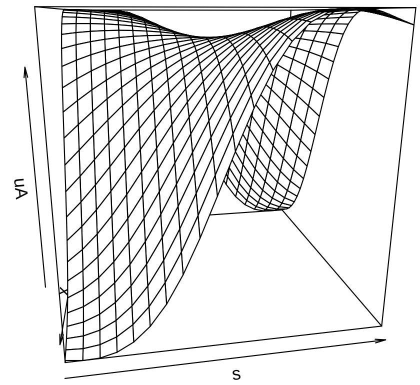


Figure H.4: Agent's utility in the second-best minus that in the first-best, effort = 0.4

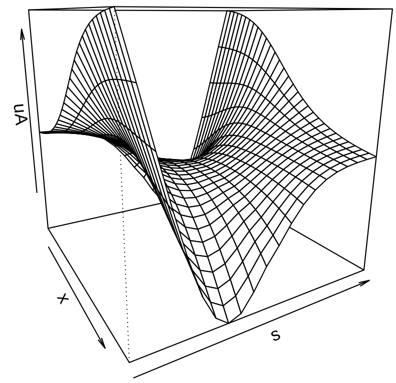


Figure H.5: Agent's utility in the third-best minus that in the second-best, effort = 0.4

The intuition behind this is straightforward. In order to induce an effort to generate a quality signal, the second-best contract overexposes the agent to the signal's optimal portfolio. However in the third-best, an agent who could misuse the signal would therefore have incentive to use a more conservative signal in order to partially undo this exposure. To address this problem, the third-best contract must provide an additional component of compensation that rewards the agent for reporting more extreme signals.

The utility of the principal is displayed in Figure H.6. As the optimal budget share of the principal in (5.31) consists of a linear sharing plus a belief arbitrage, we can see that utility has a similar shape for the principal given both agent and principal have a CARA utility.

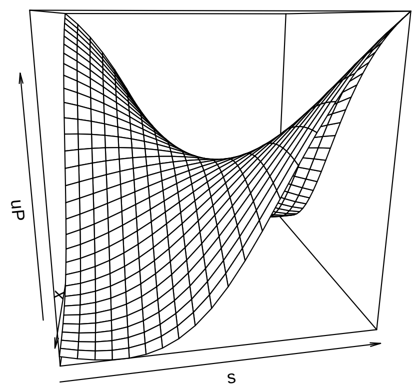


Figure H.6: Principal's utility in the second-best, effort=0.4

# I Consumption Share

In this part, we look at the consumption share for the principal and agent with and without different beliefs, and investigate how belief difference affect the budget sharing.

**The case without different beliefs.** Figure I.7 - I.9 plot the agent's consumption share of the total portfolio when $k = 1$ and $e = 0.3$ for the first-, second- and third-best respectively. The horizontal axis is the total consumption and the vertical axis is the agent's consumption.

We plot the budget share for different signals. Remark D.3 tells us that when there is no belief difference, the first-best optimal consumption of the agent is a linear function of the total consumption, and budget share $sh^A(s) \equiv B^A/W_0 = \frac{PW_0+\ln A\lambda_P-\ln P}{(A+P)W_0}$ is a constant. However in the second- and third-best, the agent's consumption share is close to but not equal to linearity.

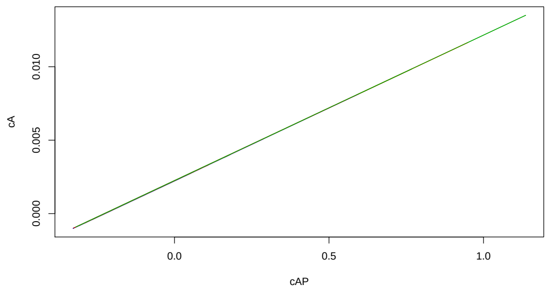


Figure I.7: Agent's consumption versus total consumption, first-best, effort=0.3

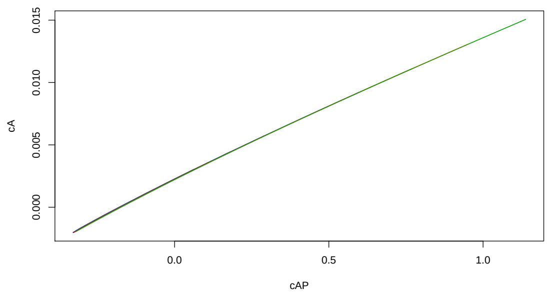


Figure I.8: Agent's consumption versus total consumption, second-best, effort=0.3

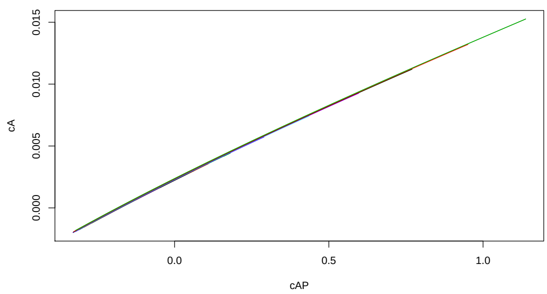


Figure I.9: Agent's consumption versus total consumption, third-best, effort=0.3

**The case with different beliefs.** Figure I.10 - I.12 plot the agent's consumption share of the total portfolio when $k = 100$ and $e = 0.3$ for the first-, second- and third-best respectively. The horizontal axis is the total consumption and the vertical axis is the agent's consumption.

We plot the budget share for different signals. Each curve, except for $s = 0$, represents two different signal values, since $s$ and $-s$ are symmetric cases swapping long futures and short futures. In the first- and second-best, the highest curve is for the neutral signal $s = 0$. No matter what is the actual signal, the manager would like to report the neutral signal if faced with the first- or second-best signal in a third-best world. For the third-best, the curves are no longer ordered and the agent can self-select to the correct curve corresponding to the signal.

The budget sharing between the principal and agent is still somewhat close to linearity, based on which more implications of the optimal budget sharing will be explored and heuristic contracts will be designed accordingly in the next section.

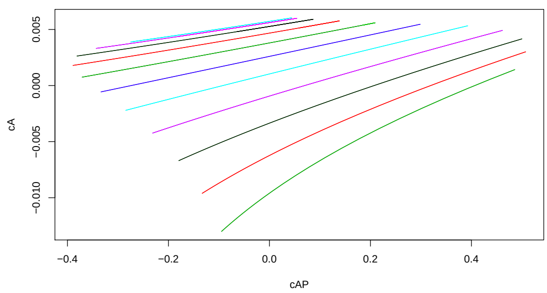


Figure I.10: Agent's consumption versus total consumption, first-best, effort=0.3

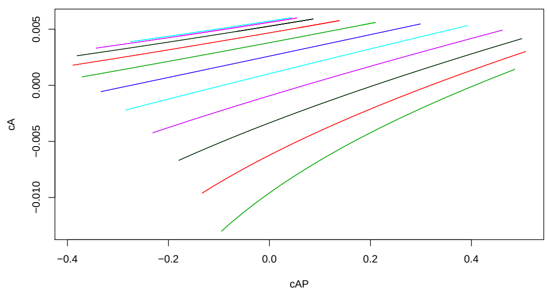


Figure I.11: Agent's consumption versus total consumption, second-best, effort=0.3

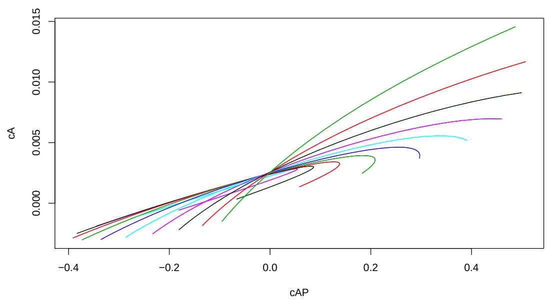


Figure I.12: Agent's consumption versus total consumption, third-best, effort=0.3

# J Optimal Effort

We search for the optimal effort for agent in each case. The cost function we take is

$$c(e) = e^2/(1-e) \tag{J.63}$$

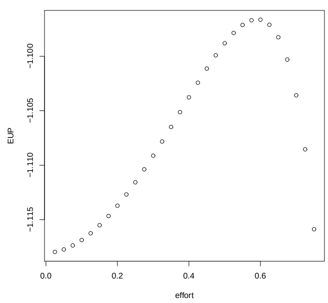


Figure J.13: first-best case

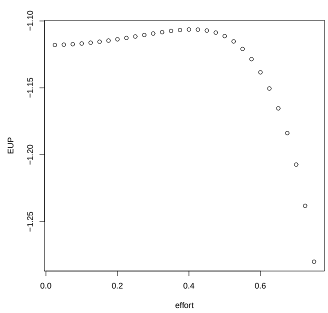


Figure J.14: second-best case

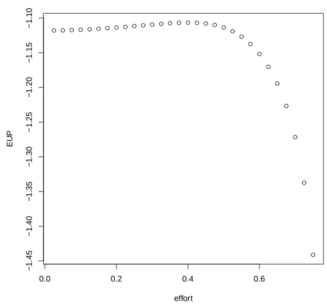


Figure J.15: third-best case

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